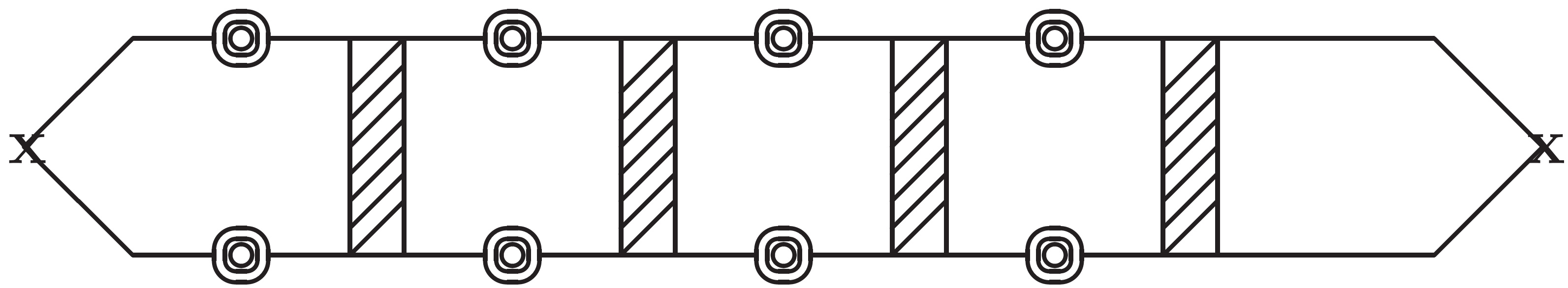
Some remarks on transport

- Boltzmann, transport, diagrams
- Higher order in couplings–what will happen?
- Higher order in gradients–2'nd order hydro
- Nonperturbative regime; story of Bulk Viscosity

Weak coupling

Weak coupling means long-lived quasiparticles.

Diagrammatically, leads to domination by ladder graphs:



Pairs of propagators act like propagating particles.

Once you know this you should never talk about graphs again.

Boltzmann equation

Leading order hydro described by distrib func $f(p,x)$.
Consider $f(p,x) = f(\beta, u^\mu, p_\mu)$ and derivatives.
Write as series expansion in number of derivatives:

$$f(\beta, u^\mu, p_\mu) = f_0(-\beta u^\mu p_\mu) + f_1(\beta, u, \partial u, p) + f_2(\beta, u, \partial^2(u,\beta), p)$$

Put in Boltzmann equation (written here with $2 \leftrightarrow 2$ only)
and expand order by order in total $\#$ of deriv's

$$2p^\mu \partial_\mu \; (f_0(p) + f_1 + \ldots) = -\mathcal{C}[p; f_0, f_1, f_2]\,,$$

$$-C[p;f] \; = \int_{kp'k'} (2\pi)^4 \delta^4(P+K-P'-K') |\mathcal{M}|^2 [p,k;p',k'] \times$$
$$\Big(f(p) f(k) [1 \pm f(p')][1 \pm f(k')] - (p,k \leftrightarrow p',k') \Big)$$

First order: defining ($\Delta_{\mu\nu} \equiv g_{\mu\nu} + u_\mu u_\nu$)

$$\sigma_{\mu\nu} = \Delta_{\mu\alpha}\Delta_{\nu\beta}\left(\partial^\alpha u^\beta + \partial^\beta u^\alpha - \frac{2}{3}g^{\alpha\beta}\partial_\gamma u^\gamma\right) ,$$

$$\begin{aligned} p^\mu \partial_\mu f_0(-\beta u^\nu p_\nu) \;&=\; \beta f_0' \times \Big(\frac{1}{2}p^i p^j \sigma_{ij}\beta f_0' + E^2(\partial_t \ln\beta - \partial_i u_i/3) \\ &\qquad + E p_i(\partial_i \ln\beta - \partial_t u_i)\Big) \end{aligned}$$

Last two lines determine $\partial_t(\beta, u_i)$, then cancel.

Expanding collision term to first order,

$$\begin{aligned} \frac{p_i p_j \sigma_{ij}\beta}{2} f_0' \;&=\; \int_{kp'k'} \ldots f_0(p) f_0(k)[1 \pm f_0(p')][1 \pm f_0(k')] \times \\ &\qquad \left(\frac{f_1(p)}{f_0(p)[1 \pm f_0(p)]} + (\ldots k) - (\ldots p') - (\ldots k')\right) \end{aligned}$$

Solution guaranteed to be of form

$$f_1(p) \equiv \mathcal{C}_1^{-1} \frac{p_i p_j \sigma_{ij} \beta}{2} f_0' = p_i p_j \sigma_{ij} \beta^3 \chi(-\beta p_\mu u^\mu)$$

Stress tensor

$$\begin{aligned} T_{ij} &= \int_p 2 p_i p_j f(p, \beta, u) = T_{ij,\mathrm{eq}} + \Pi_{ij} \, , \\ \Pi_{ij} &= \int_p 2 p_i p_j f_1 = \sigma_{lm} \int_p 2 p_i p_j p_l p_m \beta^3 \chi \\ &= \frac{2\sigma_{ij}}{15} \int_p 2 p^4 \beta^3 \chi \end{aligned}$$

is proportional to σ_{ij}. Proportionality is η,

$$\eta = -\frac{2}{15} \int_p 2 p^4 \beta^3 \chi(\beta E_p)$$

Next order?

Second-order in gradients terms in Boltzmann are

$$p^\mu \partial_\mu f_1(p,\beta,u,\partial) + \mathcal{C}_{11} = -\mathcal{C}_2$$

with $\mathcal{C}_{11}$ the collision term to quad order in f_1
and $\mathcal{C}_2$ the term to linear order in f_2.

We only need parts of f_2 contributing to $\int_p p_i p_j f_2$,
that is, $\ell = 2$ spherical harmonic terms.

Derivative term has several structures but straightforward:

$$\begin{aligned} p^\mu \partial_\mu f_1 &= p_i p_j \beta^3 \chi \left(\partial_t \sigma_{ij} + \frac{1}{3}\sigma_{ij}\partial_k u_k - \sigma_{ik}\sigma_{jk} - 2\sigma_{ik}\Omega_{ik} \right) \\ &\quad + p_i p_j p_k p_l \sigma_{ij}\sigma_{kl}\beta^4 \chi' \end{aligned}$$

General structure of terms

The departure f_2 will involve the same tensor structures; so

$$\begin{aligned}\Pi_{ij,2} &= \tau_\Pi \eta \left(\partial_t \sigma_{ij} + \frac{1}{3} \sigma_{ij} \partial_k u_k \right) \\ &\quad + \lambda_1 (\sigma_{ik} \sigma_{jk} - \mathrm{trace}) + \lambda_2 (\sigma_{ik} \Omega_{jk} - \mathrm{trace})\end{aligned}$$

Most general allowed form except missing Ω^2 term.

All coeff's but λ_1 determined by $p^\mu \partial_\mu f_1$.

These are easily computed with known techniques: obey

$$\lambda_2 = -2\tau_\Pi \eta$$

C_{11} terms structurally simpler, harder to analyze:

$$C_{11}[p, f] \propto p_i p_j \left(\sigma_{ik}\sigma_{jk} - \frac{1}{3}\delta_{ij}\sigma_{lm}\sigma_{lm}\right) \beta^4 \chi_{(2)}(-\beta p^\mu u_\mu)$$

Need to do $\int_{pp'kk'} |\mathcal{M}|^2$ integrals to find it!

Worse: matrix element depends on f_1!

$$\mathcal{M} \sim p^\mu (G_0^{-1\mu\nu} + \Pi^{\mu\nu}[f])^{-1} k^\nu$$

One contribution to C_{11} is from

$$\mathcal{M} \supset p^\mu \frac{1}{Q^2 + \Pi} \Pi_1[f_1] \frac{1}{Q^2 + \Pi} k^\nu$$

Lets in interesting physics, eg, $1/Q^2 + \Pi$ unscreened for $\omega \to 0$ magnetic exchange but Π_1 nonzero there.

Sizes and signs

Don't think η/s, think

$$\frac{\eta}{\epsilon + P} \equiv l_\eta \quad \text{a length [time?] scale}$$

roughly t for local relax to equil. ($\Gamma_{\rm sound} = 1/l_\eta k^2$.)

$$\frac{\tau_\Pi \eta}{\epsilon + P} = l_{\tau_\Pi}^2 \,, \qquad \frac{\lambda_1}{\epsilon + P} = l_{\lambda_1}^2$$

Expect dimensionless double ratio

$$\frac{(\epsilon + P)\lambda_1}{\eta^2} = \mathcal{O}(1)$$

If it's numerically large, convergence worse than expected.

First order in ∂, NLO in g

What can we say about higher order in g effects?

$\mathcal{O}(g)$ corrections can arise from gT momentum propagators.

Basically, $\mathcal{O}(g)$ soft corrections to Hard thermal loops.

Example: Fraction g of scatt. is from $k \sim gT$ particles.

Generic form of corrections:

$$(\eta^{-1}, \kappa, \ldots) = A g^n \left(\ln(T/m_D) + B + C \frac{N_c g^2 T}{m_D} \right)$$

Where computed, NLO coeff $C \sim 1$.

Computed $\mathcal{O}(g)$ corrections

- Heavy quark diffusion ($C = 1.4$) [Caron-Huot and Moore]
- Asymptotic mass m_∞^2: $C = 1$ [Caron-Huot unpublished]
- Heavy quark thermal mass shift [Chesler Vuorinen Caron-Huot]
- Transverse mom diff $\hat{q}$ [Caron-Huot unpublished]

Notice a pattern?

What $\mathcal{O}(g)$ corrections seem to imply

- Corrections are large: pert theory doesn't work well.
- Problem is strong coupling at gT scale, eg, gT, g^2T not well separated.
- Generally increase rate of scattering, lowering η...
- Looks like we need a resummation technique!