Initial conditions in heavy ion collisions

IV – Final state evolution, Thermalization

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General outline

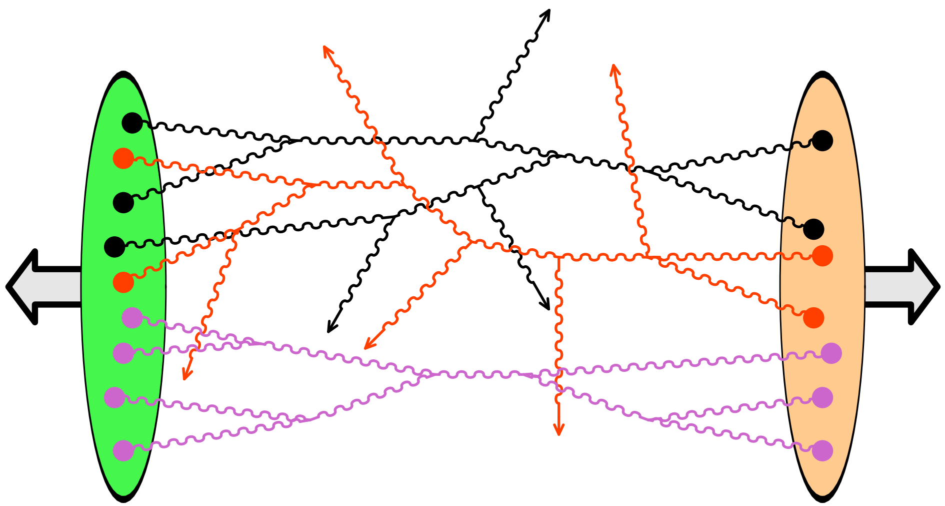
- Lecture I : Gluon production by external sources
- Lecture II : Leading Order description (T. Lappi)
- Lecture III : Next to Leading Order, Factorization (T. Lappi)
- Lecture IV : Final state evolution, Thermalization

Lecture IV : Final state evolution

- Reminder on gluon production
- Probing early dynamics via correlations
- Glasma instabilities
- Possible thermalization scenario
- Resummation of unstable fluctuations
- Open issues

Reminder: gluon production

Relevant graphs in the saturated regime



- Dilute regime : one parton in each projectile interact
- Dense regime : multiparton processes become crucial (+ pileup of many simultaneous scatterings)

Single gluon spectrum at LO

Krasnitz, Nara, Venugopalan (1999 – 2001), Lappi (2003)

- Expansion in g^2 :

$$\frac{dN}{d^3\vec{\boldsymbol{p}}} = \frac{1}{g^2}\left[c_0 + c_1\, g^2 + c_2\, g^4 + \cdots\right]$$

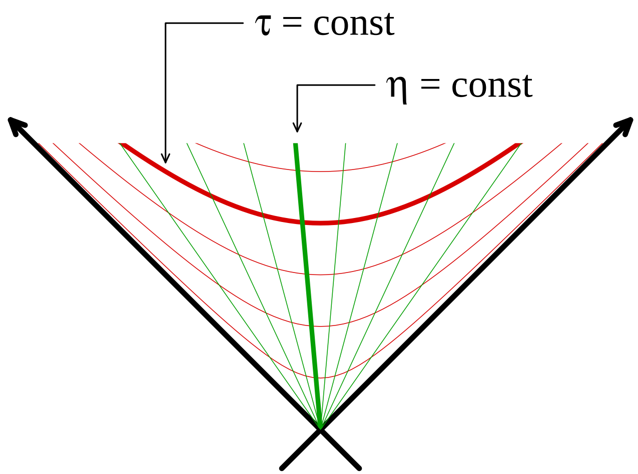
- The gluon spectrum at LO is given by :

$$\left.\frac{dN}{d^3\vec{\boldsymbol{p}}}\right|_{\rm LO} \equiv \frac{c_0}{g^2} \propto \int_{x,y} e^{ip\cdot(x-y)} \cdots \mathcal{A}_\mu(x)\mathcal{A}_\nu(y)$$

- $\mathcal{A}_\mu(x)$ is the retarded solution of Yang-Mills equations :

$$\begin{cases} \left[\mathcal{D}_\mu, \mathcal{F}^{\mu\nu}\right] = J^\nu \\ \lim\limits_{t\to-\infty} \mathcal{A}^\mu(t,\vec{\boldsymbol{x}}) = 0 \end{cases}$$

Boost invariance



- Initial values at $\tau = 0^+$: the initial fields $\mathcal{A}_{\rm in}$ do not depend on the rapidity η

  ▷ they remain independent of η at all times (invariance under boosts in the z direction)

  ▷ numerical resolution performed in $1+2$ dimensions

JIMWLK factorization

- Naive loop expansion :

$$\frac{dN}{d^3\vec{\boldsymbol{p}}} = \frac{1}{g^2}\left[c_0 + c_1\, g^2 + c_2\, g^4 + \cdots\right]$$

- Problem : $c_{1,2,\dots}$ contain logarithms of $1/x_{1,2}$:

$$\begin{aligned} c_1 &= c_{10} + c_{11}\ln\left(\frac{1}{x_{1,2}}\right) \\ c_2 &= c_{20} + c_{21}\ln\left(\frac{1}{x_{1,2}}\right) + \underbrace{c_{22}\ln^2\left(\frac{1}{x_{1,2}}\right)}_{\text{Leading Log terms}} \end{aligned}$$

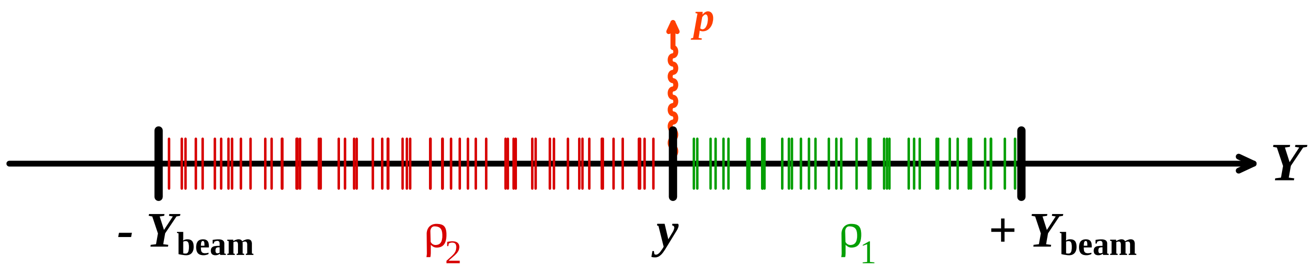
- At small $x_{1,2}$, these logs are large, and one should resum all the terms that have as many logs as powers of g^2

JIMWLK factorization

- For the single gluon spectrum in AA collisions, one can establish a formula such as :

$$\left\langle \frac{dN}{d^3\vec{\boldsymbol{p}}} \right\rangle \underset{\rm LLog}{=} \int \big[D\rho_1\, D\rho_2\big]\; W_{Y_{\rm beam}-y}[\rho_1]\; W_{y+Y_{\rm beam}}[\rho_2]\;\; \frac{dN}{d^3\vec{\boldsymbol{p}}}\Big|_{\rm LO}$$

with $\frac{\partial}{\partial Y} W_Y = \mathcal{H}\, W$



  - All the leading logs of $1/x_{1,2}$ are absorbed in the $W's$
  - The $W's$ obey the JIMWLK evolution equation

NLO corrections

- The NLO corrections can be written as :

$$\left.\frac{dN}{d^3\vec{\boldsymbol{p}}}\right|_{\rm NLO} = \Big[\frac{1}{2}\int_{\vec{\boldsymbol{u}},\vec{\boldsymbol{v}}\in\Sigma} \mathcal{G}(\vec{\boldsymbol{u}},\vec{\boldsymbol{v}})\,\mathbb{T}_{\boldsymbol{u}}\mathbb{T}_{\boldsymbol{v}} + \int_{\vec{\boldsymbol{u}}\in\Sigma} \boldsymbol{\beta}(\vec{\boldsymbol{u}})\cdot\mathbb{T}_{\boldsymbol{u}}\Big]\;\left.\frac{dN}{d^3\vec{\boldsymbol{p}}}\right|_{\rm LO}$$

- The operator $\mathbb{T}_{\boldsymbol{u}}$ is the generator of shifts of the initial value of the fields on the light-cone :

$$\mathcal{F}[\mathcal{A}_{\rm initial}+a] \equiv \exp\Big[\int_{\vec{\boldsymbol{u}}\in\Sigma} a(u)\cdot\mathbb{T}_{\boldsymbol{u}}\Big]\;\mathcal{F}[\mathcal{A}_{\rm initial}]$$

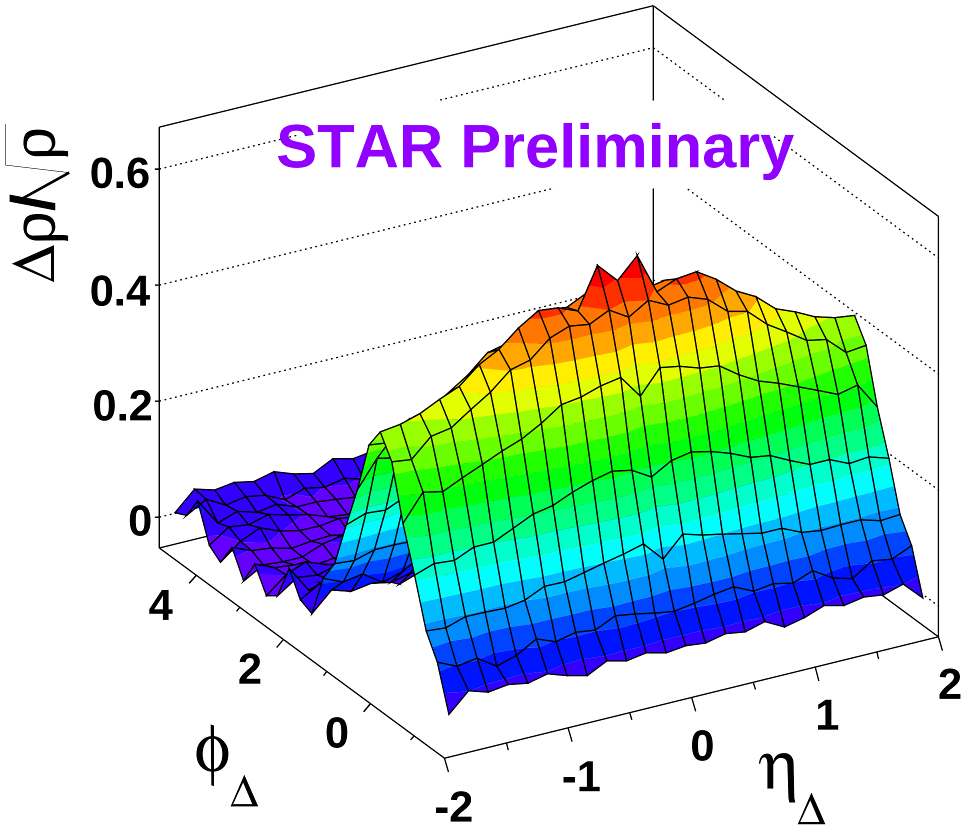
- It can be used to express fluctuations in terms of their initial value :

$$a^\mu(x) = \underbrace{\Big[\int_{\vec{\boldsymbol{u}}\in\Sigma} a(u)\cdot\mathbb{T}_{\boldsymbol{u}}\Big]}_{\text{initial condition}}\;\mathcal{A}^\mu(x)$$

Probing early dynamics via correlations

What is the ridge?

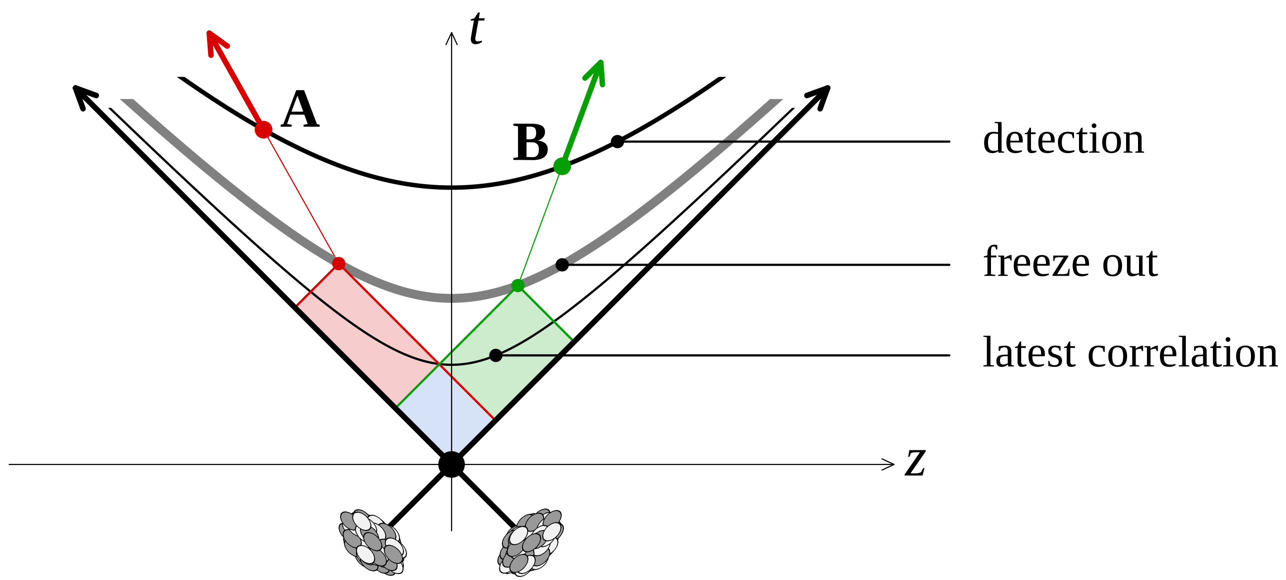
- 2-hadron correlation function in AA collisions :



- Narrow correlation in $\Delta\varphi$
- Long range correlation in $\Delta\eta$

Super-horizon correlations

- Long range correlations in rapidity probe early dynamics :

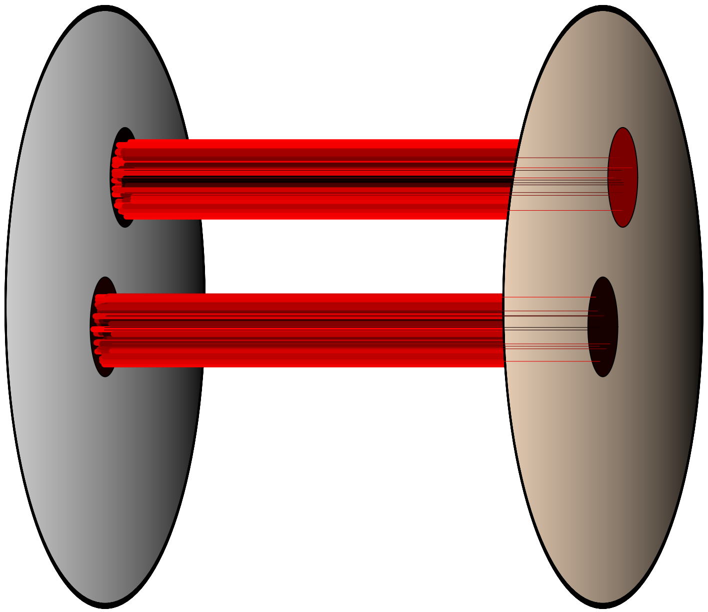


- If a correlation is measured between particles separated by ΔY, the process that caused the correlation must have occured before

$$\tau_{\rm max} = \tau_{\rm freeze\ out}\ \ e^{-\frac{1}{2}|\Delta Y|}$$

Glasma interpretation

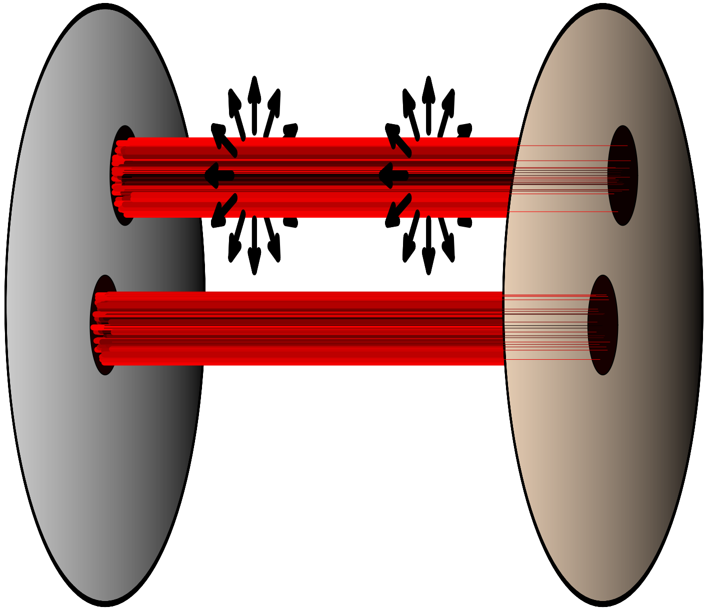
- Was there something independent of η at early times?
  - ▷ the chromo-$\vec{\boldsymbol{E}}$ and $\vec{\boldsymbol{B}}$ fields produced in the collision



- The color correlation length in the transverse plane is Q_s^{-1}
  - ▷ flux tubes of diameter Q_s^{-1}, filling up the transverse area

Glasma interpretation

- η-independent fields lead to long range correlations in the 2-particle spectrum :



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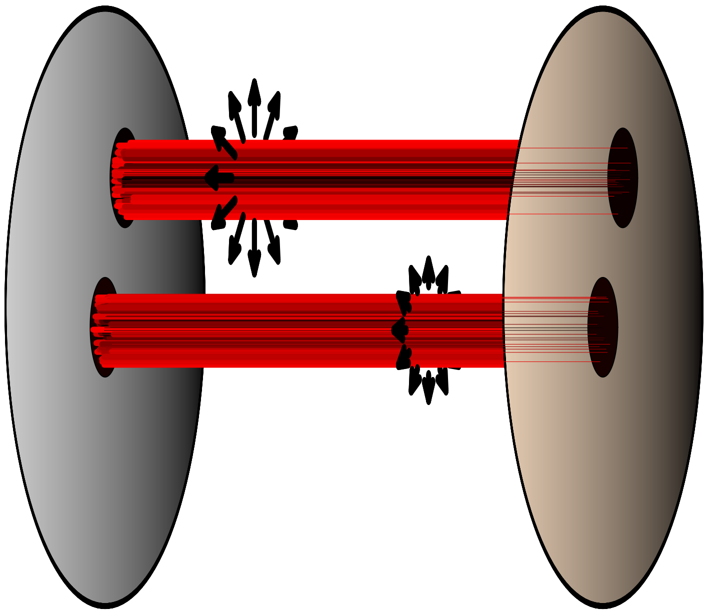

- Particles emitted by different flux tubes are not correlated. Therefore, $(R_A Q_s)^{-2}$ sets the strength of the correlation

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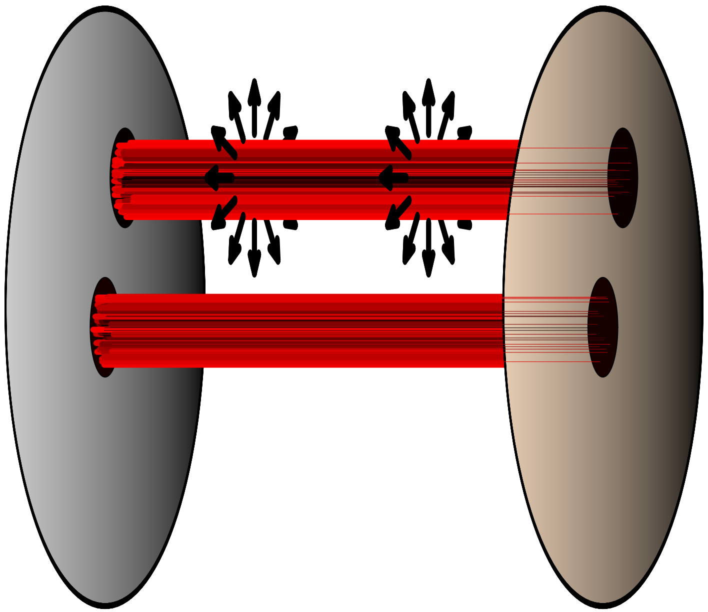

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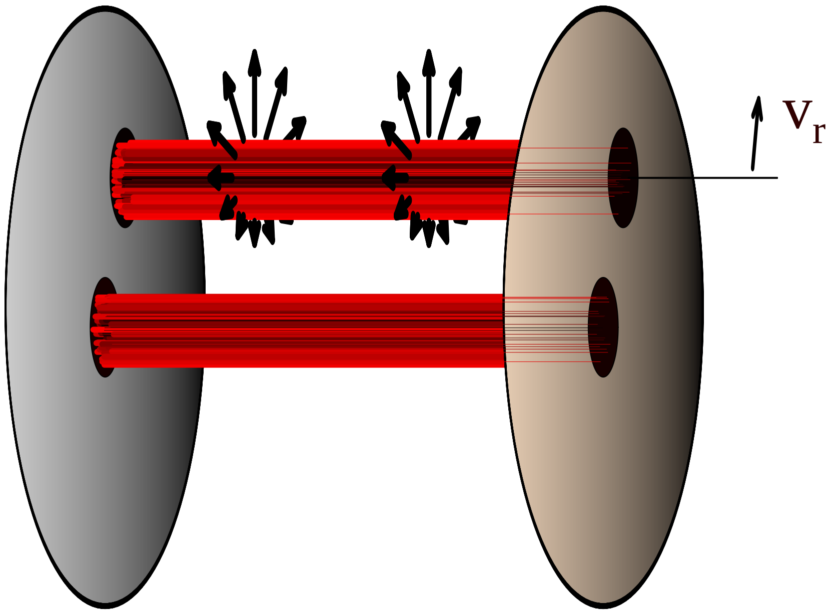


- Particles emitted by different flux tubes are not correlated. Therefore, $(R_A Q_s)^{-2}$ sets the strength of the correlation
- At early times, the correlation is flat in $\Delta\varphi$
  A collimation in $\Delta\varphi$ is produced later by radial flow

How to calculate it?

- The JIMWLK factorization (lecture III) can be extended easily to the 2-gluon spectrum :

$$\frac{dN_2}{d^3\vec{\boldsymbol{p}}d^3\vec{\boldsymbol{q}}} = \int [D\rho_1 D\rho_2]\; W_1[\rho_1]\; W_2[\rho_2]\; \frac{dN_1}{d^3\vec{\boldsymbol{p}}}\; \frac{dN_1}{d^3\vec{\boldsymbol{q}}}$$

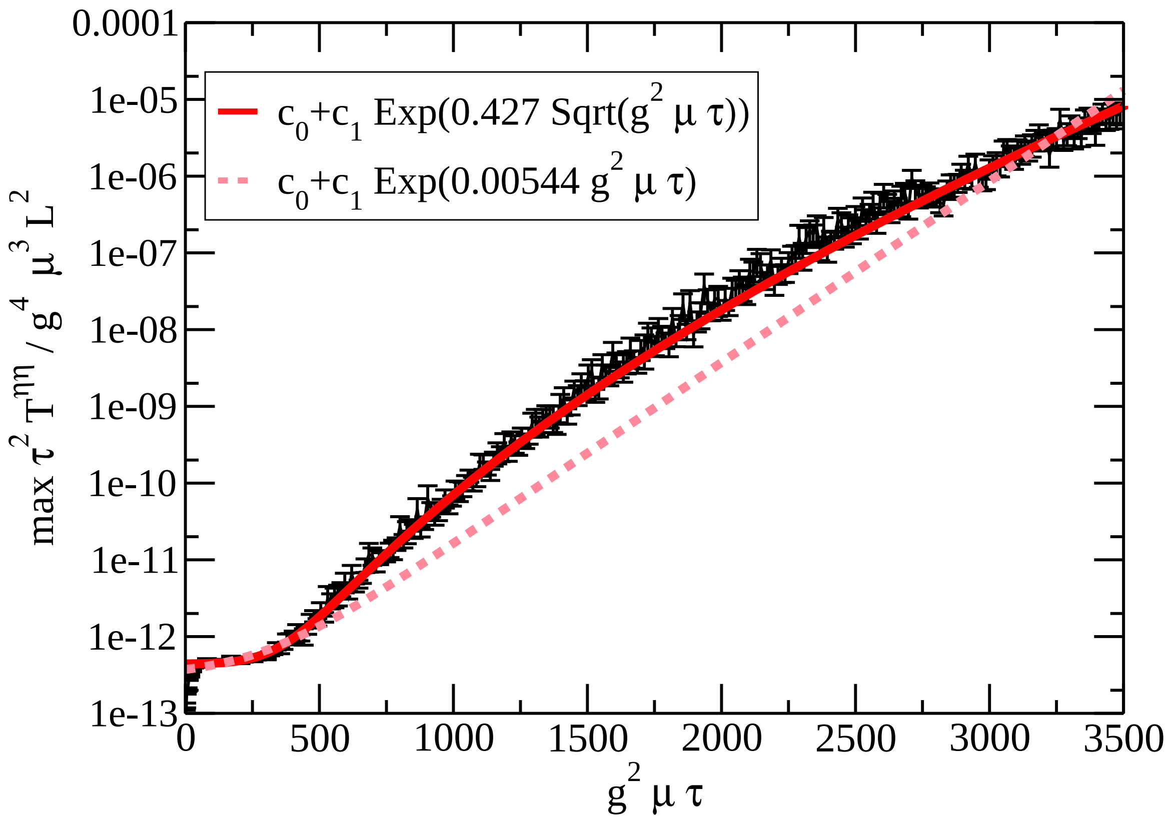
- Notes :
  - this formula needs corrections for large ΔY's
  - it describes only the early times - the effect of the radial flow is not yet included
- Semi-quantitative study : Dumitru, FG, McLerran, Venugopalan (2008)
- Quantitative study : Gavin, McLerran, Moschelli (2008)

Glasma instabilities

Numerical results

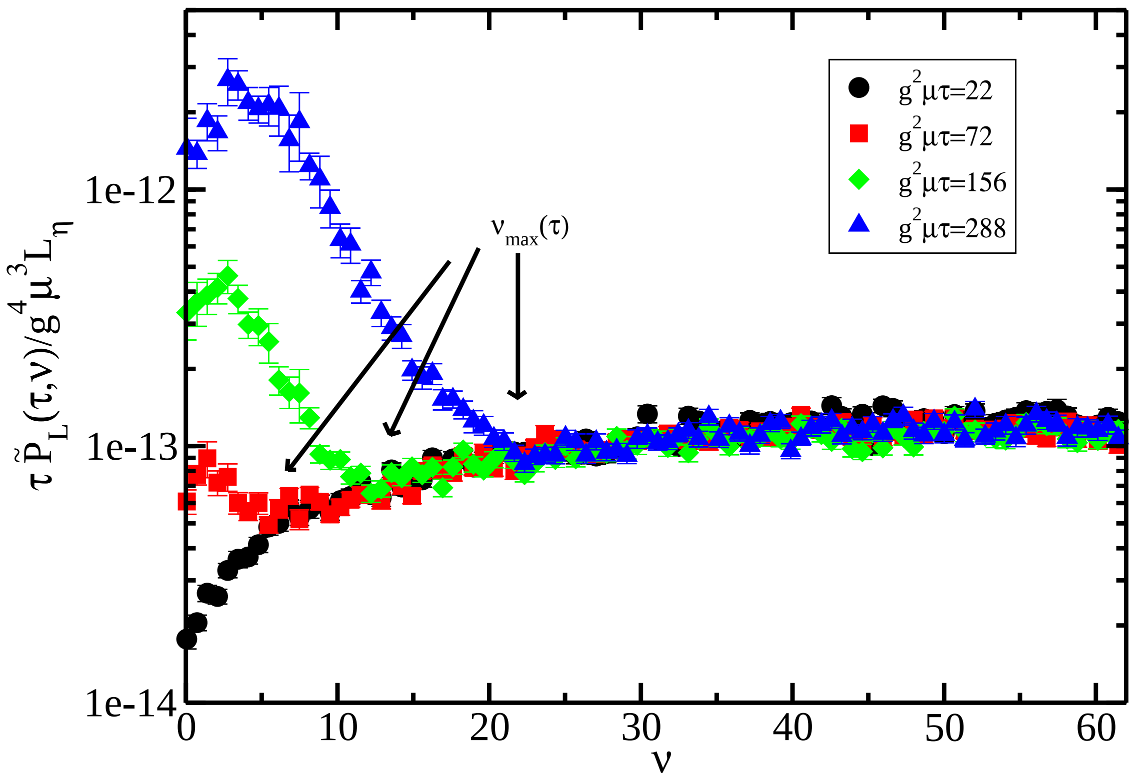
Romatschke, Venugopalan (2005)

- Rapidity dependent perturbations to the classical fields grow like $\exp(\sqrt{Q_s\tau})$ until the non-linearities become important :



Numerical results

- Fastest growing modes (ν = Fourier conjugate of η) :



▷ the zero mode grows slower than the others

Unstable modes

- This numerical analysis tells us that the small field fluctuation equation of motion,

$$\frac{\delta^2 \mathcal{S}_{YM}}{\delta \mathcal{A}^2} \cdot a = 0 \; ,$$

has runaway solutions if the initial condition depends on η :

$$a(\tau, \eta, \vec{\boldsymbol{x}}_\perp) \underset{\tau \to \infty}{\sim} e^{\sqrt{Q_s \tau}}$$

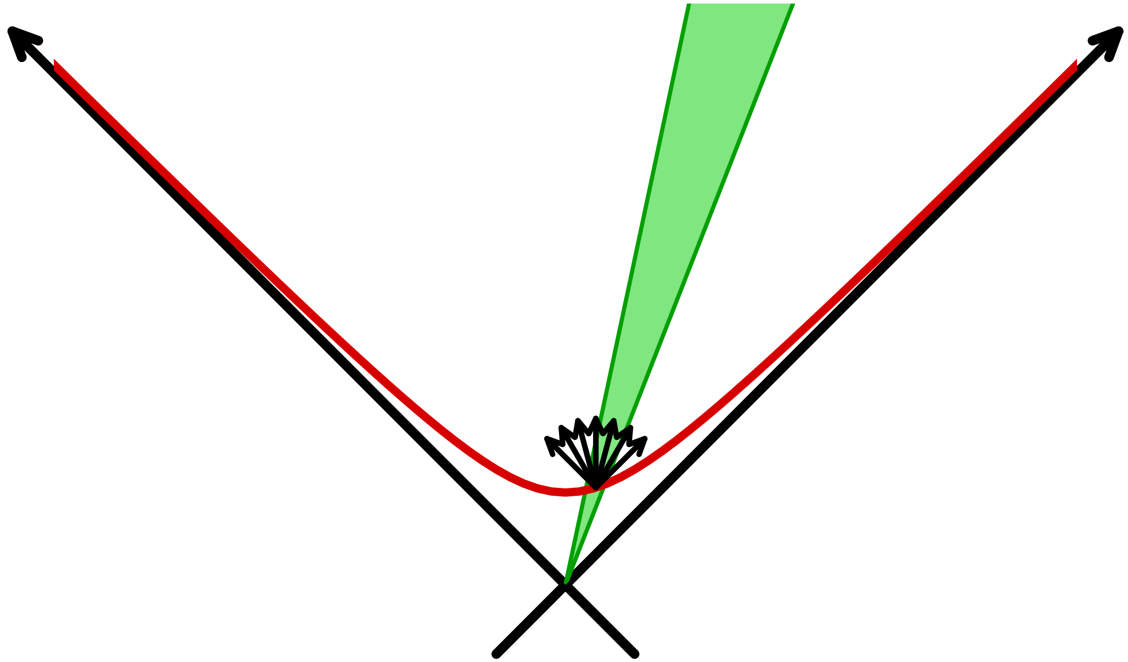
(see also : Fujii, Itakura (2008); Iwazaki (2008))

- Note : the square root is due to the longitudinal expansion (Rebhan, Romatschke (2006))

Possible thermalization scenario

Longitudinal expansion

- If nothing else happened, the distribution of produced particles would quickly become very anisotropic :



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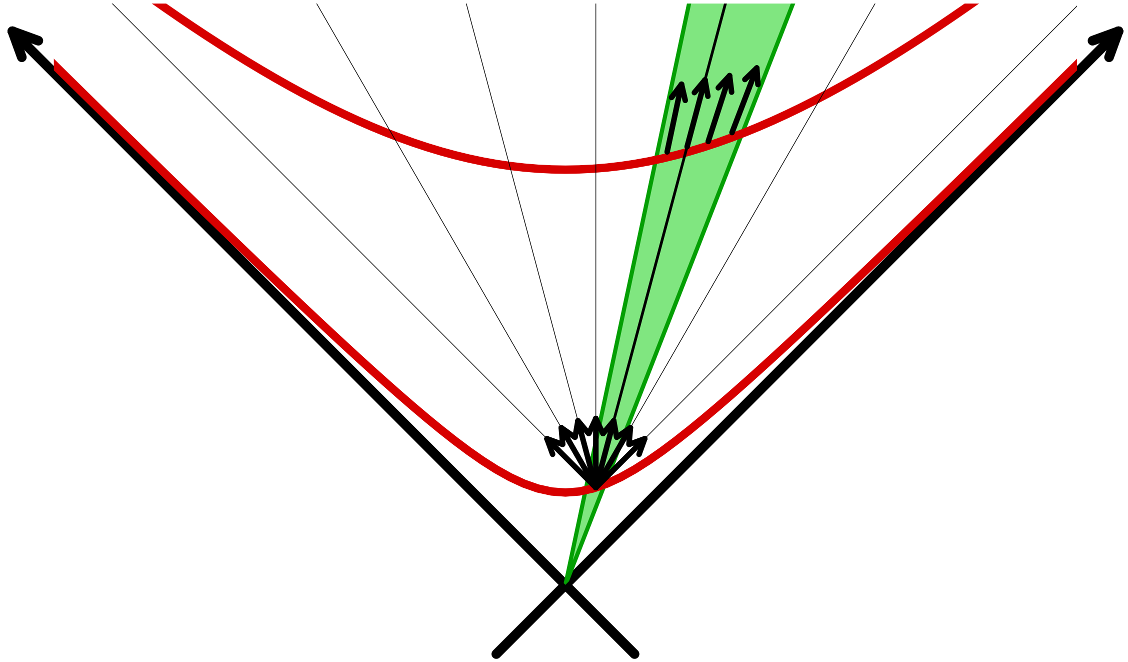
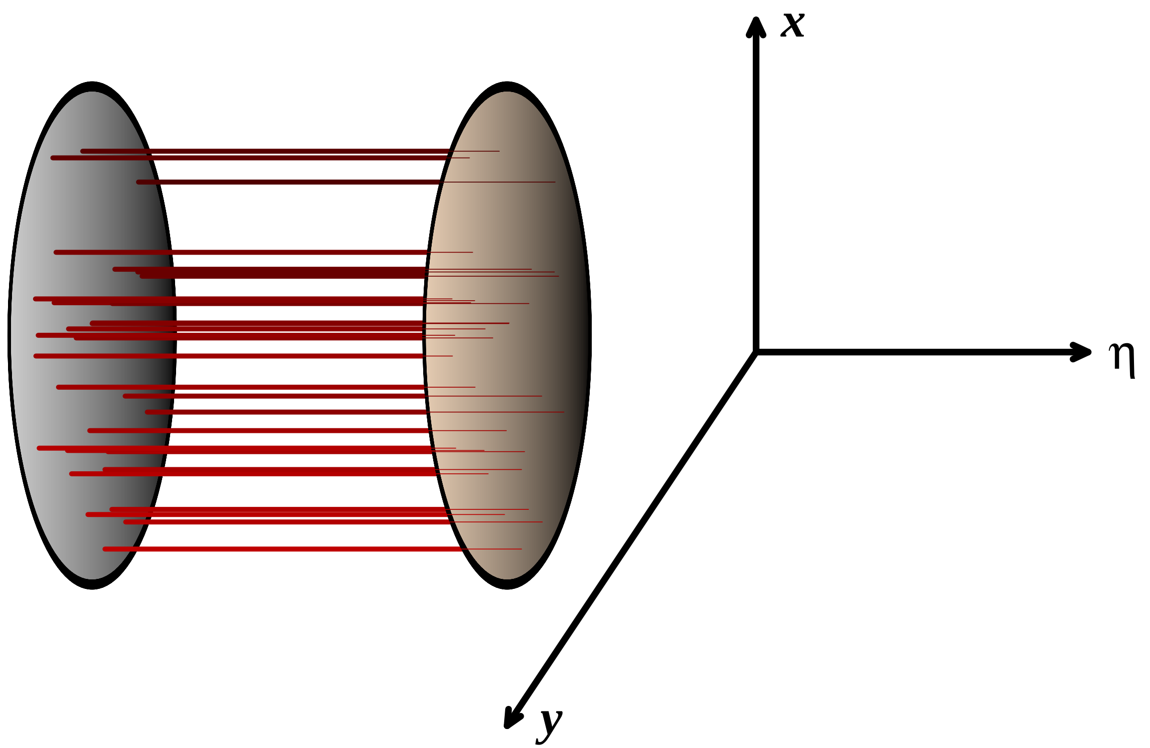


▷ if particles fly freely, only one longitudinal velocity can exist at a given η : $v_z = \tanh(\eta)$

▷ the longitudinal expansion of the system is the main obstacle to local isotropy

Glasma instability

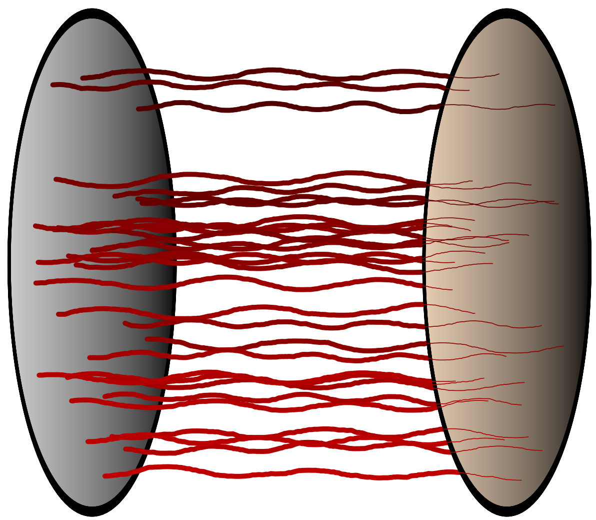
- Leading order magnetic fields at $\tau = 0^+$:



- At $\tau = 0^+$, the classical chromo-electric and chromo-magnetic fields are longitudinal
- They are also boost invariant (independent of η)

Glasma instability

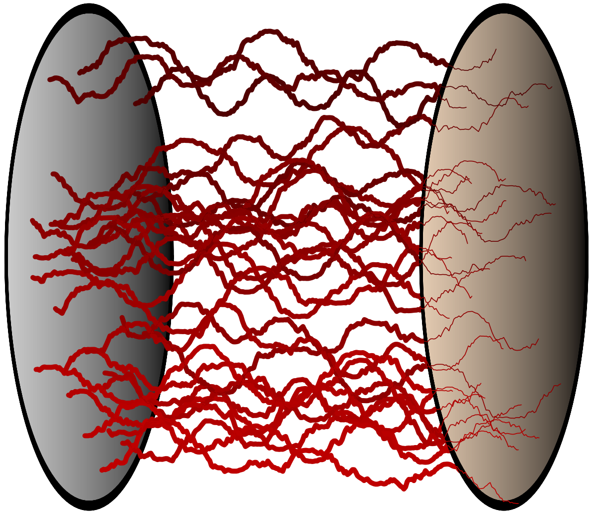
- Leading order + quantum fluctuations at $\tau = 0^+$:



  - Loop corrections bring quantum fluctuations in this picture
  - In the weak coupling regime, they are small corrections

Glasma instability

- Effect of the instability :



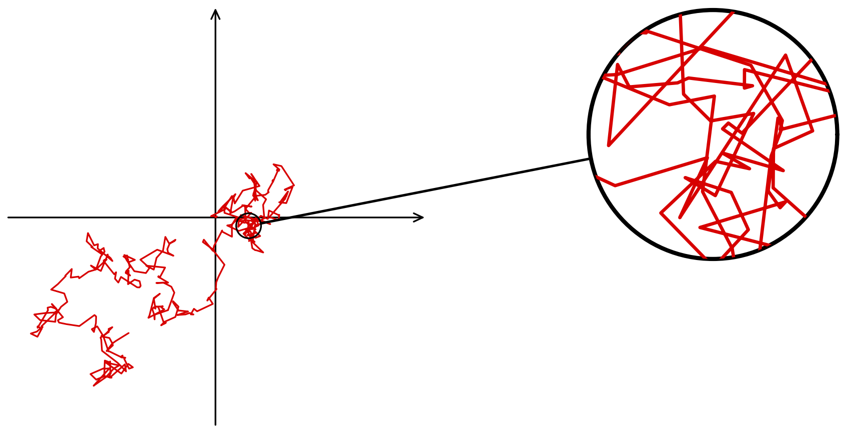
  - η-dependent perturbations grow quickly in time
  - Breakdown of the CGC approach at $\tau_{\max} \sim Q_s^{-1} \ln^2(g^{-2})$?
  - Outcome : disordered configurations of color fields

Lower bound for viscosity/entropy

- $\eta \sim \lambda\epsilon$ ($\lambda =$ mean free path, $\epsilon =$ energy density). Thus,

$$\frac{\eta}{s} \sim \lambda \underbrace{\frac{\epsilon}{s}}_{\text{energy per particle}}$$

- Heisenberg inequalities forbid the mean free path to be smaller than the De Broglie wavelength of the particles. Scatterings by an $\mathcal{O}(1)$ angle can occur only every $\lambda_{\rm Broglie}$ at most :



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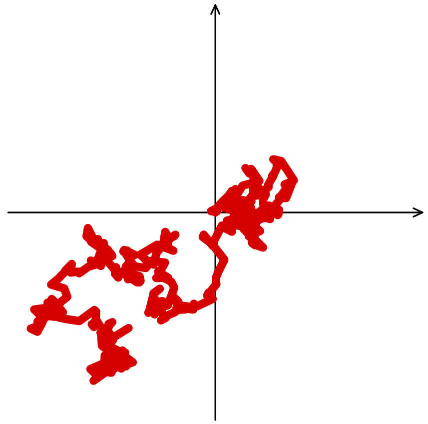

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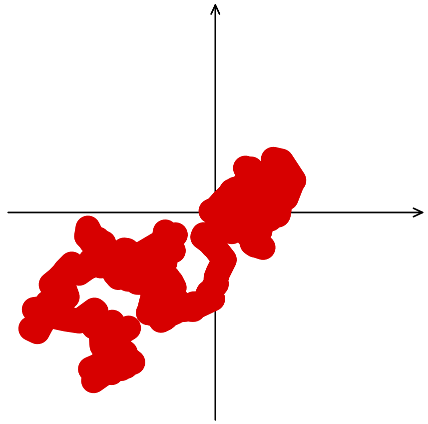


- Hence, $\frac{\eta}{s} \geq \mathcal{O}(1)$

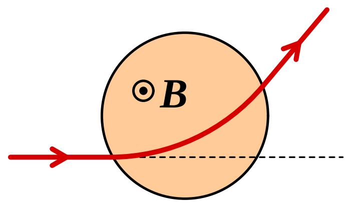
Anomalous transport

Asakawa, Bass, Muller (2006)

- Assume that $\alpha_s = \dfrac{g^2}{4\pi} \ll 1$
- Consider a domain of size Q_s^{-1}, in which the magnetic field is uniform and large, of order $B \sim Q_s^2/g$
- Let a particle of energy $E \sim Q_s$ go through this domain. The Lorenz force deflects its trajectory by an angle of order unity :

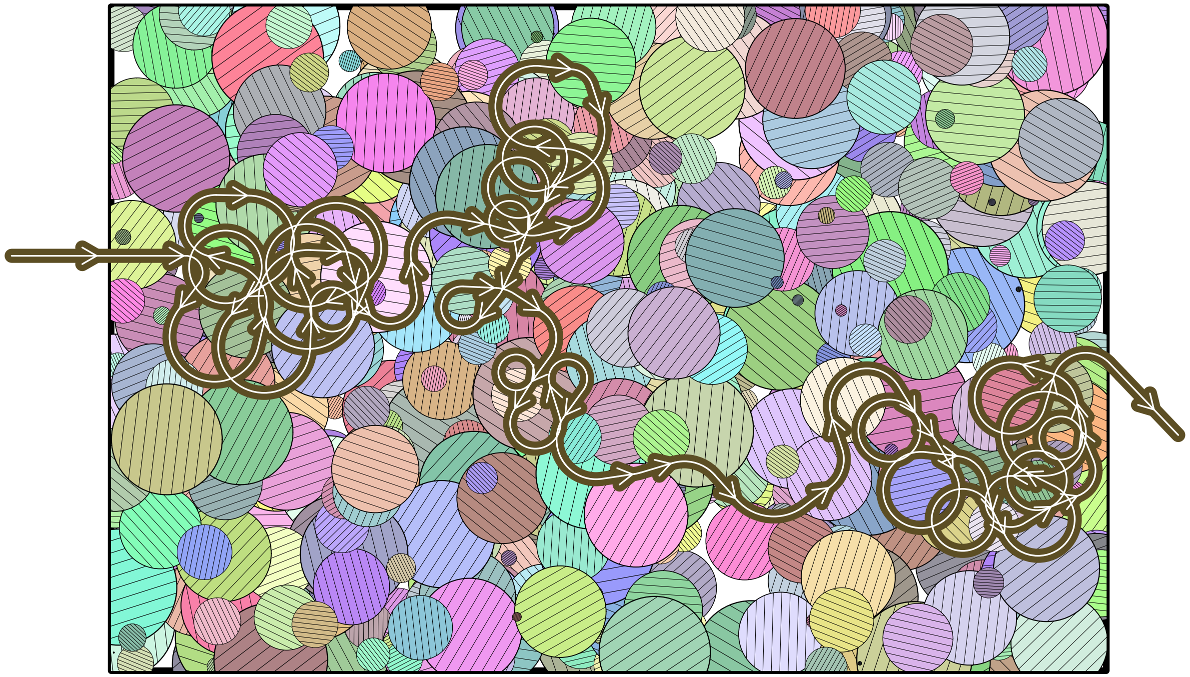
$$\frac{d\vec{\boldsymbol{p}}}{dt} = g\,\vec{\boldsymbol{v}} \times \vec{\boldsymbol{B}} \quad \Rightarrow \quad \dot{\theta} = \frac{gB}{E} \sim Q_s$$

time spent in the domain : $\delta\tau \sim Q_s^{-1}$



Anomalous transport

- Consider now a region filled with such domains, with random orientations for the magnetic field in each domain



▷ In such a medium, the mean free path of a particle of energy Q_s is of order Q_s^{-1}, i.e. as low as permitted by the uncertainty principle ▷ fast thermalization?

Resummation of unstable modes

Power counting

- Unstable modes are a problem in loop corrections, because

$$\mathbb{T}_{\boldsymbol{u}}\mathcal{A}(x) \sim \frac{\delta\mathcal{A}(x)}{\delta\mathcal{A}_{\rm initial}(u)} \underset{\tau\to\infty}{\sim} e^{\sqrt{Q_s\tau}}$$

- If we do not resum these unstable fluctuations, the CGC approach will break down at a time $\tau_{\rm max} \sim Q_s^{-1}\ln^2(1/g)$

- Power counting :
  - Naively : $\mathcal{A} \sim g^{-1}$, $\mathcal{A}_{\rm initial} \sim g^{-1}$, $\mathbb{T}_{\boldsymbol{u}}\mathcal{A}(x) \sim 1$
  - In reality : $\mathbb{T}_{\boldsymbol{u}}\mathcal{A}(x) \sim e^{\sqrt{Q_s\tau}}$

- Note : the term $[\beta(u)\mathbb{T}_{\boldsymbol{u}}]\mathcal{A}(x)$ is not subject to this instability, because β is rapidity independent (1-point function in a boost invariant background)

  ▷ the unstable fluctuations come via terms with at least second derivatives, such as $[\mathcal{G}(u,v)\mathbb{T}_{\boldsymbol{u}}\mathbb{T}_{\boldsymbol{v}}]\mathcal{A}(x)$

Power counting

- So far, we have assumed that :

$$\frac{dN}{d^3\vec{\boldsymbol{p}}} = \frac{1}{g^2}\left[c_0 + c_1\, g^2 + c_2\, g^4 + \cdots\right]$$

$$\begin{aligned} c_1 &= & & & c_{10} &+ c_{11}\ \ln\left(\frac{1}{x_{1,2}}\right) \\ c_2 &= c_{20} &+ c_{21}\ \ln\left(\frac{1}{x_{1,2}}\right) & &+ c_{22}\ \ln^2\left(\frac{1}{x_{1,2}}\right) \end{aligned}$$

with all the c_{np} coefficients are of order one

- We have resummed all the terms that have one $\ln(1/x)$ for each extra g^2, and we have shown that all these leading log terms can be absorbed in the evolved distributions of sources $W[\rho_{1,2}]$

Power counting

- Gluon production
- Initial correlations
- Glasma instabilities
- Possible scenario
- Resummation
  - Power counting
  - Leading contributions
  - Resummation
  - Initial Gaussian fluctuations
- Open issues

- Because of the instabilities, we should write instead :

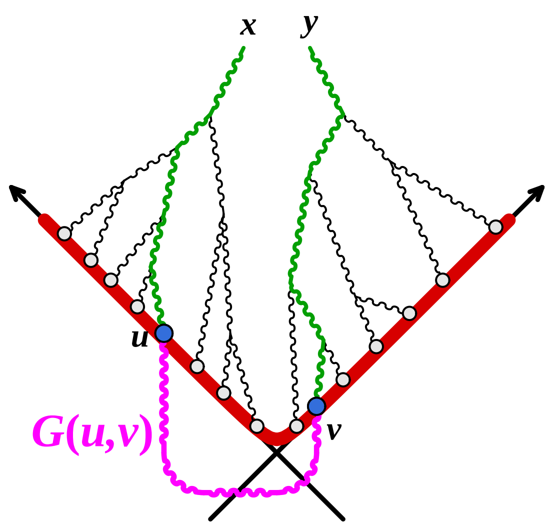
$$
\begin{aligned}
c_1 &= c_{100} + c_{110}\,\ln\left(\frac{1}{x_{1,2}}\right) + \underline{c_{101}\,e^{2\sqrt{Q_s\tau}}}\\
c_2 &= c_{200} + c_{210}\,\ln\left(\frac{1}{x_{1,2}}\right) + c_{201}\,e^{2\sqrt{Q_s\tau}}\\
&\quad + c_{220}\,\ln^2\left(\frac{1}{x_{1,2}}\right) + \underline{c_{211}\,\ln\left(\frac{1}{x_{1,2}}\right)e^{2\sqrt{Q_s\tau}}} + \underline{c_{202}\,e^{4\sqrt{Q_s\tau}}}
\end{aligned}
$$

- Note : because the logs of $1/x$ come from the zero η-modes, while the unstable terms come from the non-zero modes, there are only terms c_{npq} with $p+q\le n$
- Resummation of leading logs : keep all the c_{nn0} terms
- Improved resummation : keep all the c_{npn-p} terms

  (these are the terms for which each g^2 is compensated by a large log(1/x) or a factor $e^{2\sqrt{Q_s\tau}}$)

Leading contributions

- The instabilities are triggered by the 2-point function :

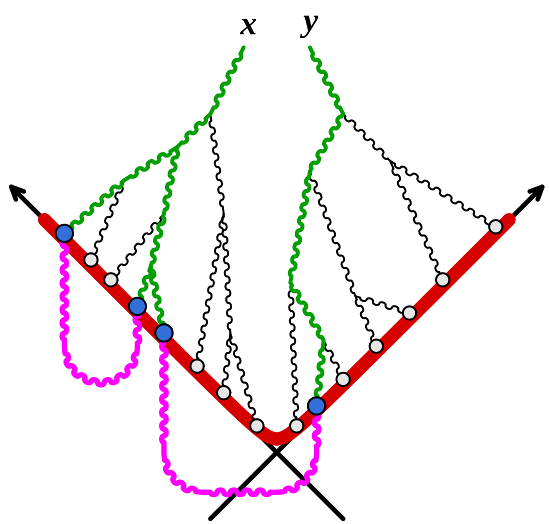


- Power counting : $\mathcal{G} \sim \mathcal{O}(1)$, $\bullet \sim \mathcal{O}(g\ e^{\sqrt{Q_s \tau}})$
- This 1-loop term is of order $g^2\ e^{2\sqrt{Q_s \tau}}$ relative to the LO contribution to the gluon spectrum

Leading contributions

- At n-loop order, one must pick the terms that have the fastest growth in time

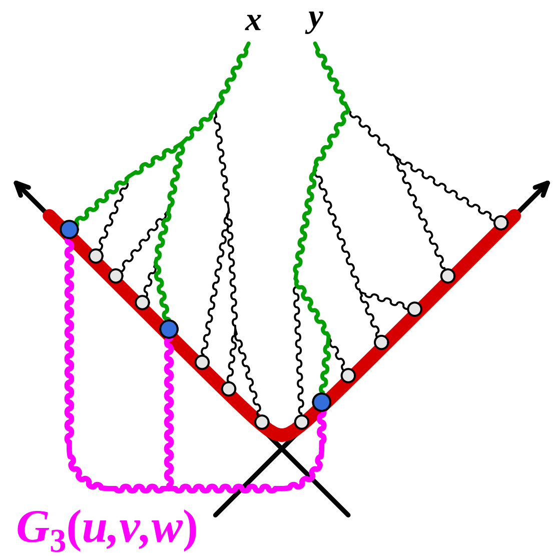
  ▷ one must maximize the number of locations where the initial field is perturbed on the light-cone, while minimizing the powers of α_s



- This 2-loop term is of order $g^4\, e^{4\sqrt{Q_s\tau}}$ relative to the LO contribution to the gluon spectrum

Leading contributions

- Non-Gaussian correlations are suppressed :



- Power counting : $\mathcal{G}_3 \sim \mathcal{O}(g)$, ● $\sim \mathcal{O}(g\ e^{\sqrt{Q_s\tau}})$
- This 2-loop term is of order $g^4\ e^{3\sqrt{Q_s\tau}}$ relative to the LO contribution to the gluon spectrum ▷ subleading

Resummation

- 1-loop contributions :

$$\left.\frac{dN}{d^3\vec{\boldsymbol{p}}}\right|_{\rm NLO} = \Big[\ln\left(\frac{\Lambda^+}{p^+}\right)\mathcal{H}_1 + \ln\left(\frac{\Lambda^-}{p^-}\right)\mathcal{H}_2 + \frac{1}{2}\int\limits_{\vec{\boldsymbol{u}},\vec{\boldsymbol{v}}\in\Sigma} \mathcal{G}_{\nu\neq 0}(\vec{\boldsymbol{u}},\vec{\boldsymbol{v}})\,\mathbb{T}_{\boldsymbol{u}}\mathbb{T}_{\boldsymbol{v}}\Big] \left.\frac{dN}{d^3\vec{\boldsymbol{p}}}\right|_{\rm LO}$$

  - $\mathcal{G}_{\nu\neq 0}(\vec{\boldsymbol{u}},\vec{\boldsymbol{v}})$ does not contain the zero η-mode
  - This formula does not make sense beyond $\tau_{\rm max}$

- Assume that the resummation of these terms to all orders can be written as :

$$\sum_{n=0}^{\infty}\left.\frac{dN}{d^3\vec{\boldsymbol{p}}}\right|_{{\rm N}^n{\rm LO}} = \mathcal{U}_1\,\mathcal{U}_2\;\; F[\mathbb{T}_{\boldsymbol{v}}]\;\left.\frac{dN}{d^3\vec{\boldsymbol{p}}}\right|_{\rm LO}$$

where $\mathcal{U}_{1,2}$ are evolution operators for the JIMWLK Hamiltonians (factorization is due to the non-mixing of the various divergences)

Resumming the unstable terms

- Introduce the "Laplace transform" of $F[\mathbb{T}_{\boldsymbol{u}}]$,

$$F[\mathbb{T}_{\boldsymbol{u}}] \equiv \int [Da(\vec{\boldsymbol{u}})]\; F[a(\vec{\boldsymbol{u}})]\; \underbrace{\exp \int_\Sigma a(\vec{\boldsymbol{u}}) \cdot \mathbb{T}_{\boldsymbol{u}}}_{\text{translation operator for the initial classical field}}$$

- The effect of $F[\mathbb{T}_{\boldsymbol{u}}]$ is :

$$F[\mathbb{T}_{\boldsymbol{v}}] \quad \left.\frac{dN}{d^3\vec{\boldsymbol{p}}}\right|_{\rm LO} = \int [Da(\vec{\boldsymbol{u}})]\; F[a(\vec{\boldsymbol{u}})] \quad \left.\frac{dN}{d^3\vec{\boldsymbol{p}}}[\mathcal{A}+a]\right|_{\rm LO}$$

▷ resumming the unstable modes amounts to add a fluctuating field to the initial value of the classical field on the light-cone, with a distribution $F[a(\vec{\boldsymbol{u}})]$

Resumming the unstable terms

- Summing both the large logs of $1/x_{1,2}$ and the unstable terms, we get :

$$\left\langle \frac{dN}{d^3\vec{\boldsymbol{p}}} \right\rangle \underset{\substack{\text{improved}\\ \text{resummation}}}{=} \int [D\rho_1 \, D\rho_2] \, W_{Y_1}[\rho_1] \, W_{Y_2}[\rho_2]$$

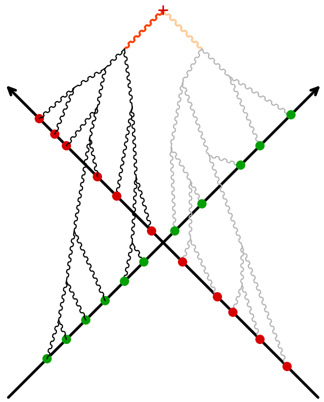
$$\times \int [Da(\vec{\boldsymbol{u}})] \; F[a(\vec{\boldsymbol{u}})] \;\; \frac{dN}{d^3\vec{\boldsymbol{p}}}[\mathcal{A}+a]\Big|_{\mathrm{LO}}$$

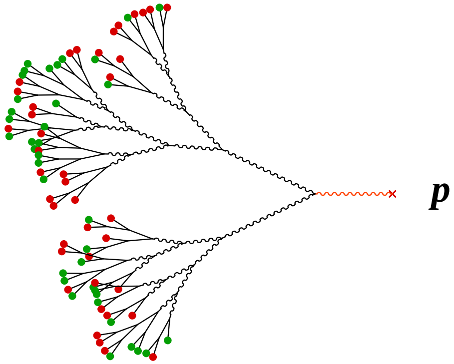
- Note : after this resummation, the instabilities do not lead to divergences when $\tau \to +\infty$

  (the quantum fluctuations are now absorbed in the initial condition of the non-linear YM equations – instead of being treated in the linear approximation)

Instabilities and gluon splitting

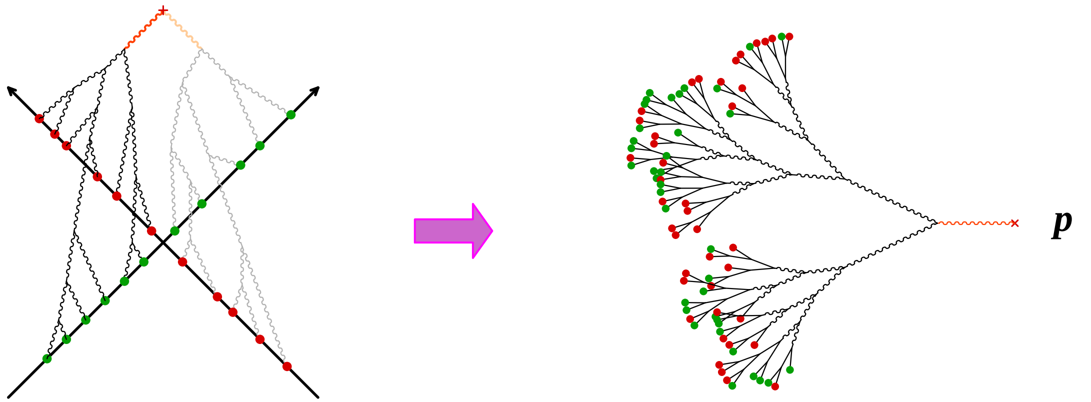
- Tree level :



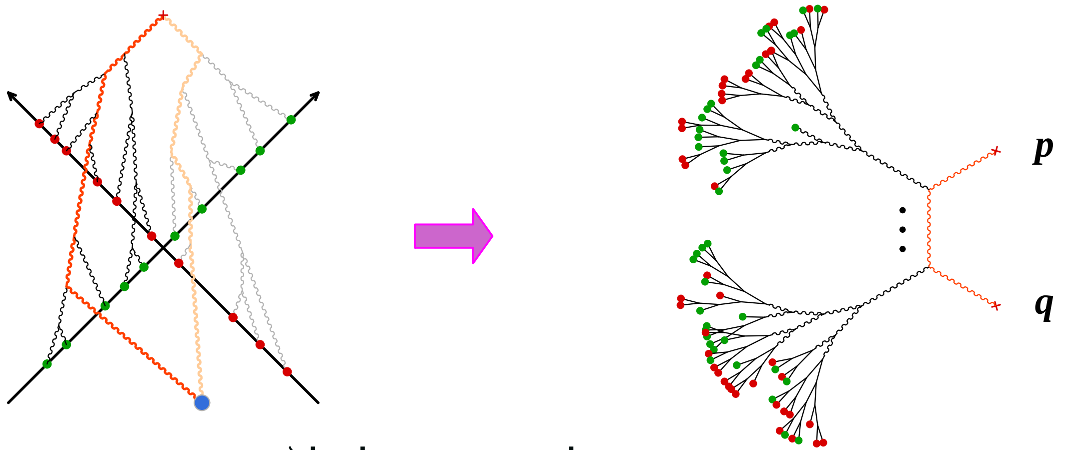


Instabilities and gluon splitting

- Tree level :



- One loop ▷ gluon pairs :



▷ The momentum $\vec{q}$ is integrated out

▷ If $\alpha_s^{-1} \lesssim |y_{\boldsymbol{p}} - y_{\boldsymbol{q}}|$, the correction is absorbed in $W[\rho_{1,2}]$

▷ If $|y_{\boldsymbol{p}} - y_{\boldsymbol{q}}| \lesssim \alpha_s^{-1}$: gluon splitting in the final state

Initial Gaussian fluctuations

- Some numerical results have been obtained with a toy model for the distribution of initial fluctuations

- With some approximations, one can obtain a spectrum of Gaussian fluctuations characterized by :

$$\big\langle a_i(\eta,\vec{\boldsymbol{x}}_\perp)\,a_j(\eta',\vec{\boldsymbol{x}}'_\perp)\big\rangle =$$
$$= \frac{1}{\tau\sqrt{-(\partial_\eta/\tau)^2-\boldsymbol{\partial}_\perp^2}}\left[\delta_{ij}+\frac{\partial_i\partial_j}{(\partial_\eta/\tau)^2}\right]\delta(\eta-\eta')\,\delta(\vec{\boldsymbol{x}}_\perp-\vec{\boldsymbol{x}}'_\perp)$$

(Fukushima, FG, McLerran (2006))

Open issues

Choice of the initial time?

- The τ, η system of coordinates is pathological at $\tau = 0$

  The field equations of motion contain terms in $\tau^{-1}\partial_\eta$

  $\triangleright$ these derivatives become a nuisance when one resums the η-dependent fluctuations

- Solution : start the time evolution at $\tau_0 > 0$

- Constraint : we should not affect the physics measured at later times

Choice of the initial time?

- This implies that the spectrum of initial fluctuations should depend on the time τ_0

$$\begin{aligned}\langle\mathcal{O}\rangle &= \int[D\rho_1 D\rho_2]\, W_{Y_1}[\rho_1]\, W_{Y_2}[\rho_2] \\ &\times \int[Da(\vec{\boldsymbol{u}})]\; F_{\tau_0}[a(\vec{\boldsymbol{u}})]\; \mathcal{O}[\underbrace{\mathcal{A}_{\infty,\tau_0}}[\mathcal{A}_{\tau_0}[\rho_{1,2}]+a]]\end{aligned}$$

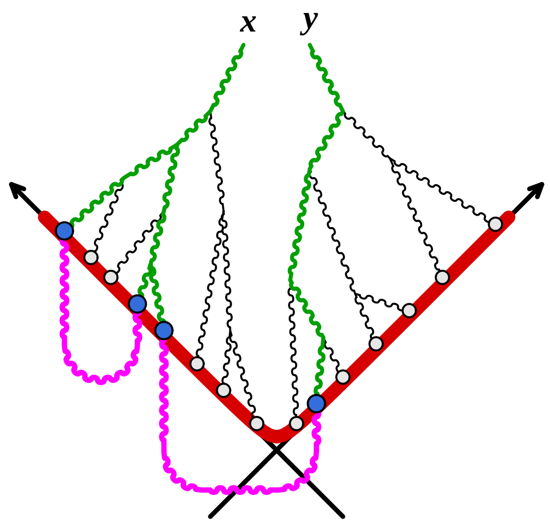
classical field at $\tau\to\infty$

in terms of the initial value at τ_0

- From the Yang-Mills equation, one knows how the classical field changes if one changes the time τ_0 at which the initial condition is specified
- One also knows how the boost invariant part $\mathcal{A}_{\tau_0}$ changes
- One must change the fluctuation a in such a way that the value of $\mathcal{A}_\infty$ remains unchanged
- This can be achieved by changing the distribution $F[a]$

Choice of the initial time?

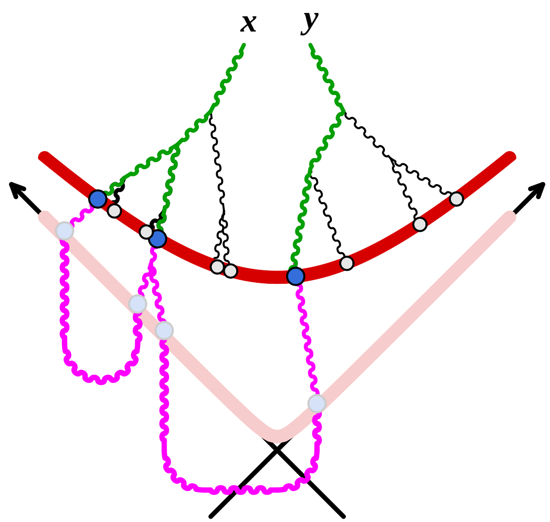
- Even if the distribution $F[a]$ is Gaussian at $\tau_0 = 0$, it does not remain Gaussian at later times



▷ only 2-point correlations among the initial fluctuations

Choice of the initial time?

- Even if the distribution $F[a]$ is Gaussian at $\tau_0 = 0$, it does not remain Gaussian at later times



▷ non trivial higher-point correlations appear due to the mergings between the fluctuations

Ultraviolet divergences

- The summation of the small fluctuations on the initial surface is equivalent to the summation of some loop corrections
- The final result is plagued by ultraviolet divergences, due to the fact that the momentum running in these loops has no upper bound
- In a lattice resolution of the classical EOM, this problem will show up as an infinite limit $a \to 0$

  $\triangleright$ one must properly renormalize the calculation

- Difficulty : one is performing a resummation of terms of various loop-orders

  $\triangleright$ not as simple as the renormalization of a fixed order calculation

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