Thermalisation in the early universe

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The SM description of the early universe is a hot radiation-dominated plasma which is close to an ideal gas except near phase transitions ... verified upto $T \sim \text{MeV}$.

We believe that the hot phase was preceded by accelerated expansion driven by an ‘inflaton’ field which then released its potential energy thus (re)heating the universe ... some time between the EW and GUT scales.
Scalar metric Fluctuations from Inflation
\[ ds^2 = -(1 - 2\Phi)dt^2 + (1 + 2\Phi)a^2d\bar{x}^2 \]

Initial conditions from inflation ➔

Random Gaussian Field \( \Phi(\bar{x}) \)

\[ a = e^{Ht} \]

\[ \Omega_{tot} = 1 \]
\[ k^3\Phi_k^2 \rightarrow P_s = A_s k^{n_s-1} \]
\[ P_T = \frac{H^2}{M_p^2} k^{n_T} \]
\[ N = 62 - \ln \frac{10^{16} Gev}{V_{inf}^{1/4}} \]

Cold deSitter ➔ Hot FRW

Is there a possible analogy to heavy ion collisions? - RV

(courtesy: L. Kofman)
Early universe inflation

After inflation ends, the field oscillates about its minimum and transfers its energy density into other fields \( \cdots \) the longer this goes on the greater is the energy loss due to Hubble expansion \( \rightarrow T_{\text{reh}} \) is always less than the temperature at the start of inflation.

**Scale factor** \( a(t) \)

**Equation of State** \( t \leq 10^{-35} \text{ sec} \)

\[ p \approx -\epsilon \quad \text{slow roll} \quad \dot{\phi}^2 \ll V \]

**Inflation** \( a(t) \approx e^{Ht} \)

**Scalar field**

\[ \ddot{\phi} + 3H\dot{\phi} + V_{,\phi} = 0 \]

\[ p = \frac{1}{2} \dot{\phi}^2 - V \]

\[ \epsilon = \frac{1}{2} \dot{\phi}^2 + V \]
What is the evidence that the early universe was thermalised?

The blackbody spectrum of the CMB testifies to our hot, dense past … also implies that there was *negligible* energy release after the thermalisation epoch of $t \sim 10^5$ s.

However we know of no other *thermal* relic from the Big Bang … the baryonic matter certainly resulted from an out-of-equilibrium process and we do not yet know the nature or origin of the dark matter.
To create a Planck spectrum (with $\mu = 0$), we need $2 \rightarrow 3$ processes

\[
\frac{\partial \eta}{\partial t} |_{DC} = \frac{1}{t_{\gamma e}} \left[ \frac{4 \alpha}{3 \pi} \left( \frac{T_e}{m_e} \right)^2 I(t) \right] \frac{1}{x^3} \left[ 1 - \eta (e^x - 1) \right]
\]

\[
I(t) = \int x^4 (1 + \eta) \eta \, dx.
\]

\[
\frac{\partial \eta}{\partial t} |_B = \frac{1}{t_{\gamma e}} \left[ \frac{Q g(x)}{e^x} \right] \frac{1}{x^3} \left[ 1 - \eta (e^x - 1) \right],
\]

\[
Q = 2 \sqrt{2 \pi} \left( \frac{m_e}{T_e} \right)^{1/2} \alpha \left( \sum n_{\text{ion}} Z_{\text{ion}}^2 \right) T_e^{-3}
\]

So the full equation is:

\[
\frac{\partial \eta}{\partial t} = \frac{\partial \eta}{\partial t} |_C + \frac{\partial \eta}{\partial t} |_B + \frac{\partial \eta}{\partial t} |_{DC}
\]
Relevant time scales

Spectral evolution
The expected spectral distortions are not observed … thus setting strong constraints on a possible photon chemical potential:

\[ \mu \approx 1.4 \Delta \rho_\gamma / \rho_\gamma < 3.3 \times 10^{-4} \]

We can then infer the maximum allowed energy release back to the critical redshift where the radiative processes became efficient:

\[ \int_{t(z_{\text{crit}})}^{t_0} \frac{dt}{t_{\text{scat}}} = \int_{0}^{z_{\text{crit}}} \frac{dt}{dz} \frac{dz}{t_{\text{scat}}} = 1 \]
... so the CMB was fully thermalised before $z \sim 5 \times 10^7 \Rightarrow t \sim 10^5$ s

(Sarkar & Cooper 1984; Hu & Silk 1985)
Can similarly consider the problem of thermalisation following inflaton decay $\rightarrow$ (re)heating of the universe

Need to know decay rate $\Gamma$ of inflaton field into SM fields ($\chi$)

$$T_{\text{reh}} \sim (\Gamma H)^{1/2}$$

... essentially by energy conservation

(NB: decay through parametric resonance does not affect this)

As before, $2 \rightarrow 3$ processes must be faster than the Hubble expansion rate to ensure true thermalisation

The timescale to produce a particle number density of $O(T_{\text{reh}}^3)$ is the inelastic energy loss timescale:

$$\sim \left( \alpha^3 T_{\text{reh}}^2 / E_\chi \right)^{-1}$$

The Universe will be thermalised within a Hubble time at $T_{\text{reh}}$ if

$$E_\chi \lesssim \alpha^3 M_P, \quad \text{Davidson & Sarkar [hep-ph/0009078]}$$
How hot did the universe ever get?

It is not well known that the maximum temperature in the standard Big Bang cosmology cannot have reached even the GUT scale

Enqvist & Sirkka [hep-ph/9304273]

This is because particle interactions are asymptotically free … so $2 \leftrightarrow 2$ processes must have a $\#-secn \propto \alpha^2(\mu)/s \sim \alpha^2(T)/T^2$ at high energies well above the particle masses involved.

Thus the annihilation rate will go as: $\Gamma \sim \alpha^2 T$ (since $n \sim T^3$) while the Hubble expansion rate goes as: $H \sim \sqrt{g_\ast T^2}/M_P$

so thermal equilibrium cannot be attained above:

$T \sim \alpha^2 M_P/\sqrt{g_\ast} \sim 2 \times 10^{14}$ GeV, taking $\alpha \sim 1/24, g_\ast \sim 915/4$
A careful estimate gives: $T_{\text{therm}} = 2.5 \times 10^{14}$ GeV

To lowest order, consider only $q\bar{q} \rightarrow gg$

$q\bar{q} \rightarrow ggg$ contributes just 3% more)

$$
\Gamma_{\text{ann}}(q\bar{q} \rightarrow gg) = \frac{1}{n_q} \int \frac{d^3p_1}{(2\pi)^3 E_1} \frac{d^3p_2}{(2\pi)^3 E_2} f(E_1/T) f(E_2/T) \sigma(q\bar{q} \rightarrow gg) v_{\text{rel}} p_1 \cdot p_2
$$

$$
\sigma(q\bar{q} \rightarrow gg) = \frac{2\pi \alpha_s^2}{N^2 s} \left[ \left( \frac{2x^2 + 2x - 1}{x(x-1)} \ln(\sqrt{x} + \sqrt{x-1}) - \frac{x+1}{\sqrt{x(x-1)}} \right) B \\
+ \left( \frac{1}{2x(x-1)} \ln(\sqrt{x} + \sqrt{x-1}) - \frac{1}{12} \frac{4x+5}{\sqrt{x(x-1)}} \right) A \right]
$$

Where $x = s/4m^2$ and the $SU(N)$ colour factors are:

$$
A = C_A T_F (N^2 - 1) = N/2(N^2 - 1) \\
B = N C_F^2 = 1/(4N)(N^2 - 1)^2
$$
At high temperatures, use the quark/gluon masses in the plasma:

\[ m_{q}^2(T) = \frac{2}{3} g_s^2 T^2, \quad m_{g}^2(T) = \left( \frac{1}{6} g_s^2 + \frac{3}{32} g_W^2 + \frac{1}{288} g_Y^2 \right) T^2 \]

\[ g_s^{-2}(\mu, T) = g_s^{-2}(\mu_0, 0) + \frac{1}{16\pi^2} \left[ 7 \ln \left( \frac{\mu}{\mu_0} \right)^2 + a_0(T/\mu) - a_0(0) \right] \]

and take into account that the early universe gas is interacting, which modifies the ideal gas value (but only by a few percent):

\[ \rho_{SM} = \frac{\pi^2 T^4}{30} \left( g_*(T) - \frac{5}{2\pi} \left( 84\alpha_s + \frac{57}{2}\alpha_W + \frac{25}{12}\alpha_Y \right) \right) \]

It is clear that chemical equilibrium cannot be maintained in the QCD gas above: \( T = 2.5 \times 10^{14} \text{ GeV} \)

... i.e. should reconsider GUT symmetry restoration by thermal effects (monopole problem)
Conclusions

... many interesting connections - we should talk more!