Thermalisation in the early universe

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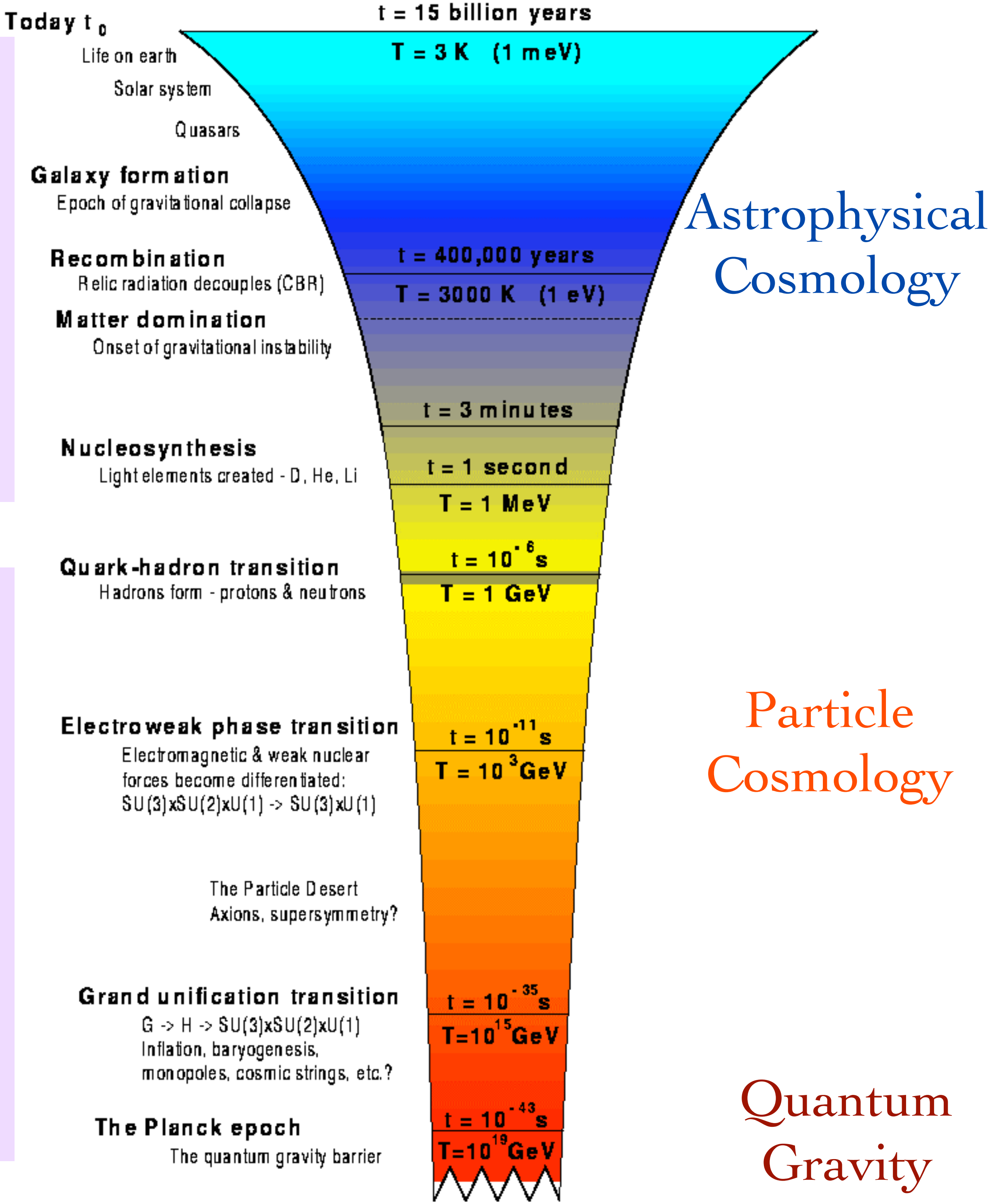
ICTS workshop on 'Initial conditions in heavy ion collisions', Dona Paula, 4 September 2008

The SM description of the early universe is a hot radiation-dominated plasma which is close to an *ideal gas* except near phase transitions … verified upto T ~ MeV

We believe that the hot phase was preceeded by accelerated expansion driven by an 'inflaton' field which then released its potential energy thus (re)heating the universe … some time between the EW and GUT scales



Scalar metric Fluctuations from Inflation

$$ds^2 = -(1-2\Phi)dt^2 + (1+2\Phi)a^2 d\vec{x}^2$$

$$\Omega_{tot} = 1$$

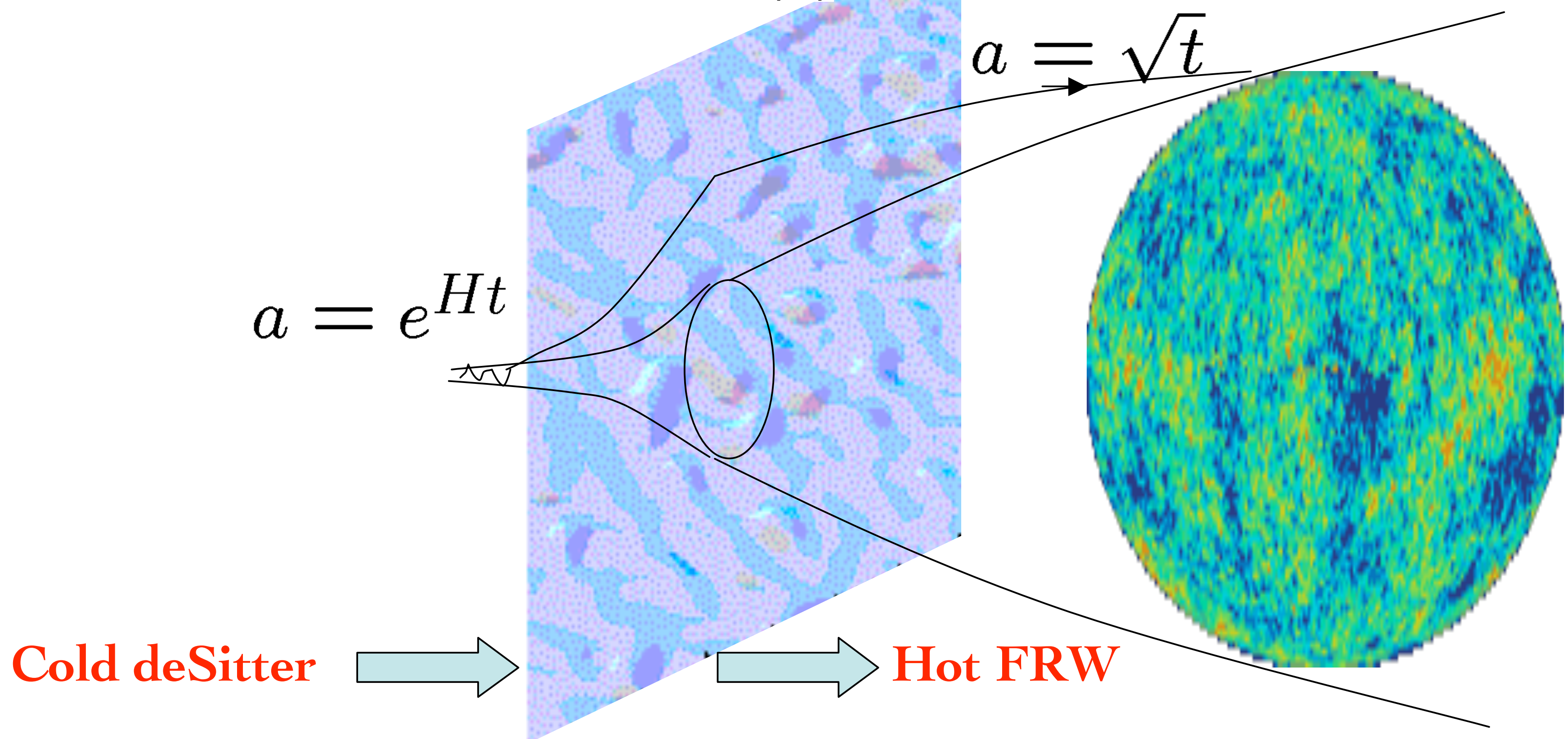
$$k^3 \Phi_k^2 \rightarrow P_s = A_s k^{n_s - 1}$$

$$P_T = \frac{H^2}{M_p^2} k^{n_T}$$

$$N = 62 - \ln \frac{10^{16} Gev}{V_{inf}^{1/4}}$$

Initial conditions from inflation

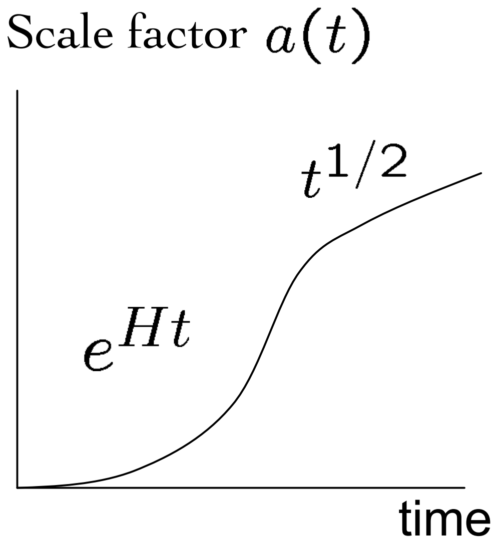
Random Gaussian Field $\Phi(\vec{x})$



(courtesey: L. Kofman)

Is there a possible analogy to heavy ion collisions? - RV

Early universe inflation



Equation of State $t \leq 10^{-35}$ sec

$p \approx -\epsilon$ slow roll $\dot{\phi}^2 \ll V$

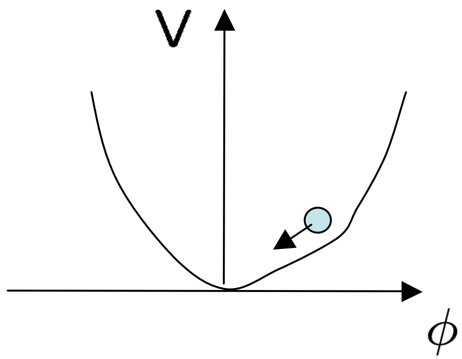
Inflation $a(t) \approx e^{Ht}$

Scalar field

$$\ddot{\phi} + 3H\dot{\phi} + V_{,\phi} = 0$$

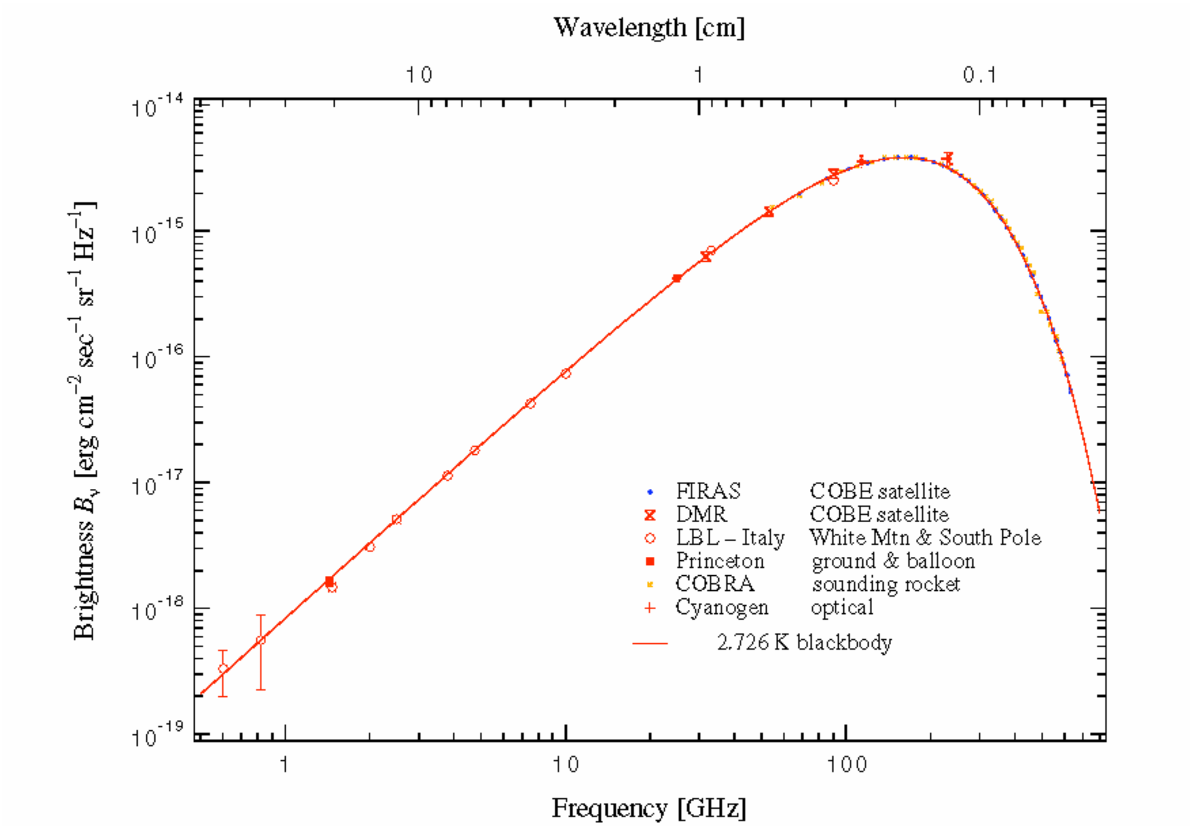
$$p = \frac{1}{2}\dot{\phi}^2 - V$$

$$\epsilon = \frac{1}{2}\dot{\phi}^2 + V$$



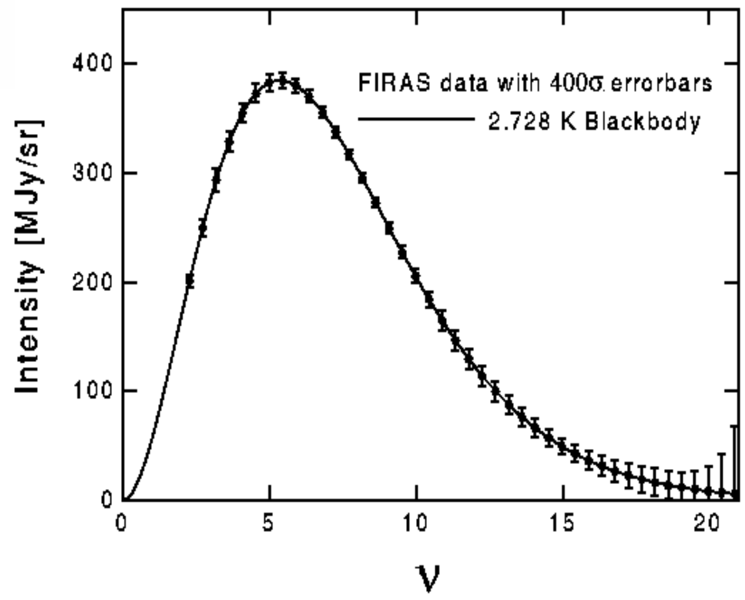
After inflation ends, the field oscillates about its minimum and transfers its energy density into other fields … the longer this goes on the greater is the energy loss due to Hubble expansion → T_{reh} is always *less* than the temperature at the start of inflation

What is the evidence that the early universe was thermalised?



The blackbody spectrum of the CMB testifies to our hot, dense past … also implies that there was *negligible* energy release after the thermalisation epoch of t ~ 10^5 s

However we know of no other *thermal* relic from the Big Bang … the baryonic matter certainly resulted from an out-of-equilibrium process and we do not yet know the nature or origin of the dark matter



Kompaneets equation

$$\frac{\partial \eta}{\partial t}|_{\rm C} = \frac{1}{t_{\gamma {\rm e}}} \frac{T_{\rm e}}{m_{\rm e}} \frac{1}{x^2} \frac{\partial}{\partial x} \left[x^4 \left(\frac{\partial \eta}{\partial x} + \eta + \eta^2 \right) \right], \quad x \equiv \frac{\omega}{T_{\rm e}}. \qquad t_{\gamma {\rm e}} = \frac{1}{(n_{\rm e} \sigma_{\rm T})}$$

… has as a general solution the Bose-Einstein form: $\eta_{\rm BE} = \frac{1}{({\rm e}^{x+\mu} - 1)}$

To create a Planck spectrum (with $\mu = 0$), we need 2 → 3 processes

$$\frac{\partial \eta}{\partial t}|_{\rm DC} = \frac{1}{t_{\gamma {\rm e}}} \left[\frac{4\alpha}{3\pi} \left(\frac{T_{\rm e}}{m_{\rm e}} \right)^2 I(t) \right] \frac{1}{x^3} \left[1 - \eta({\rm e}^x - 1) \right]$$

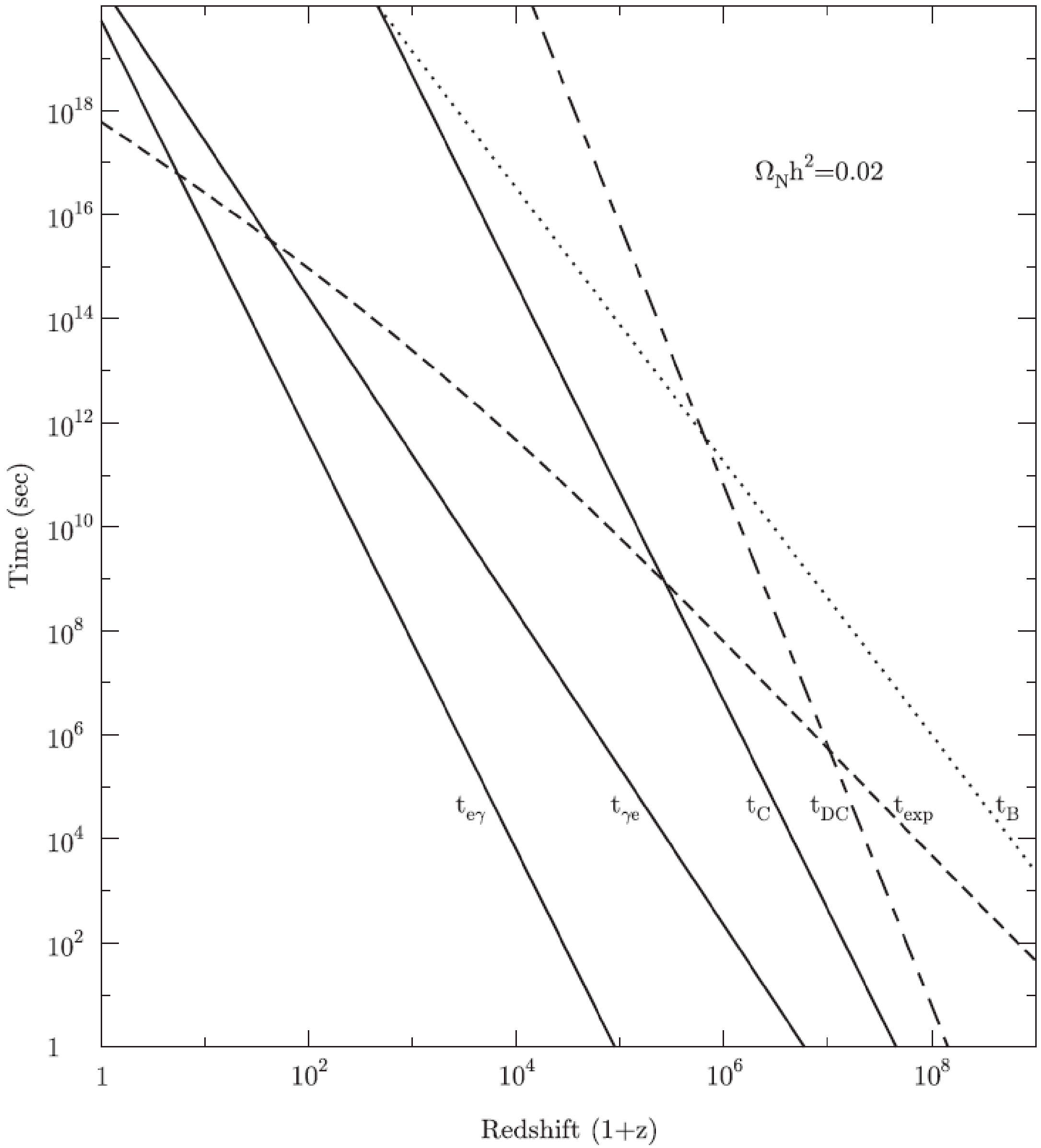
$$I(t) = \int x^4 (1 + \eta)\, \eta \, {\rm d}x.$$

$$\frac{\partial \eta}{\partial t}|_{\rm B} = \frac{1}{t_{\gamma {\rm e}}} \left[\frac{Q g(x)}{{\rm e}^x} \right] \frac{1}{x^3} \left[1 - \eta({\rm e}^x - 1) \right],$$

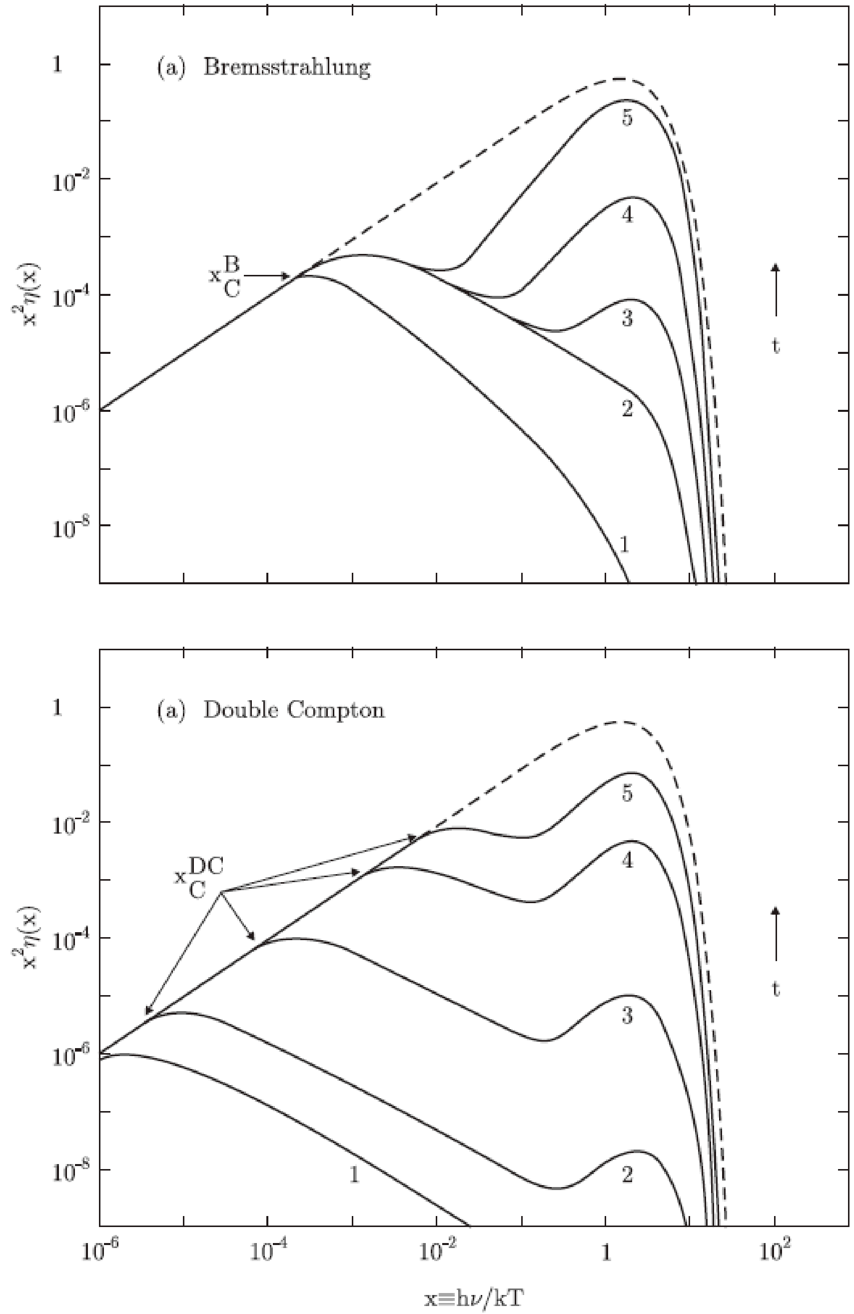
$$Q = 2\sqrt{2\pi} \left(\frac{m_{\rm e}}{T_{\rm e}} \right)^{1/2} \alpha \left(\sum n_{\rm ion} Z_{\rm ion}^2 \right) T_{\rm e}^{-3}$$

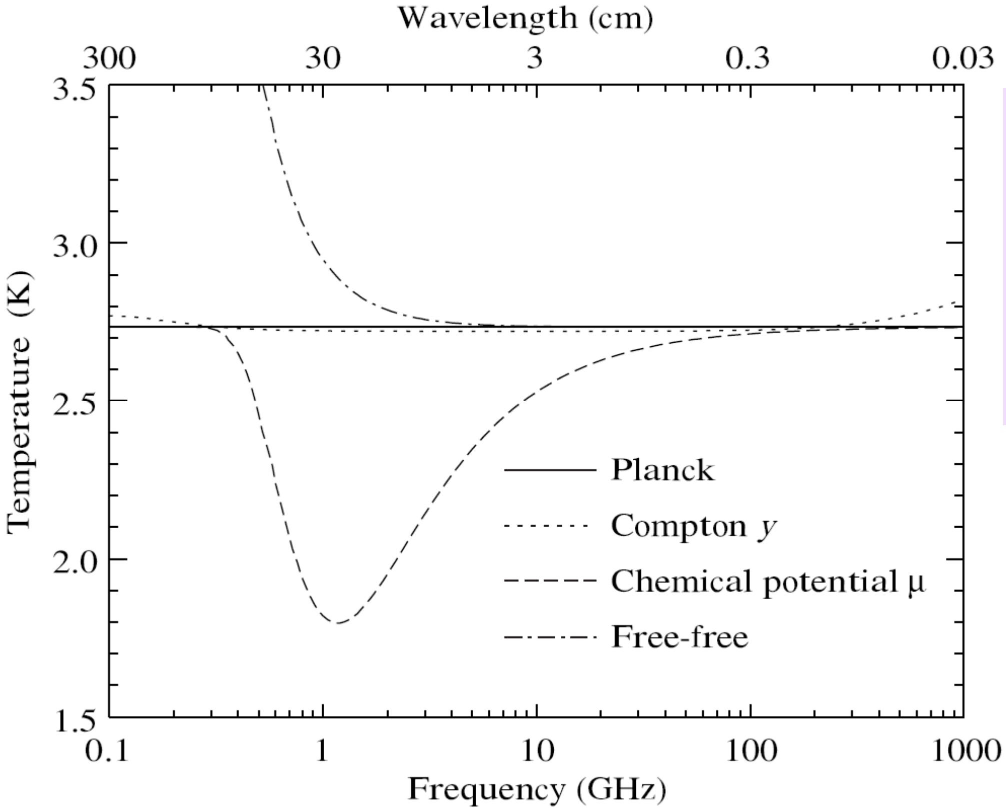
So the full equation is: $\frac{\partial \eta}{\partial t} = \frac{\partial \eta}{\partial t}|_{\rm C} + \frac{\partial \eta}{\partial t}|_{\rm B} + \frac{\partial \eta}{\partial t}|_{\rm DC}$

Relevant time scales



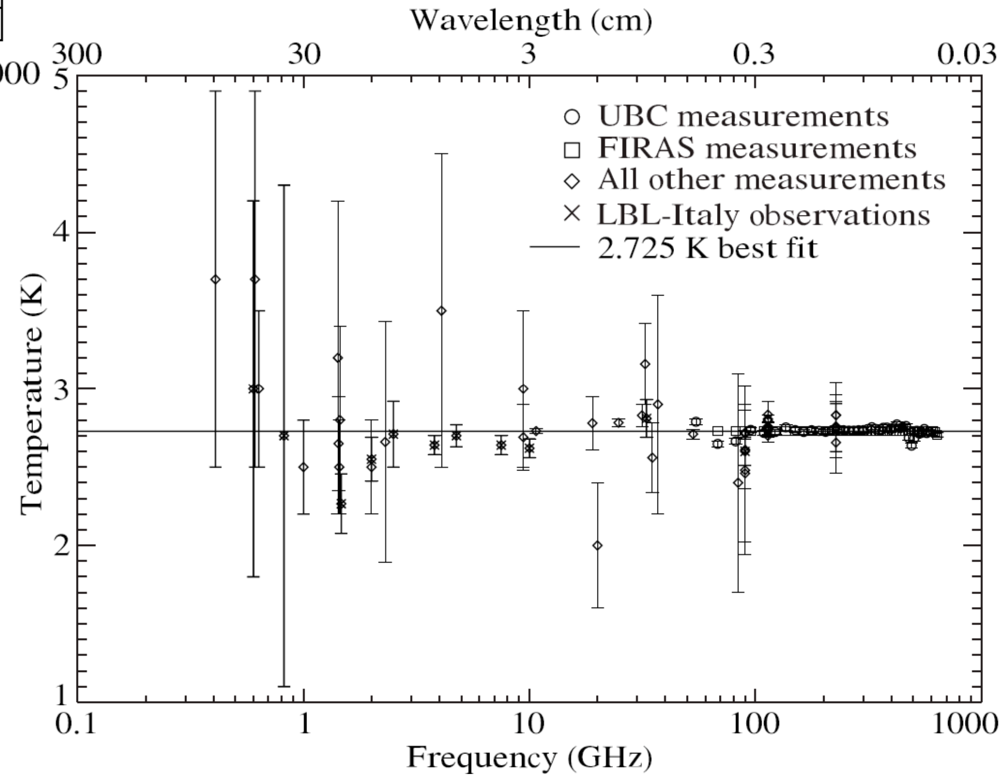
Spectral evolution





The expected spectral distortions are *not* observed … thus setting strong constraints on a possible photon chemical potential:

$$\mu \simeq 1.4\ \Delta\rho_\gamma/\rho_\gamma \ < 3.3 \times 10^{-4}$$

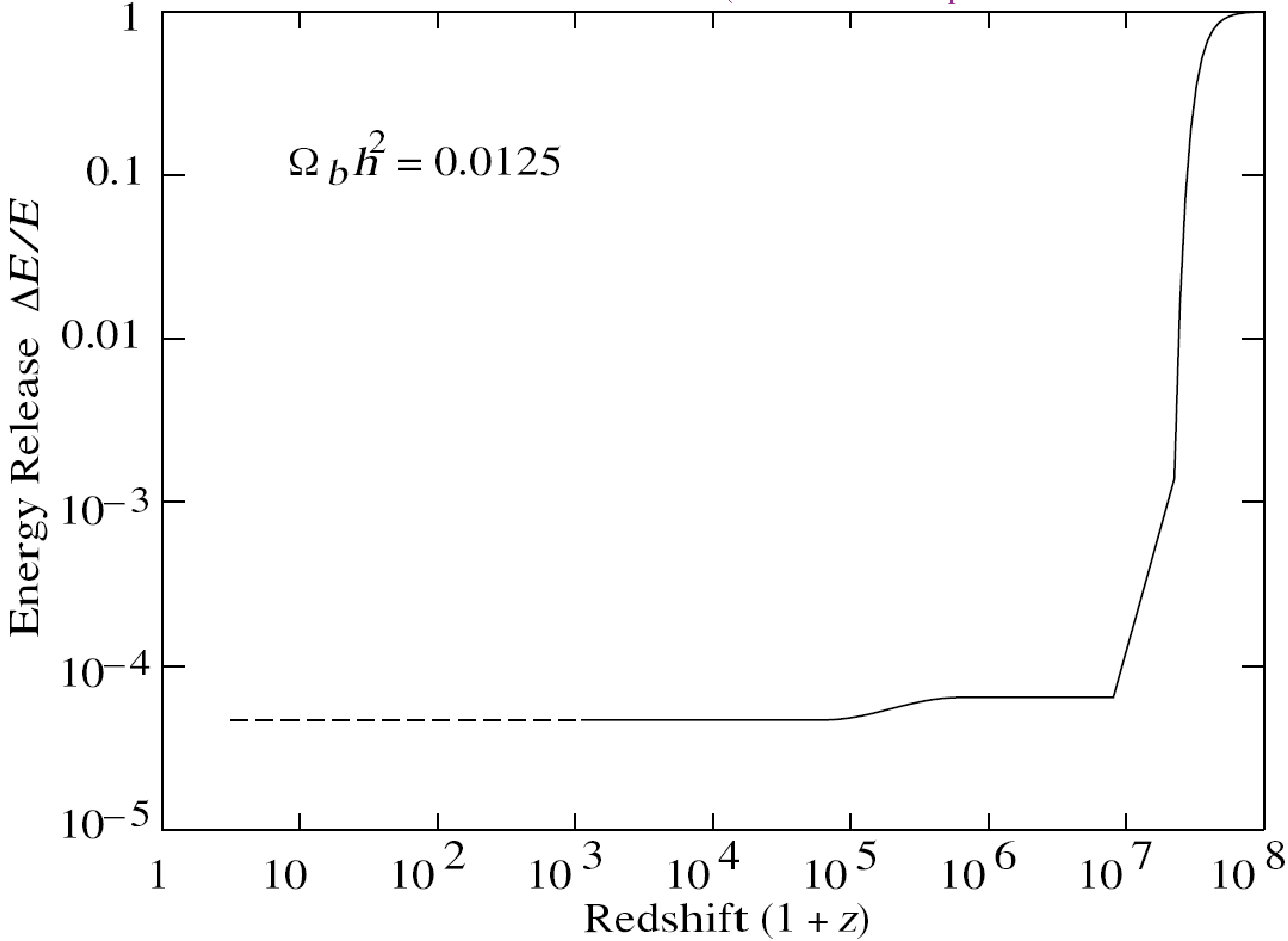


We can then infer the maximum allowed energy release back to the critical redshift where the radiative processes became efficient:

$$\int_{t(z_{\rm crit})}^{t_0} \frac{{\rm d}t}{t_{\rm scat}} = \int_0^{z_{\rm crit}} \frac{{\rm d}t}{{\rm d}z}\frac{{\rm d}z}{t_{\rm scat}} = 1$$

… so the CMB was fully thermalised before $z \sim 5 \times 10^7 \Rightarrow t \sim 10^5$ s

(Sarkar & Cooper 1984; Hu & Silk 1985)



Can similarly consider the problem of thermalisation following inflaton decay → **(re)heating of the universe**

Need to know decay rate Γ of inflaton field into SM fields (χ)
⇒ $T_{reh} \sim (\Gamma H)^{1/2}$ … essentially by energy conservation

(NB: decay through parametric resonance does *not* affect this)

As before, 2 →3 processes must be faster than the Hubble expansion rate to ensure true thermalisation

The timescale to produce a particle number density of $\mathcal{O}(T_{\rm reh}^3)$
is the *inelastic* energy loss timescale: $\sim (\alpha^3 T_{\rm reh}^2 / E_\chi)^{-1}$

The Universe will be thermalised within a Hubble time at T_{reh} if

$$E_\chi \lesssim \alpha^3 M_{P\ell}$$

Davidson & Sarkar [hep-ph/0009078]

How hot did the universe ever get?

It is not well known that the maximum temperature in the standard Big Bang cosmology *cannot* have reached even the GUT scale

Enqvist & Sirkka [hep-ph/9304273]

This is because particle interactions are *asymptotically free* …
so 2 ↔ 2 processes must have a #-secn $\propto \alpha^2(\mu)/s \sim \alpha^2(T)/T^2$
at high energies well above the particle masses involved

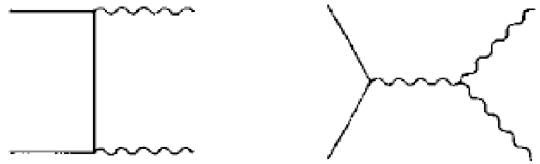
Thus the annihilation rate will go as: $\Gamma \sim \alpha^2 T$ (since $n \sim T^3$)
while the Hubble expansion rate goes as: $H \sim \sqrt{g_*} T^2/M_P$

so thermal equilibrium *cannot* be attained above:
$T \sim \alpha^2 M_P/\sqrt{g_*} \sim 2 \times 10^{14}$ GeV, taking $\alpha \sim 1/24$, $g_* \sim 915/4$

A careful estimate gives: T_{therm} = 2.5 x 10^{14} GeV

To lowest order, consider only $q\bar{q} \longrightarrow gg$

$q\bar{q} \longrightarrow ggg$ contributes just 3% more)



$$\Gamma_{\rm ann}(q\bar{q} \to gg) = \frac{1}{n_q} \int \frac{d^3p_1}{(2\pi)^3 E_1} \frac{d^3p_2}{(2\pi)^3 E_2} f(E_1/T) f(E_2/T) \sigma(q\bar{q} \to gg) v_{rel} p_1 \cdot p_2$$

$$\sigma(q\bar{q} \to gg) = \frac{2\pi\alpha_s^2}{N^2 s} \left[\left(\frac{2x^2+2x-1}{x(x-1)} \ln(\sqrt{x} + \sqrt{x-1}) - \frac{x+1}{\sqrt{x(x-1)}} \right) B + \left(\frac{1}{2x(x-1)} \ln(\sqrt{x} + \sqrt{x-1}) - \frac{1}{12} \frac{4x+5}{\sqrt{x(x-1)}} \right) A \right]$$

Where $x = s/4m^2$ and the $SU(N)$ colour factors are:

$$A = C_A T_F (N^2 - 1) = N/2(N^2-1) \qquad B = N C_F^2 = 1/(4N)(N^2-1)^2$$

At high temperatures, use the quark/gluon masses in the plasma:

$$m_g^2(T) = \frac{2}{3} g_s^2 T^2, \quad m_q^2(T) = \left(\frac{1}{6} g_s^2 + \frac{3}{32} g_W^2 + \frac{1}{288} g_Y^2\right) T^2$$

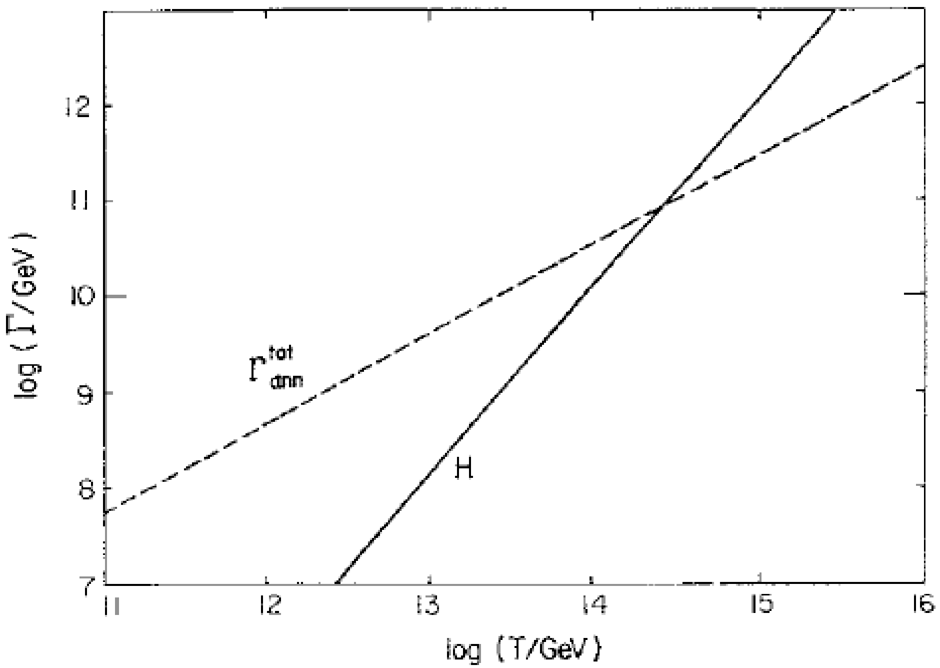
$$g_s^{-2}(\mu, T) = g_s^{-2}(\mu_0, 0) + \frac{1}{16\pi^2}\left[7 \ln\left(\frac{\mu}{\mu_0}\right)^2 + a_0(T/\mu) - a_0(0)\right]$$

and take into account that the the early universe gas is *interacting*, which modifies the ideal gas value (but only by a few percent):

$$\rho_{\rm SM} = \frac{\pi^2 T^4}{30}\left(g_*(T) - \frac{5}{2\pi}\left(84\alpha_s + \frac{57}{2}\alpha_W + \frac{25}{12}\alpha_Y\right)\right)$$

It is clear that chemical equilibrium cannot be maintained in the QCD gas above: T = 2.5 x 10^{14} GeV

… i.e. should reconsider GUT symmetry restoration by thermal effects (→monopole problem)



Conclusions

… many interesting connections - we should talk more!