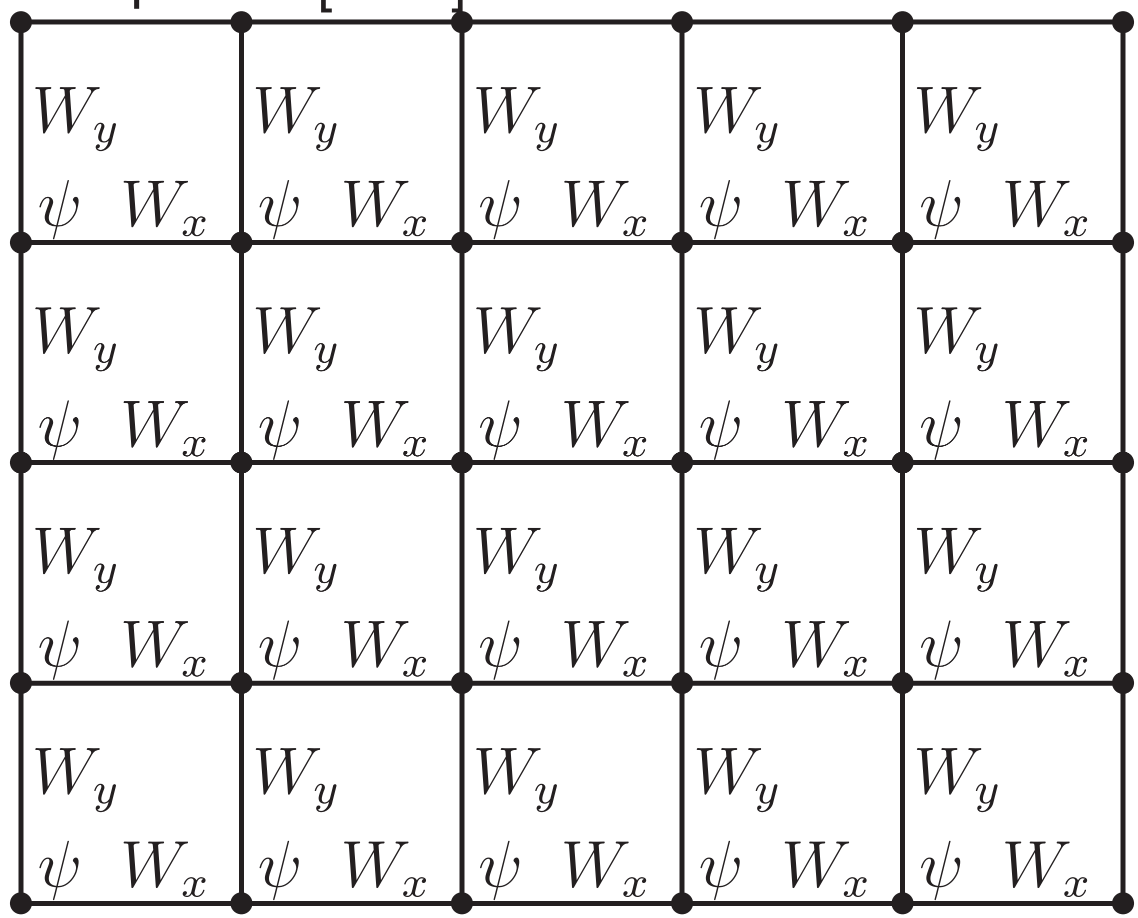
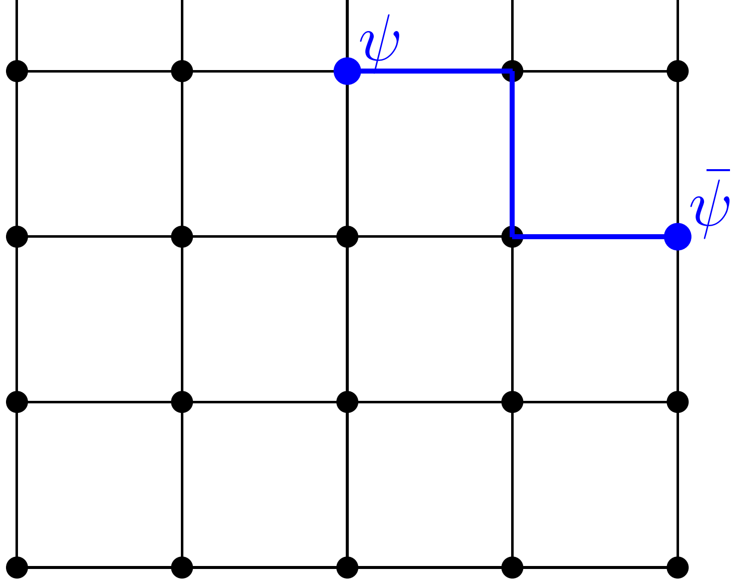
Classical lattice gauge theory 5

Wilson's insight: must base implementation on W, not A.

Fields live on points [sites]: W lives on links



Making gauge invariant objects is easy:
Always connect gauge variant things with W's until you "tie off" all indices.



Fermion bilinear

$$\bar{\psi}_{\bar{b}}(y) W_{b\bar{a}}(y, x) \psi_a(x)$$

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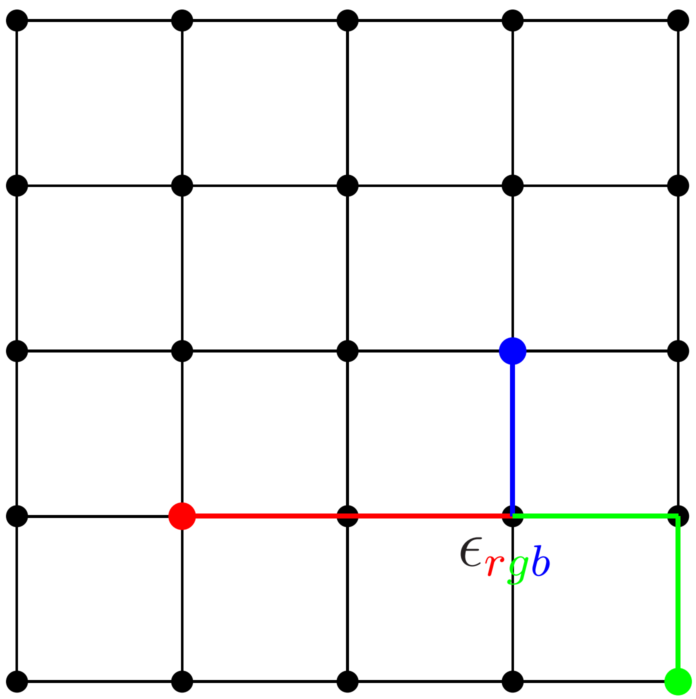

Baryon

$$\psi_a(x)\psi_b(y)\psi_c(z)W_{d\bar{a}}(w,x)W_{e\bar{b}}(w,y)W_{f\bar{c}}(w,z)\epsilon^{def}$$

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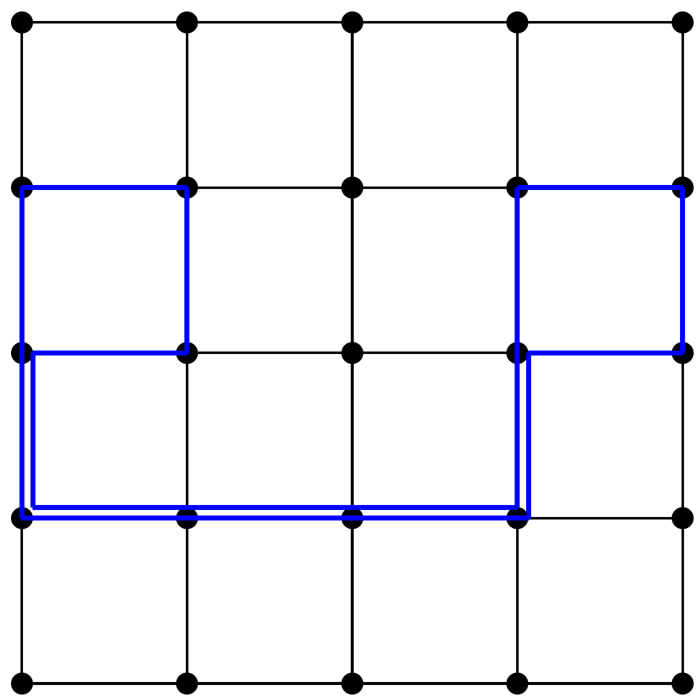


Purely gluonic correlator (B-B connected by double Wilson line)

$$W_{C:a\bar{a}}(x,x) = \mathrm{Tr}\, W_C(x,x)$$

Application to classical field thy

Kogut/Susskind PRD11:395(1975), Ambjørn *et al* NuclPhysB353:346(1991)

IR description: $n_b \gg n_f$, no fermions!

Make spacetime a lattice, $a_t \ll a$ (1/20 in practice)

Write action which generates all dim. 4 IR terms:

$$
\begin{aligned}
S_{\rm contin} &= \int d^3x dt \frac{1}{2g^2}(B^2 - E^2) \\
&= \frac{1}{g^2} \int d^3x dt \left(\sum_{i<j} {\rm Tr}\, F_{ij}F_{ij} - \sum_i {\rm Tr}\, F_{0i}F_{0i} \right) \\
S_{\rm latt} &= \frac{2a^3 a_t}{g^2} \sum_x \left(\sum_{i<j} a^{-4} {\rm Tr}\, \Box_{ij} - \sum_i a^{-2} a_t^{-2} {\rm Tr}\, \Box_{i0} \right)
\end{aligned}
$$

$\Box_{ij}$ is prod of 4 W's in a square in i,j direc ("plaquette")

Remarks:

- $\mathrm{Tr}\ \Box$ is what? Roughly, $\Box \sim 1 - a^2 F^A_{\mu\nu} T^A$ curvature integrated over area of box. Unitarity (SU(3)) requires $a^4 F^2/2$ contribution.

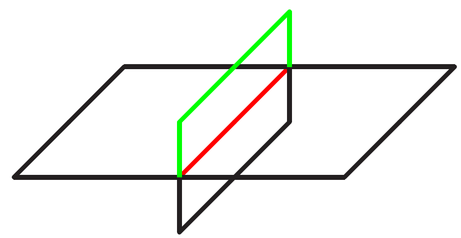
- Unlike Euclidean, E and B terms ($0i$ and ij) have different signs

- Different coefficients, a^{-4} versus $a^{-2}\ a_t^{-2}$. Makes E fluct. "stiffer" corresponding to smaller lattice spacing (asymm lattice)

- Overall coefficient $1/g^2$ doesn't matter if we force $\delta S/\delta W_i(x) = 0$ strictly!

Update rule

Variation of a spatial link:



Line in red varied: lines in green unknown.

Variation means $\delta W \equiv -i\epsilon_A T^A \, W$

So $\delta\Box \sim (-i\epsilon_B T^B)(1 + ia^2 F)$

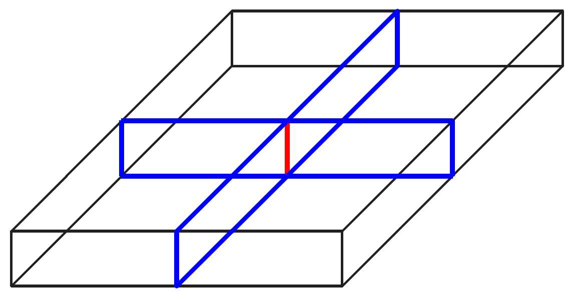
Tracing gives $a^2 F^B_{\mu\nu}$. Forward-backward difference is D_μ.

Corresponds to $D_i F_{ij} = D_0 F_{0j}$ or $D_t E = D \times B$

Determines green lines uniquely except for W_0's

Initial value problem

Two derivatives: fields+time deriv's, or values on two time slices. But $\delta S/\delta W_0$ a constraint!



$$0 = \delta S/\delta W_0 = \sum_i (E_i(x) - E_i(x - a\hat{i}) \sim D_i E_i$$

Gauss' Law (expected from $\delta S/\delta A_0$)

Values of W_0 NEVER determined! But that's gauge freedom.

So does it work?

- Good: thermodynamics exactly same as QCD in Dimensional Reduction (see Vuorinen talk)
- Good: fast, exact thermalization algorithms known GM hep-ph/9603384, Krasnitz hep-lat/9507025
- Mixed: strange role of g^2. Thermalization determines combination $g^2 aT$ only
  ($g^2/\hbar$ dimensionless: a a length, T energy, $g^2 T$ inverse length.)
- Bad: dynamics do not have simple $a \to 0$ limit!

Dynamics: problem or interesting physics?

Short wavelength lattice excitations act like "particles."

So describe them using (collisionless) Vlasov theory:

$$2p^\mu D_\mu n = -p^\nu \Big\{ F_{\nu\mu} \, , \, \partial_p^\mu n \Big\}$$

Here $n = n_{a\bar{a}}$ is in $R \times \bar{R}$ rep, reducible!

$$n_{a\bar{a}} = n_s \delta_{a\bar{a}} + n_A T^A_{a\bar{a}} + \ldots$$

Expand in $n_s \gg n_A \gg \ldots$ (justification...)

$$2p^\mu D_\mu n_A = -p^\nu F_{\nu\mu} \partial_p^\mu n_s(p)$$

Dynamics: Hard (Thermal) Loops

Repeating:

$$2p^\mu D_\mu n_A = -p^\nu F_{\nu\mu} \partial_p^\mu n_s(p)$$

Solving (formally):

$$n_A(x,p) = -\frac{1}{2p^\mu D_\mu}\, p^\nu F_{\nu\mu} \partial_p^\mu n_s(p)$$

which means

$$n_A(x,p) = \int_0^\infty dy W_{AB}(0,-y\hat{p}) \frac{-p^\nu}{2E} F^B_{\nu\mu}(-y\hat{p}) \partial_p^\mu n_s(p,-y\hat{p})$$

Here $\int dy$ is over line backwards in p direction

W is adjoint Wilson line along that path

Interpretation and importance

$F_{\mu\nu}$ acts on colorless mixture of part. to give "net color"

$$\frac{d[\text{color}]}{dt dp} \sim p^\nu F^A_{\nu\mu} \partial^\mu_p n_s(p)$$

Particles propagate: coloration at x is $\int_{\rm past}$ of color source.

Current

$$J^\mu_A(x) = \sum_{\rm species} g^2 T_{\rm R} \int_p p^\mu n_A(p,x)$$

$$\text{Approx. size} \sim \int \frac{d^3p}{p} p^\mu \partial_p n_s \sim \int \frac{d^3p}{p} p \frac{1}{p^2}$$

(since $n_s \sim T/p$). Linear UV divergent!

Hard (thermal) loops

Current dominated by hard scales–class. field approx fails

For $D_\mu \sim g^2 T$, J dominates $D_\mu F^{\mu\nu}$ by $\mathcal{O}(1/g^2)$

Dynamics actually *dominated* by these "hard thermal" effects on scales up to gT, $1/g$ larger than nonpert. scale

Need to get them right!

But aren't these effects already there on the lattice?
Well, yes and no....

Lattice dispersion: dirty little secret

We saw that for IR fields, our latt action acts like $B^2 - E^2$ as it should. Therefore $\omega_p^2 = p^2$ as usual.

But now fluct. with $ka \sim 1$ are important. For them

$$\omega_p^2 = \sum_i \frac{4}{a^2} \sin^2 \frac{k_i a}{2} = \sum_i 2 - 2\cos k_i a$$

which is different.

Can change (make more complicated) action. But cannot fix! ω_p^2 must be periodic, $p_x \in [-\pi/a, \pi/a)$.

This enters in Hard Loops found above: p^μ....

Hard loops from latt modes are "wrong"

Need to put in "right" hard loops

Find a way to introduce a current obeying

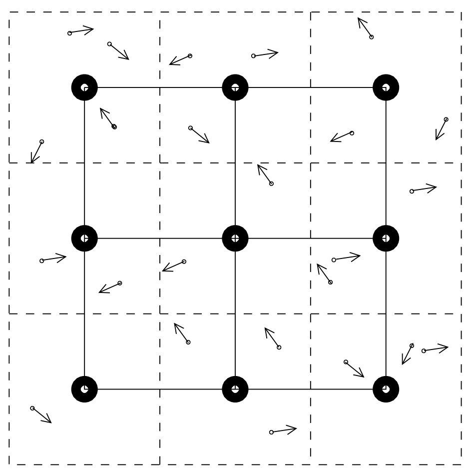
$$\begin{aligned} J^\mu_A(x) &= \sum_{\rm species} g^2 T_{\rm R} \int_p p^\mu n_A(p,x) \\ &= g^2 T_{\rm R} \sum_{\rm sp} \int \frac{d^3p}{\omega_p^2} \int dy W(0, y\hat{p}) p^\mu p^\nu F_{\nu\alpha} \partial_p^\alpha n_s \end{aligned}$$

Wish we could also *subtract* (false) lattice contrib. with wrong ω_p and d^3p range. But this appears hopeless.

Two techniques known: "particles" and "fields"

Particles method

GM, Hu Müller hep-ph/9710436



Add "particles":

position x continuous (!)

momentum p with $E = |p|$

charge q^A adjoint valued

q^A transforms as chg at nearest site

Vol nearest a site is dual cell...

Free propagation $dx/dt = p/E$, $dp/dt = 0$ within dual cell

$D_i E_i = Q$ with Q sum of q's of part. within dual cell

On crossing dual face i, q^A parallel transports, E_i (link dual to face) changes by $-q^A$, p changes to conserve tot. energy

Particles: good and bad

- Good: (almost) no change to equil thermodynamics; therm easy.
- Good: reproduces Hard (thermal) Loops
- Bad: need very large number, very small charges
- Bad: CANNOT interpret literally as UV degrees of freedom!
- Bad: Fake “particle-UV lattice” interactions:
  UV latt modes dispersion
  $v = dE/dp = (1/2aE_k)\sum_i 2\sin(k_i a) < 1$ So particles, moving with $v = 1$, Cherenkov radiate. Dominates their interactions.

Useful for equilibrium, dubious nonequilibrium.

See however Dumitru Nara Strickland hep-ph/0604149

Method 2: W fields

Iancu hep-ph/9710543, Bödeker GM Rummukainen hep-ph/9907545, Arnold GM Yaffe hep-ph/0505121,

Rebhan Romatschke Strickland hep-ph/0412016,0505261

Replace $n_A(x,p)$ with $W_A(x,v) = \int d|p| n_A(x,p)$. Obey

$$
\begin{aligned}
D_\nu F_A^{\mu\nu}(x) &= j^\mu = \int_v v^\mu W_A(x,v) \\
v^\mu D_\mu W_A(x,v) &= \sum g^2 T_{\rm R} v^\nu F_{\nu\alpha} \partial_v^\alpha \Omega(v) \\
\Omega(v) &= \int \frac{4\pi p^2 dp}{(2\pi)^3} \partial_p^\alpha n_s
\end{aligned}
$$

Here $\Omega(v)$ is angular dependence of n_s, usually taken as spacetime independent. (Could compute back-reaction but no one has)

W fields on a lattice

Must make v space finite: two proposed ways:

- Spherical harmonic expansion (BMR'99):
  rewrite $W(x,v) = \sum_{lm} W_{lm}(x) Y_{lm}(v)$.
  Truncate at finite l (can cut m independently)
  Treat W_{lm} as fields.

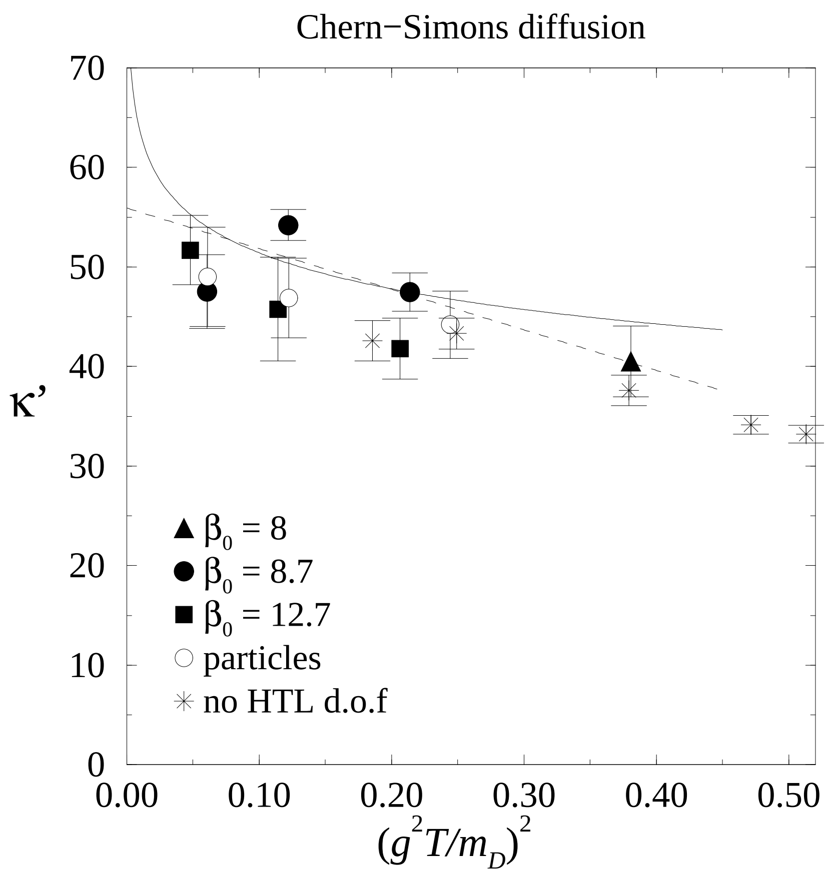
- Real-space "disco ball" discretization (RRS'04):
  tile the sphere with discrete directions v.

Systematic comparison is still lacking.

Other complications [for both]: linear derivatives...

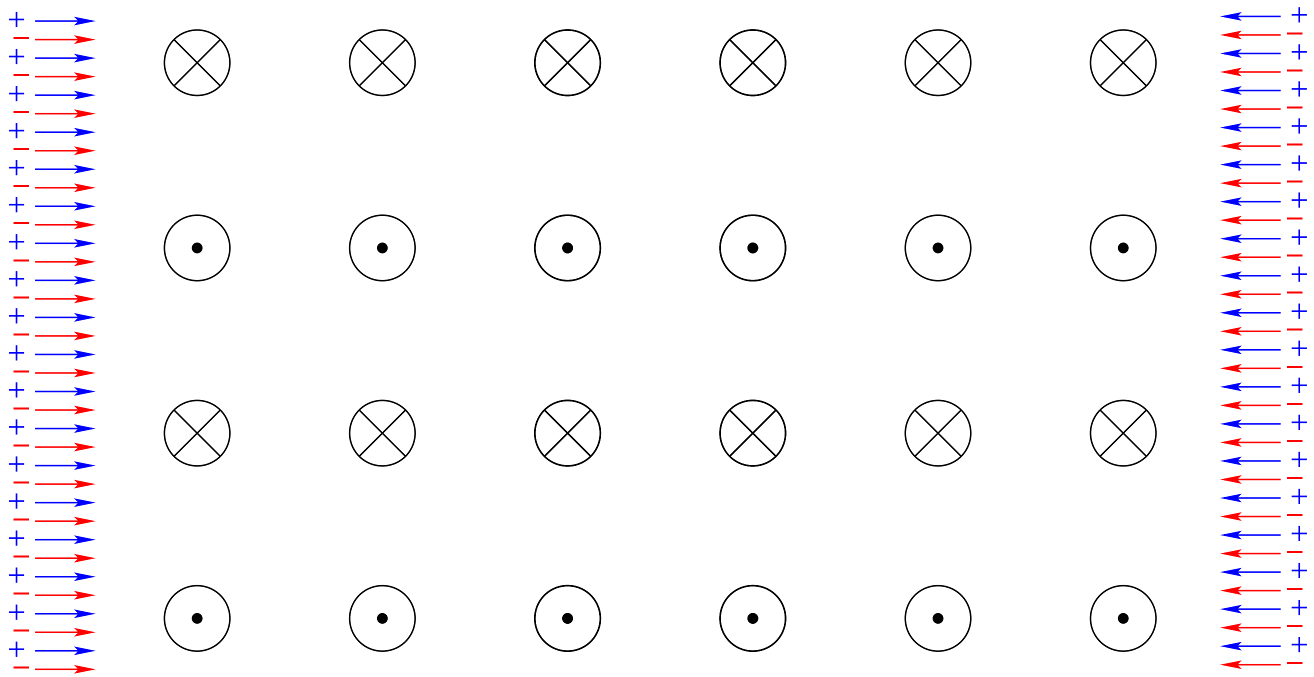
One comparison

Equilibrium studies of a quantity "sphaleron rate" sensitive only to nonperturbative IR fields' dynamics:



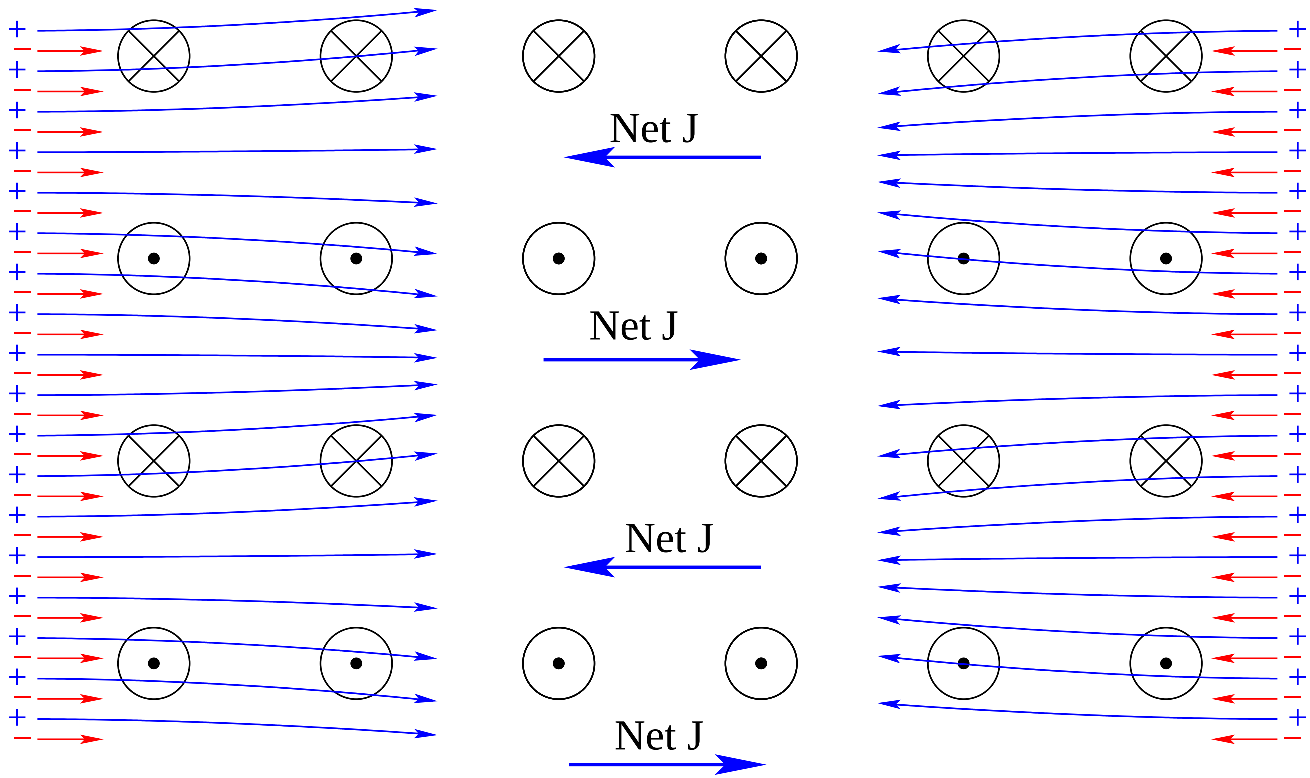
Plasma instabilities

Suppose all p are in-plane. Consider seed B: $\hat{B} \cdot \hat{p} = 0$ and $\hat{k} \cdot \hat{p} = 0$



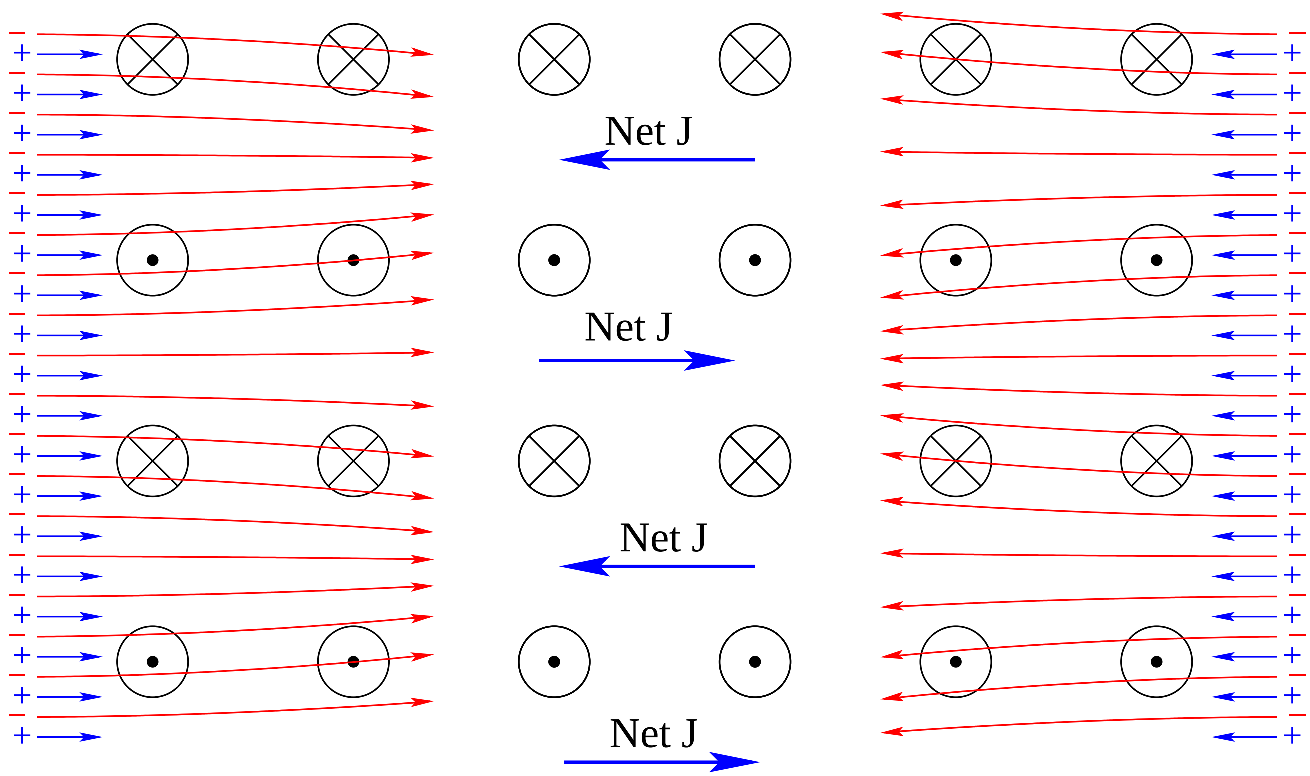
How do the particles deflect?

Positive charges:



No net ρ. Net current is induced as indicated.

Negative charges:



Induced B *adds* to seed B. Exponential Weibel instability

Linearized analysis: B grows until bending angles become large.

This instability is generic

Always occurs if

- weak coupling
- Momenta p dominating energy have $n(p) \ll 1/g^2$
- Typical momenta have $n(p)$ not isotropic
- IR occupancies not yet $1/g^2$ large

Instabilities studied using techniques we have discussed, but still not fully characterized.