Initial conditions in heavy ion collisions
Lecture II: Leading order particle production

Tuomas Lappi
IPhT, CEA/Saclay
tuomas.lappi@cea.fr

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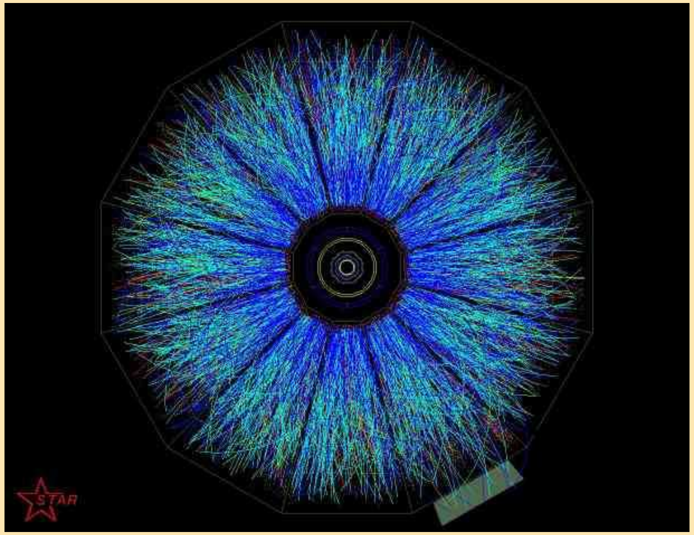
Outline

- Brief introduction: why classical fields?
- Leading order classical field, 1 nucleus
- 2 colliding nuclei
  - Initial condition
  - Weak field limit
  - Classical Hamiltonian chromodynamics on the lattice and in an expanding system
  - Some numerical results



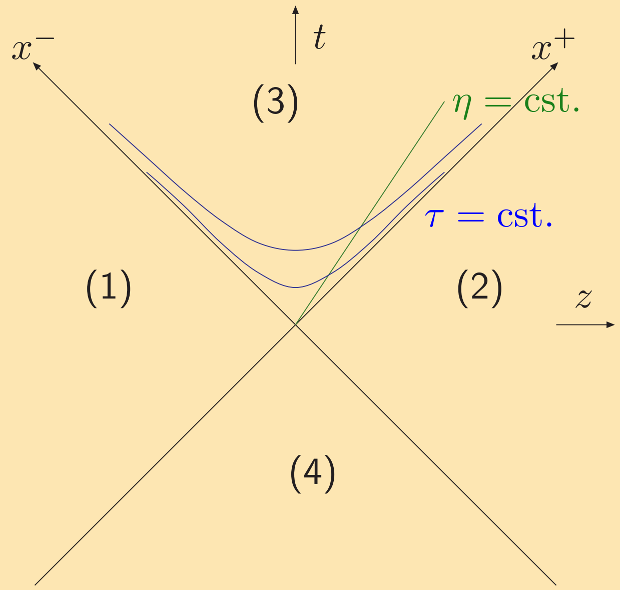
Heavy ion collision: spacetime picture

1. Initial condition	$\tau = 0$	▶	Small-x nuclear wavefunction
2. Equilibration	$\tau \lesssim \tau_0$	▶	Time-dependent, **nonequilibrium** QFT
3. Plasma (?)	$\tau_0 \lesssim \tau \lesssim 10\text{fm}$	▶	Finite-T **equilibrium** QFT + hydro
4. Hadronisation	$\tau \sim 10\text{fm}$	▶	nonperturbative QCD

Light cone coordinates: $x^{\pm} = \frac{1}{\sqrt{2}}(t \pm z)$

Proper time coordinates:

$$\tau = \sqrt{t^2 - z^2}$$
$$\eta = \frac{1}{2} \ln \frac{x^+}{x^-}$$



What happens on the line $\eta = 0$, $t > 0$?

Gluon saturation, Glass and Glasma

Gluon saturation: At large energies (small x) the hadron/nucleus wavefunction is characterized by saturation scale $Q_{\mathrm{s}} \gg \Lambda_{\mathrm{QCD}}$.

▼

At $\boldsymbol{p} \sim Q_{\mathrm{s}}$: strong gluon fields $A_\mu \sim 1/g$

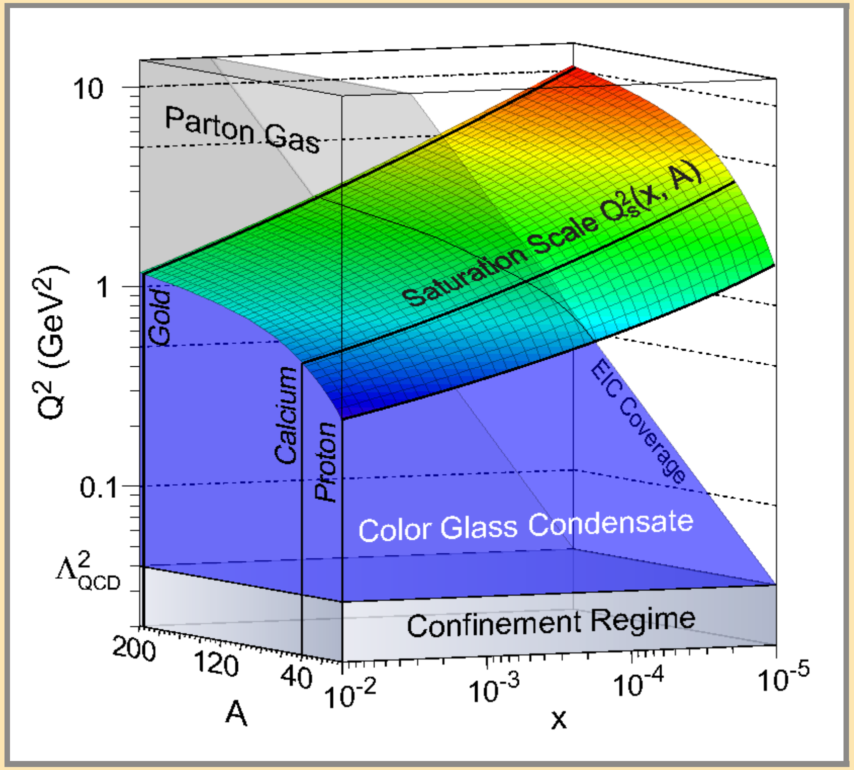
- large occupation numbers $\sim 1/\alpha_{\mathrm{s}}$
- classical field approximation.

▼

CGC: The saturated wavefunction of one hadron/nucleus
Effective theory description with large x =source, small x=radiated classical gluon field.

▼

Glasma: [1] The coherent, classical field configuration of two colliding sheets of CGC.



Pocket formula for "oomph":

$$Q_{\mathrm{s}}^2 \sim A^{1/3} x^{-0.3}$$

- To see high gluon density effects: go to small x and large nuclei.

[1] T. Lappi and L. McLerran, *Nucl. Phys.* **A772** (2006) 200 [hep-ph/0602189].

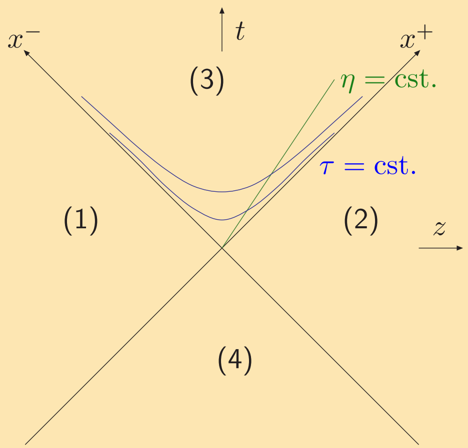
Weizsäcker-Williams color field, MV model

Separation of scales between small x and large x:

classical field **color charge**

$$[D_\mu, F^{\mu\nu}] = J^\nu$$

$$J^\mu = \delta^{\mu+}\rho_{(1)}(\underset{\sim}{\boldsymbol{x}})\delta(x^-) + \delta^{\mu-}\rho_{(2)}(\underset{\sim}{\boldsymbol{x}})\delta(x^+)$$



What is the charge density $\rho(\underset{\sim}{\boldsymbol{x}})$? A static (**glass!**) stochastic variable, distribution

$$\boxed{W_y[\rho(\underset{\sim}{\boldsymbol{x}})]}$$

E.g. MV model [2]: $$W[\rho(\underset{\sim}{\boldsymbol{x}})] \sim \exp\left[-\frac{1}{2}\int \mathrm{d}^2\underset{\sim}{\boldsymbol{x}}\rho^a(\underset{\sim}{\boldsymbol{x}})\rho^a(\underset{\sim}{\boldsymbol{x}})/g^2\mu^2\right]$$

Cannot compute $W_y[\rho(\underset{\sim}{\boldsymbol{x}})]$ from first principles, but can derive evolution equation for $y = \ln 1/x$–dependence: **JIMWLK**. Jalilian-Marian, Iancu, McLerran, Weigert, Leonidov, Kovner ▶ More in next lecture

[2] L. D. McLerran and R. Venugopalan, *Phys. Rev.* **D49** (1994) 2233 [hep-ph/9309289].

Field of one nucleus

Current $J^\mu = \delta^{\mu +}\rho(\underset{\sim}{\boldsymbol{x}}, x^-)$ (relax δ-function but still no x^+-dependence. EoM has solution

$$A^+(\underset{\sim}{\boldsymbol{x}}, x^-) = \frac{1}{{\boldsymbol{\nabla}_T}^2}\rho(\underset{\sim}{\boldsymbol{x}}, x^-), \quad A^- = A^i = 0$$

This is known the **covariant gauge** field; transform to **light cone gauge** $A^+ = 0$:

$$A^+ \quad \Rightarrow \quad U^\dagger(\underset{\sim}{\boldsymbol{x}}, x^-)\frac{\rho(\underset{\sim}{\boldsymbol{x}}, x^-)}{{\boldsymbol{\nabla}_T}^2}U(\underset{\sim}{\boldsymbol{x}}, x^-) - \frac{i}{g}U^\dagger(\underset{\sim}{\boldsymbol{x}}, x^-)\partial_- U(\underset{\sim}{\boldsymbol{x}}, x^-) = 0 \qquad (1)$$

$$A^- \quad \Rightarrow \quad -\frac{i}{g}U^\dagger(\underset{\sim}{\boldsymbol{x}}, x^-)\partial_+ U(\underset{\sim}{\boldsymbol{x}}, x^-) = 0, \text{ still}$$

$$A^i \quad \Rightarrow \quad \frac{i}{g}U^\dagger(\underset{\sim}{\boldsymbol{x}}, x^-)\partial_i U(\underset{\sim}{\boldsymbol{x}}, x^-)$$

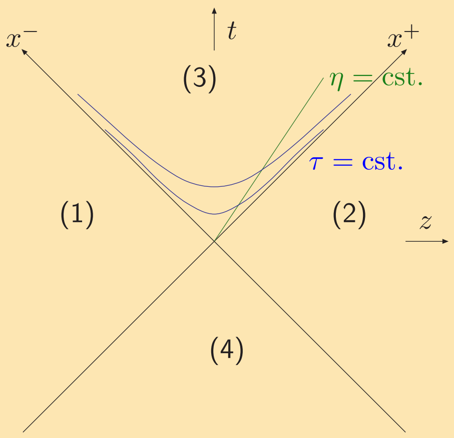
(1) is solved by a **path ordered exponential** or **Wilson line**

$$U^\dagger(\underset{\sim}{\boldsymbol{x}}, x^-) = \mathbb{P}\exp\left[-ig\int^{x^-} \mathrm{d}y^- \rho(\underset{\sim}{\boldsymbol{x}}, y^-)/{\boldsymbol{\nabla}_T}^2\right]$$

Note: A^i expressed in terms of **covariant** gauge source

Fields look very different in different gauges

Covariant: $A^+ \sim \delta(x^-)$ is singular and only lives on the light cone



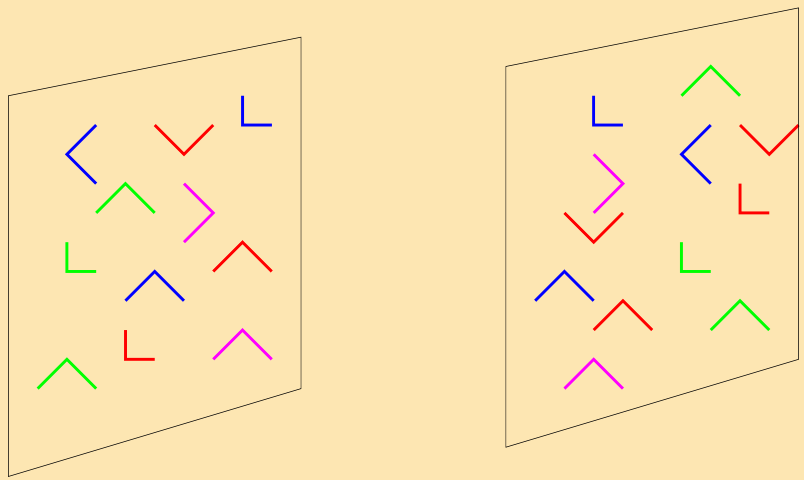
LC: $A_i \sim \theta(x^-)$ is discontinuous, but not singular, and lives above the light cone

Large extent in x^-, small k^+, small x_{Bj} ▶ uncertainty principle works in LC gauge, this is how one would quantize.

But field stregth tensor $F^{\mu\nu}$ is of course the same (up to a gauge rotation), only nonzero components

$$F^{+i}_{\text{cov.}} = \partial_i A^+_{\text{cov.}} \sim \delta(x^-)$$

Both $E_\perp$ (F^{0i}) and $B_\perp$ (F^{zi}) fields.



From glass to glasma: initial condition

Following KMW [3]: work in Fock-Schwinger/temporal gauge $A_\tau = \left(x^+ A^- + x^- A^-\right)/\tau = 0$ ▶ consistent with LC gauge solutions for both nuclei. Ansatz:

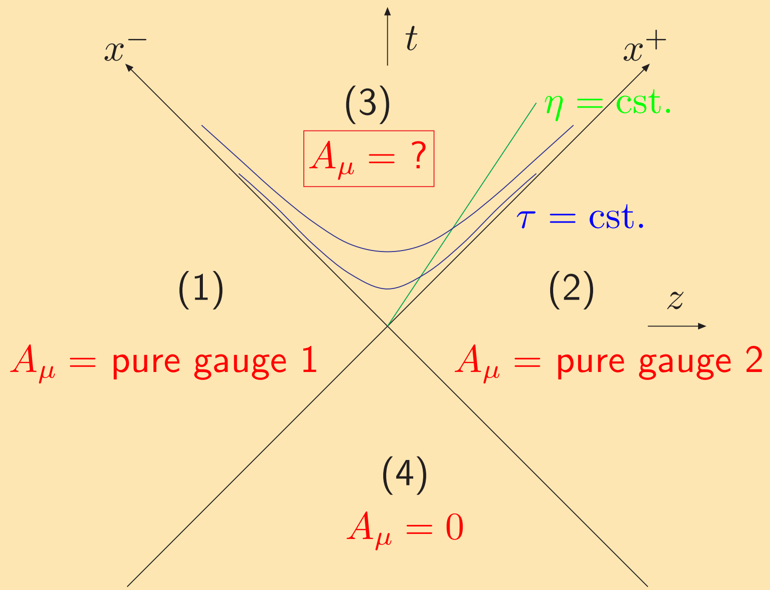
$$\begin{aligned} A_i &= \overbrace{A_i^{(1)}\theta(-x^+)\theta(x^-) + A_i^{(2)}\theta(x^+)\theta(-x^-)}^{\text{known}} + A_i^{(3)}\theta(x^+)\theta(x^-) \\ A^\pm &= \pm\theta(x^+)\theta(x^-)x^\pm A^\eta \end{aligned}$$

Insert into EoM and match δ-functions

▼

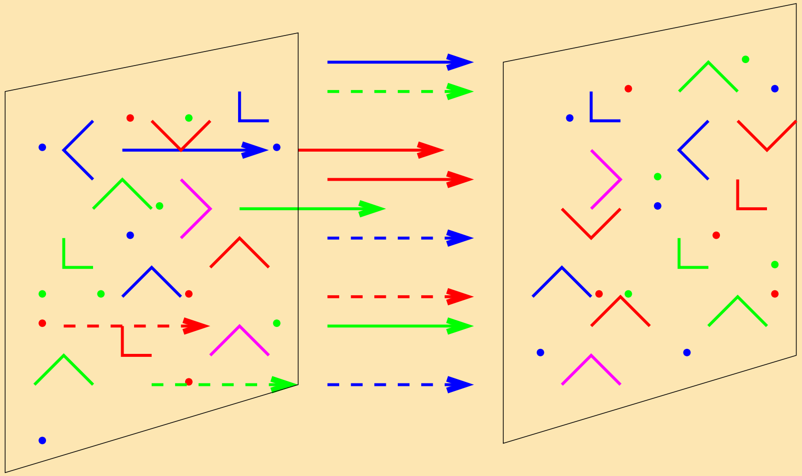
initial condition for region (3):

$$\begin{aligned} A_i^{(3)}|_{\tau=0} &= A_i^{(1)} + A_i^{(2)} \\ A^\eta|_{\tau=0} &= \frac{ig}{2}\left[A_i^{(1)}, A_i^{(2)}\right] \end{aligned}$$



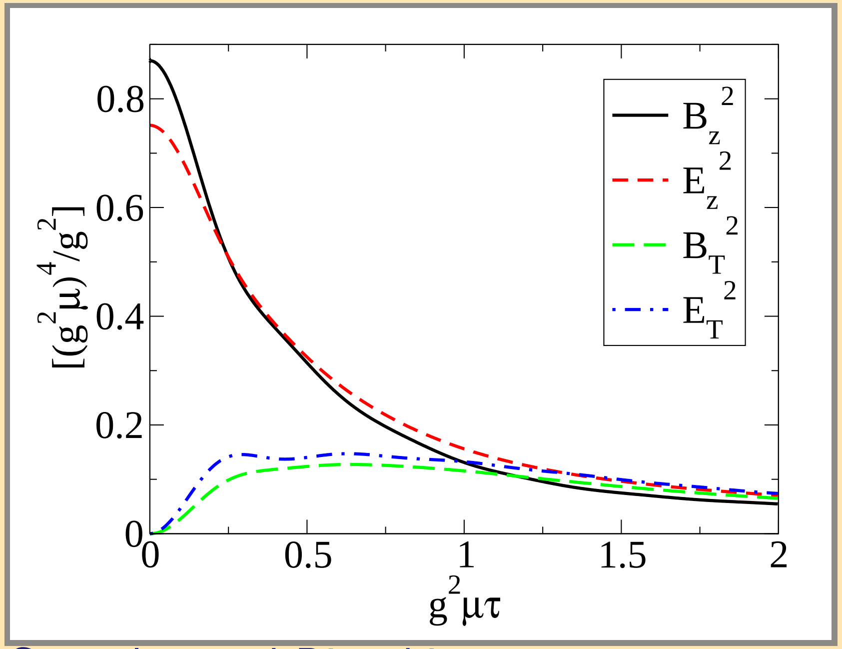
[3] A. Kovner, L. D. McLerran and H. Weigert, *Phys. Rev.* **D52** (1995) 3809 [hep-ph/9505320].

Glasma field after the collision



Initial condition is longitudinal E and B field, depending on transverse coordinate with correlation length $1/Q_{\mathrm{s}}$.

Effective electric and magnetic charge densities.



Gauss law and Bianchi:

$$[D_i, E^i] = 0, \quad [D_i, B^i] = 0$$

Separate nonabelian parts:

$$\partial_i E^i = ig[A^i, E^i], \quad \partial_i B^i = ig[A^i, B^i]$$

Digression: gluon production weak field limit

$$\text{Introduce: } \Lambda_{(m)}(\underset{\sim}{\boldsymbol{x}}) = -g\frac{\rho_{(m)}(\underset{\sim}{\boldsymbol{x}})}{\boldsymbol{\nabla}_T{}^2}, \qquad \text{i.e. } gA^+_{\text{cov}} = \delta(x^-)\Lambda_{(1)}, \quad gA^-_{\text{cov}} = \delta(x^+)\Lambda_{(2)}$$

Note: covariant gauge field Λ is dimensionless (dimension of $A^\pm$ is in $\delta(x^\mp)$); our expansion parameter.

Initial conditions:

$$\begin{aligned} A_i(0,\underset{\sim}{\boldsymbol{x}}) &= -\frac{\partial_i}{g}\left(\Lambda_{(1)}(\underset{\sim}{\boldsymbol{x}}) + \Lambda_{(2)}(\underset{\sim}{\boldsymbol{x}})\right) + \frac{i}{2g}\left[\partial_i\Lambda_{(1)}(\underset{\sim}{\boldsymbol{x}}), \Lambda_{(1)}(\underset{\sim}{\boldsymbol{x}})\right] \\ &\quad + \frac{i}{2g}\left[\partial_i\Lambda_{(2)}(\underset{\sim}{\boldsymbol{x}}), \Lambda_{(2)}(\underset{\sim}{\boldsymbol{x}})\right] \qquad (2) \\ A^\eta(0,\underset{\sim}{\boldsymbol{x}}) &= -\frac{i}{2g}\left[\partial_i\Lambda_{(1)}(\underset{\sim}{\boldsymbol{x}}), \partial_i\Lambda_{(2)}(\underset{\sim}{\boldsymbol{x}})\right]. \end{aligned}$$

Fix 2d Coulomb gauge ▶ Removes leading order "Abelian pure gauge" part and (2) becomes

$$A_i^{\text{Coul}}(0,\underset{\sim}{\boldsymbol{x}}) = \frac{i}{2g}\left(\delta_{ij} - \frac{\partial_i\partial_j}{\boldsymbol{\nabla}_T{}^2}\right)\left(\left[\Lambda_{(1)}(\underset{\sim}{\boldsymbol{x}}), \partial_j\Lambda_{(2)}(\underset{\sim}{\boldsymbol{x}})\right] + \left[\Lambda_{(2)}(\underset{\sim}{\boldsymbol{x}}), \partial_j\Lambda_{(1)}(\underset{\sim}{\boldsymbol{x}})\right]\right).$$

Weak field limit 2: Bertsch-Gunion

Linearized equations are now free wave propagation ($A_\eta \equiv -\tau^2 A^\eta$)

$$\left(\tau^2\partial_\tau^2 + \tau\partial_\tau + \tau^2 \underset{\sim}{\boldsymbol{k}}^2\right) A_i(\tau, \underset{\sim}{\boldsymbol{k}}) = 0$$
$$\left(\tau^2\partial_\tau^2 - \tau\partial_\tau + \tau^2 \underset{\sim}{\boldsymbol{k}}^2\right) A_\eta(\tau, \underset{\sim}{\boldsymbol{k}}) = 0.$$

with solutions

$$A_i(\tau, \underset{\sim}{\boldsymbol{k}}) = A_i(0, \underset{\sim}{\boldsymbol{k}}) J_0(|\underset{\sim}{\boldsymbol{k}}|\tau) \qquad A_\eta(\tau, \underset{\sim}{\boldsymbol{k}}) = \frac{\tau}{|\underset{\sim}{\boldsymbol{k}}|}\pi(0, \underset{\sim}{\boldsymbol{k}}) J_1(|\underset{\sim}{\boldsymbol{k}}|\tau).$$

Hamiltonian ▶ average over sources ρ ▶ identify $\frac{E}{\mathrm{d}\eta} = \int \mathrm{d}^2\underset{\sim}{\boldsymbol{q}} |\underset{\sim}{\boldsymbol{q}}| \frac{\mathrm{d}N}{\mathrm{d}^2\underset{\sim}{\boldsymbol{q}}\,\mathrm{d}y}$ ▶

$$\frac{\mathrm{d}N}{\mathrm{d}^2\underset{\sim}{\boldsymbol{q}}\,\mathrm{d}y} = g^2 \frac{\pi R_A^2}{(2\pi)^2} \frac{N_\mathrm{c}(N_\mathrm{c}{}^2 - 1)}{\pi} \frac{1}{\underset{\sim}{\boldsymbol{q}}^2} \int_{\underset{\sim}{\boldsymbol{k}}_1, \underset{\sim}{\boldsymbol{k}}_2} \overbrace{\frac{(g^2\mu)^2}{g^2 \underset{\sim}{\boldsymbol{k}}_1^2}}^{\varphi(\underset{\sim}{\boldsymbol{k}}_1)} \overbrace{\frac{(g^2\mu)^2}{g^2 \underset{\sim}{\boldsymbol{k}}_2^2}}^{\varphi(\underset{\sim}{\boldsymbol{k}}_2)} \delta^2(\underset{\sim}{\boldsymbol{q}} - \underset{\sim}{\boldsymbol{k}}_1 - \underset{\sim}{\boldsymbol{k}}_2)$$

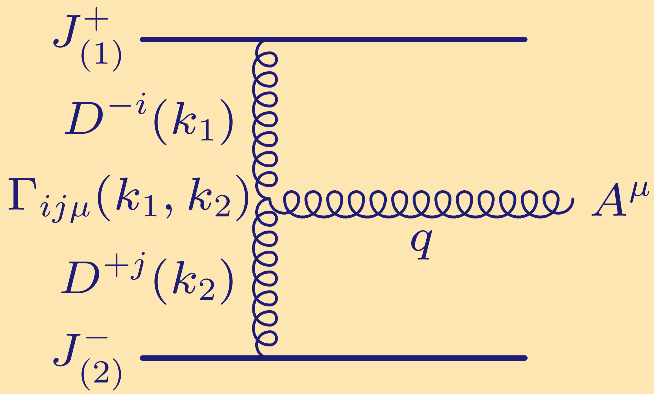
Power counting: $g^2 = 4\pi\alpha_\mathrm{s}$ in front, $g^2\mu \sim Q_\mathrm{s} \sim \alpha_\mathrm{s}^0$, $\varphi(\underset{\sim}{\boldsymbol{k}}) \sim 1/\alpha_\mathrm{s}$

Weak field limit 3: diagram in LC gauge

How about the same with diagrams?

Problem: $A_\tau = 0$-gauge Feynman rules horrible in momentum space.

Solution: Cheat and use different LC propagators for different lines. **Only justified a posteriori.**



$$k_1 = (k_1^+, 0, \underset{\sim}{\boldsymbol{k}}_1) \quad k_2 = (0, k_2, \underset{\sim}{\boldsymbol{k}}_2) \quad q = (k_1^+, k_2^-, \underset{\sim}{\boldsymbol{k}}_1 + \underset{\sim}{\boldsymbol{k}}_2), \quad q^2 = 0$$

$$D^{\mu\nu}(k) = \frac{-i}{k^2}\left(g^{\mu\nu} - \frac{n^\mu k^\nu + k^\mu n^\nu}{n \cdot k}\right) \qquad n \cdot A = 0$$

$$\Gamma_{ij\mu}(k_1, k_2) \sim g_{ij}(k_1 - k_2)_\mu + g_{j\mu}(2k_2 + k_1)_i + g_{i\mu}(-2k_1 - k_2)_j$$

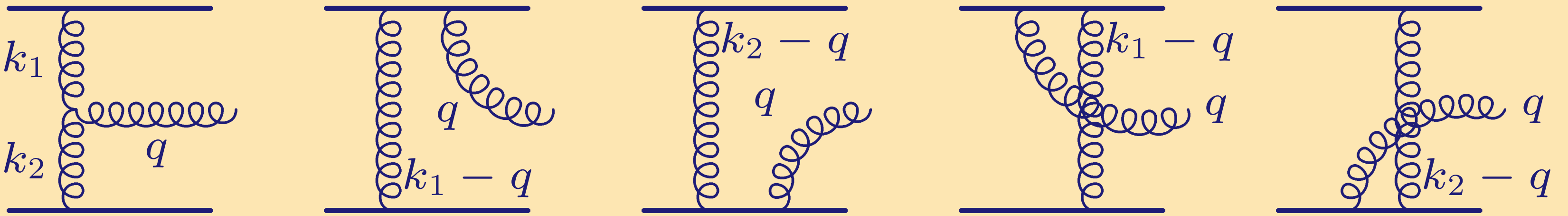
2 indep. final state polarisations μ

- $A^+(q)A^-(q)$ (cf A_η)
- $(\delta^{ij} - \frac{q^i q^j}{\underset{\sim}{\boldsymbol{q}}^2})A^i(q)$

- Only one diagram, because of gauge choice
- Vertex and propagator complicated: only funny non-diagonal part contributes
- Multi-Regge kinematics:only $\underset{\sim}{\boldsymbol{p}}$ in t-channel propagators because $k_1^- = k_2^+ = 0$ ▶ $k_1^2 = -\underset{\sim}{\boldsymbol{k}}_1^2$

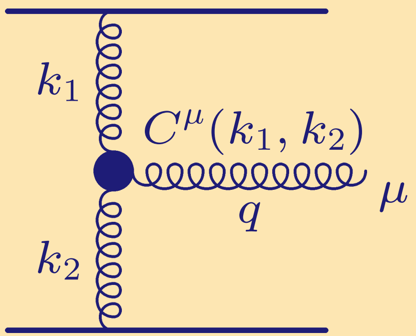
Weak field limit 4: diagrams in covariant gauge

Used for analytical/momentum space computations, Kovchegov and Rischke [4]



$$C(k_1, k_2) = \left(q^+ - \frac{\underset{\sim}{\boldsymbol{k}}_1^2}{q^-} \, , \, \frac{\underset{\sim}{\boldsymbol{k}}_2^2}{q^+} - q^- \, , \, \underset{\sim}{\boldsymbol{k}}_2 - \underset{\sim}{\boldsymbol{k}}_1 \right) \quad q^\mu C_\mu = 0 \quad C^\mu C_\mu = 4 \frac{\underset{\sim}{\boldsymbol{k}}_1^2 \underset{\sim}{\boldsymbol{k}}_2^2}{\underset{\sim}{\boldsymbol{q}}^2}$$

$$A^\mu(q) = J^+_{(1)}(k_1) J^-_{(2)}(k_2) \frac{g^{+-}}{\underset{\sim}{\boldsymbol{k}}_1^2} \frac{g^{+-}}{\underset{\sim}{\boldsymbol{k}}_1^2} C^\mu(k_1, k_2),$$

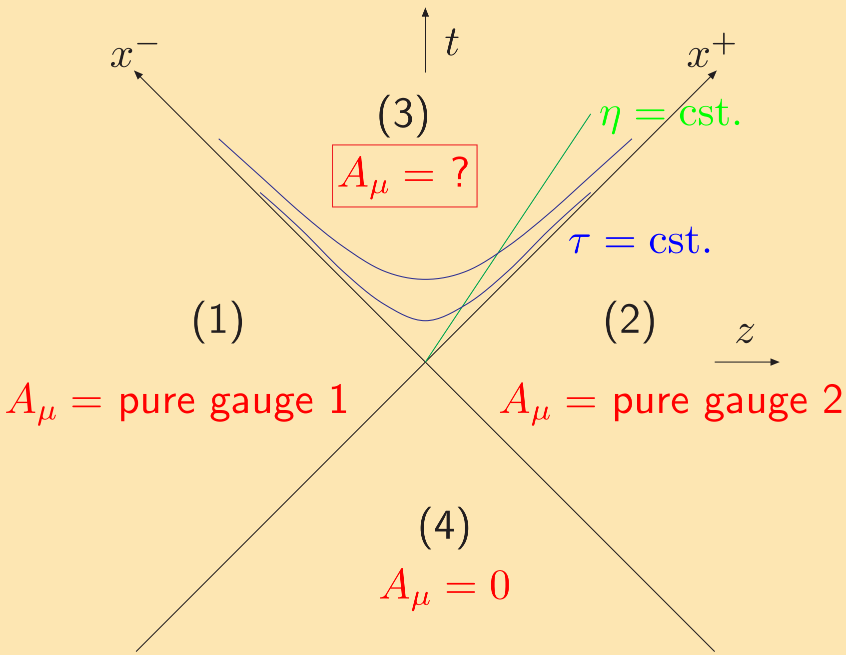


- Many diagrams, but leading high energy magically simplifies into effective **Lipatov vertex** C^μ
- Propagator simple
- Only $\pm$-component propagating down the t-channel
- $C^\mu C_\mu$ kills half of the t-channel propagators

[4] Y. V. Kovchegov and D. H. Rischke, *Phys. Rev.* **C56** (1997) 1084 [hep-ph/9704201].

Numerical solution of the eom's

Analytically known initial condition at $\tau = 0$,
▶ numerical solution from there on.
First numerical study by Krasnitz, Nara & Venugopalan, [5])



- Hamiltonian formalism
- Dimensionally reduced to 2+1d, ▶ $p_z \sim 1/\tau$
- Calculate energy (easy in Hamiltonian formalism) and multiplicity (by decomposing the field in Fourier modes)

[5] A. Krasnitz and R. Venugopalan, *Nucl. Phys.* **B557** (1999) 237 [hep-ph/9809433].

Lattice Hamiltonian formulation

Krasnitz and Venugopalan,[5] Fields independent of η ▶ $2+1$–dimensional theory with Hamiltonian (energy per unit rapidity) on a transverse lattice:

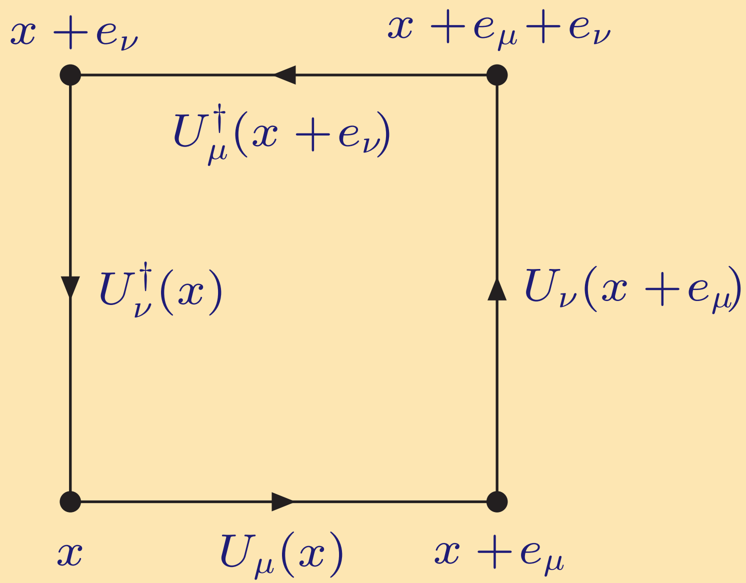
$$H = \sum_{\underset{\sim}{\boldsymbol{x}}} \left\{ \frac{g^2}{\tau} \mathrm{Tr}\, E^i E^i + \frac{2 N_c \tau}{a^2 g^2} \left(1 - \frac{1}{N_c} \mathrm{Re}\ \mathrm{Tr}\, U_\perp \right) + \frac{\tau}{a^2} \mathrm{Tr}\, \pi^2 + \frac{1}{\tau} \sum_i \mathrm{Tr} \left(\phi - \tilde{\phi}_i \right)^2 \right\}$$

$$\tilde{\phi}_i(\underset{\sim}{\boldsymbol{x}}) \equiv U_i(\underset{\sim}{\boldsymbol{x}}) \phi(\underset{\sim}{\boldsymbol{x}} + \underset{\sim}{\boldsymbol{e}}_i) U_i^\dagger(\underset{\sim}{\boldsymbol{x}}).$$

$$\phi \equiv A_\eta \quad U_i = e^{igaA_i} \quad E^i = \tau \dot{A}_i \quad \pi = \dot{\phi}/\tau$$

$$U_\perp(\underset{\sim}{\boldsymbol{x}}) \equiv U_{xy}(\underset{\sim}{\boldsymbol{x}})$$

$$U_{\mu\nu}(\underset{\sim}{\boldsymbol{x}}) = U_x(\underset{\sim}{\boldsymbol{x}}) U_y(\underset{\sim}{\boldsymbol{x}} + e_x) U_x^\dagger(\underset{\sim}{\boldsymbol{x}} + e_x + e_y) U_y^\dagger(\underset{\sim}{\boldsymbol{x}} + e_y)$$



[5] A. Krasnitz and R. Venugopalan, *Nucl. Phys.* **B557** (1999) 237 [hep-ph/9809433].

Dof count:

One has a 2+1-dimensional gauge field theory with an adjoint scalar field ϕ. The physical degrees of freedom are:

Fields	dofs	Momenta	dofs
A_i^a	16	E_i^a	16
Gauge condition	– 8	Gauss law	– 8
ϕ^a	8	π^a	8
Total	16		16

Parameters:

- coupling g
- source density μ^2
- πR_A^2

The numerics essentially depends on a single dimensionless parameter

$$g^4 \pi R_A^2 \mu^2 = \pi R_A^2 \Lambda_s^2.$$

Multiplicity: technicalities

One possibility: define particle number using only the electric field & energy equipartition

$$\begin{aligned} H &\approx 2\sum_{\underset{\sim}{\boldsymbol{x}}} \frac{g^2}{\tau} \operatorname{Tr} E^i(\underset{\sim}{\boldsymbol{x}})E^i(\underset{\sim}{\boldsymbol{x}}) + +\frac{\tau}{a^2} \operatorname{Tr} \pi(\underset{\sim}{\boldsymbol{x}})\pi(\underset{\sim}{\boldsymbol{x}}) \\ &= \frac{2}{N^2}\sum_{\underset{\sim}{\boldsymbol{k}}} \frac{g^2}{\tau} \operatorname{Tr} E^i(\underset{\sim}{\boldsymbol{k}})E^i(-\underset{\sim}{\boldsymbol{k}}) + +\frac{\tau}{a^2} \operatorname{Tr} \pi(\underset{\sim}{\boldsymbol{k}})\pi(-\underset{\sim}{\boldsymbol{k}}) \end{aligned}$$

$$n(\underset{\sim}{\boldsymbol{k}}) = \frac{2}{N^2}\frac{1}{\widetilde{k}}\left[\frac{g^2}{2\tau}E_i^a(\underset{\sim}{\boldsymbol{k}})E_i^a(-\underset{\sim}{\boldsymbol{k}}) + \frac{\tau}{2}\pi^a(\underset{\sim}{\boldsymbol{k}})\pi^a(-\underset{\sim}{\boldsymbol{k}})\right]$$

Not gauge invariant, fix Coulomb gauge.

Dispersion relation

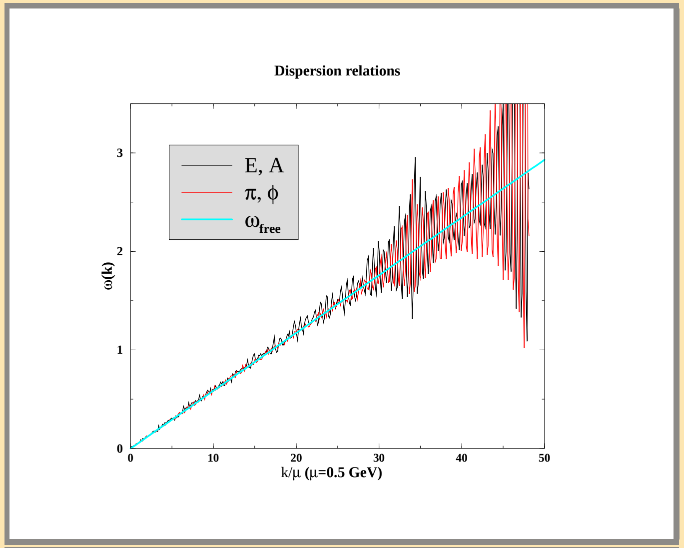
For the energy and multiplicity one can safely assume a free lattice dispersion relation[6]

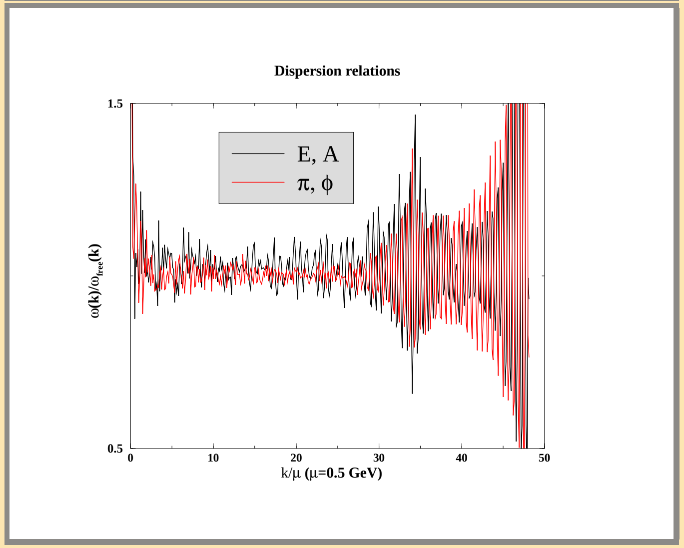
$$\omega(\underset{\sim}{\boldsymbol{k}}) = \widetilde{k} = 2\sqrt{\sin^2 k_x/2 + \sin^2 k_y/2}$$

One can verify this assumption by looking at the correlators

$$\omega_A(\underset{\sim}{\boldsymbol{k}}) = \frac{1}{\tau}\sqrt{\frac{\langle E_i^a(\underset{\sim}{\boldsymbol{k}}) E_i^a(-\underset{\sim}{\boldsymbol{k}})\rangle}{\langle A_i^a(\underset{\sim}{\boldsymbol{k}}) A_i^a(-\underset{\sim}{\boldsymbol{k}})\rangle}}$$

$$\omega_\phi(\underset{\sim}{\boldsymbol{k}}) = \tau\sqrt{\frac{\langle \pi^a(\underset{\sim}{\boldsymbol{k}}) \pi^a(-\underset{\sim}{\boldsymbol{k}})\rangle}{\langle \phi^a(\underset{\sim}{\boldsymbol{k}}) \phi^a(-\underset{\sim}{\boldsymbol{k}})\rangle}}$$



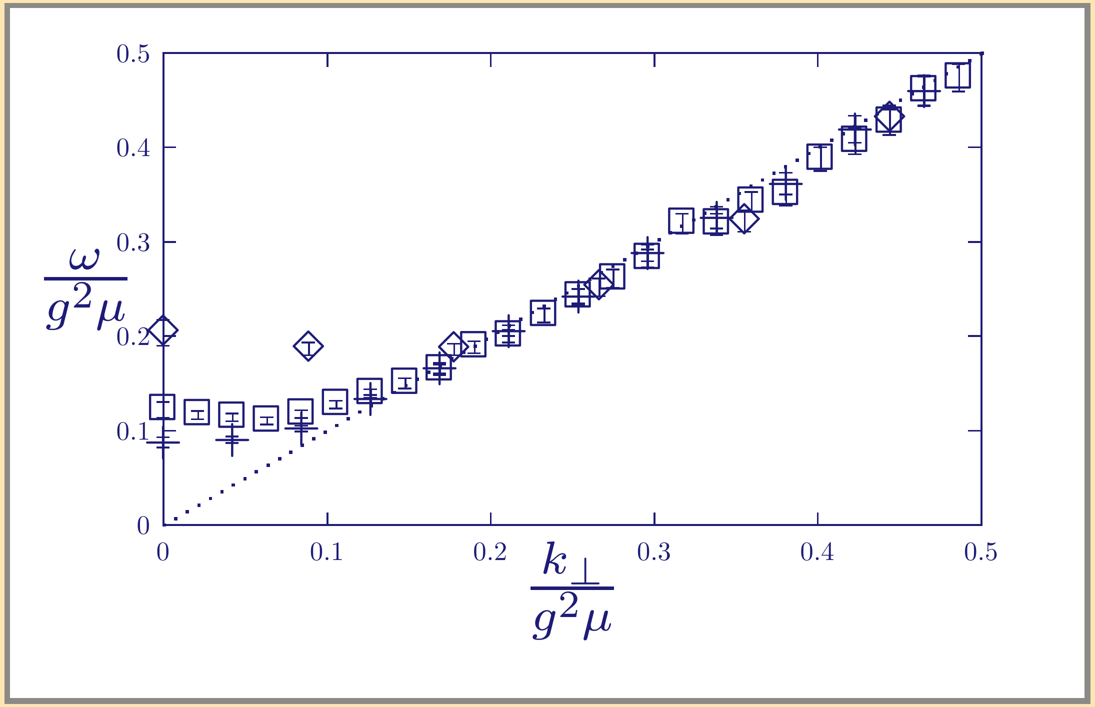


[6] T. Lappi, *Phys. Rev.* **C67** (2003) 054903 [hep-ph/0303076].

Mass gap

However the free dispersion relation does not persist down to very low $\underset{\sim}{\boldsymbol{k}}$: For small $\underset{\sim}{\boldsymbol{k}}$ there is a "plasmon mass" gap [7]

$$m^2 \sim g^2\mu/\tau.$$

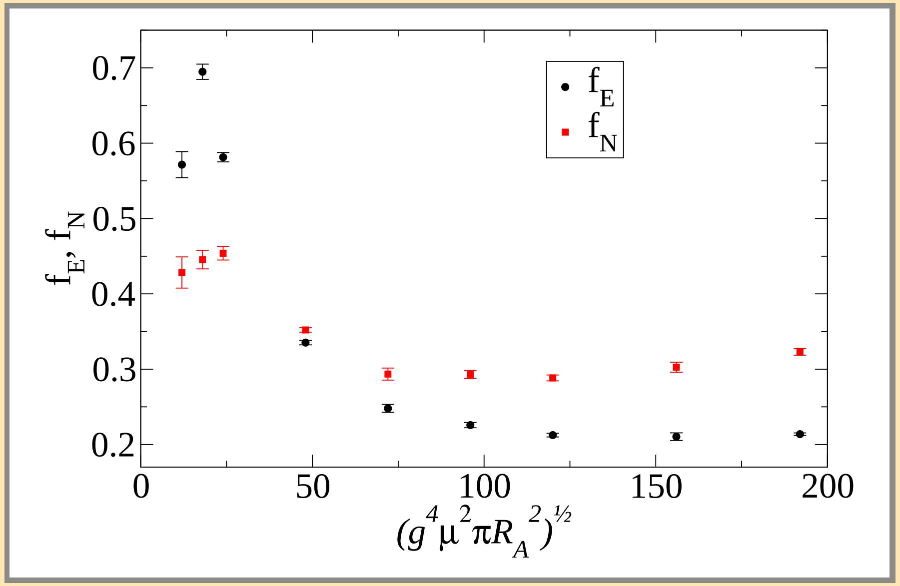


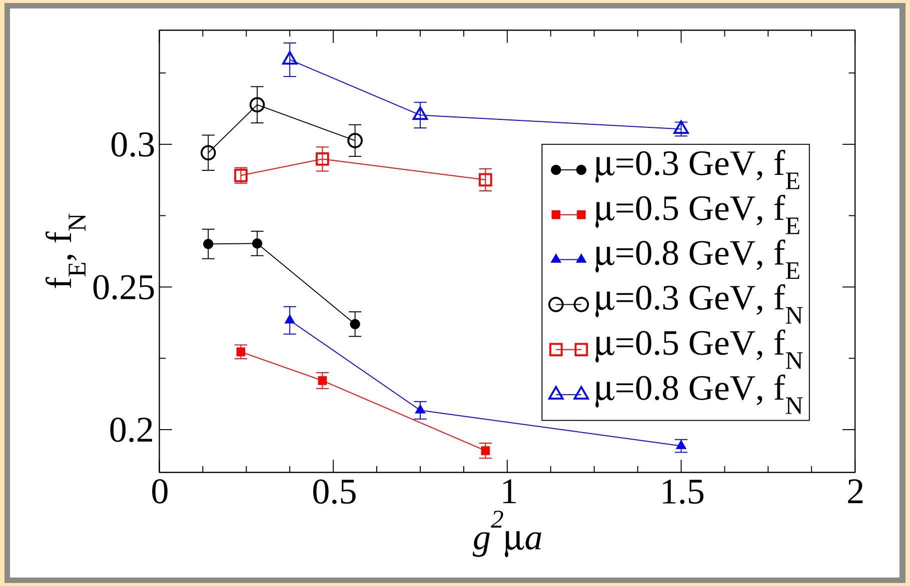
[7] A. Krasnitz and R. Venugopalan, *Phys. Rev. Lett.* **86** (2001) 1717 [hep-ph/0007108].

Results for multiplicity, energy

Dimensionless ratios
$$f_E = \frac{\mathrm{d}E/\,\mathrm{d}\eta}{g^4\mu^3\pi R_A^2} \quad f_N = \frac{\mathrm{d}N/\,\mathrm{d}\eta}{g^2\mu^2\pi R_A^2}$$

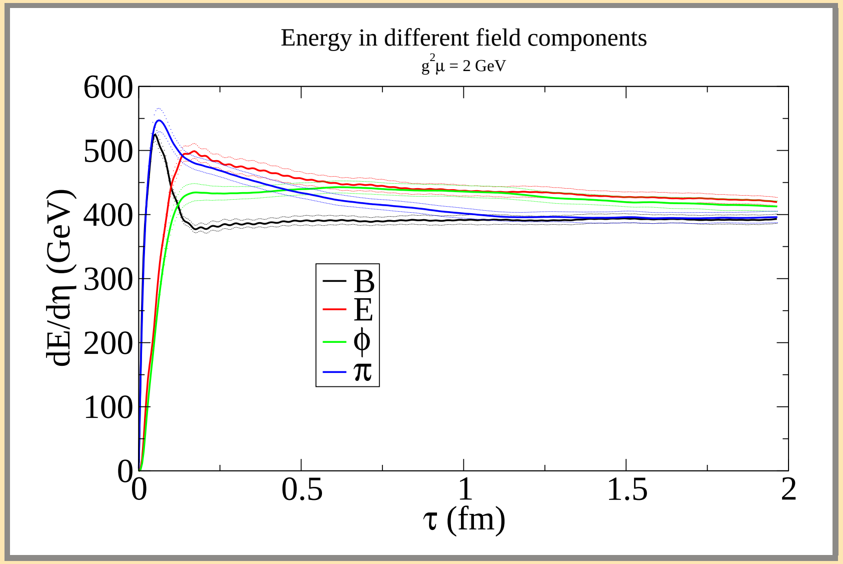
For strong enough fields $g^2\mu R_A \gtrsim 50$ these are $\sim$ constant and depend only weakly on lattice spacing (UV finite):

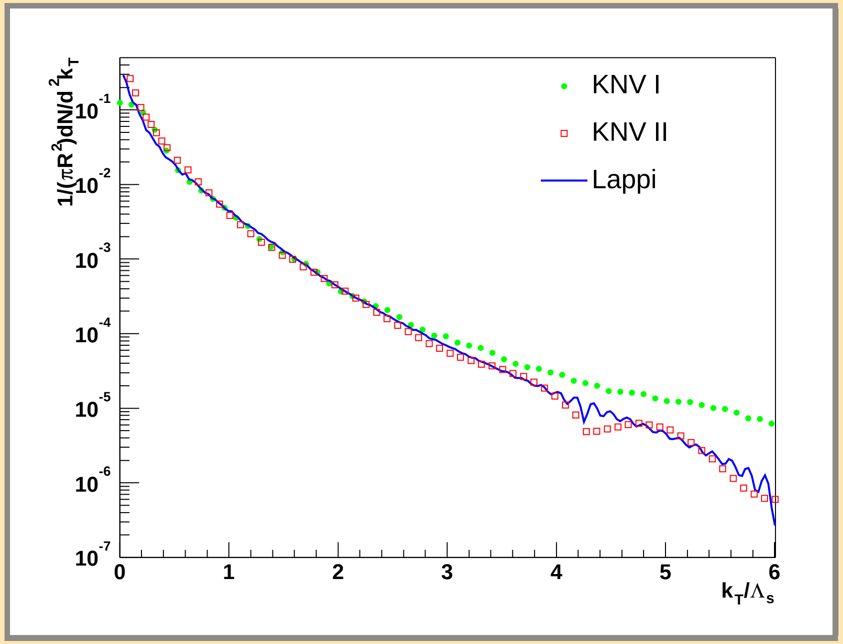




. . . Numerical results

The energy is distributed between the different field components and almost constant after a very short time $\sim \frac{1}{g^2\mu}$. ▶ This means that $\varepsilon \sim \tau^{-1}$.



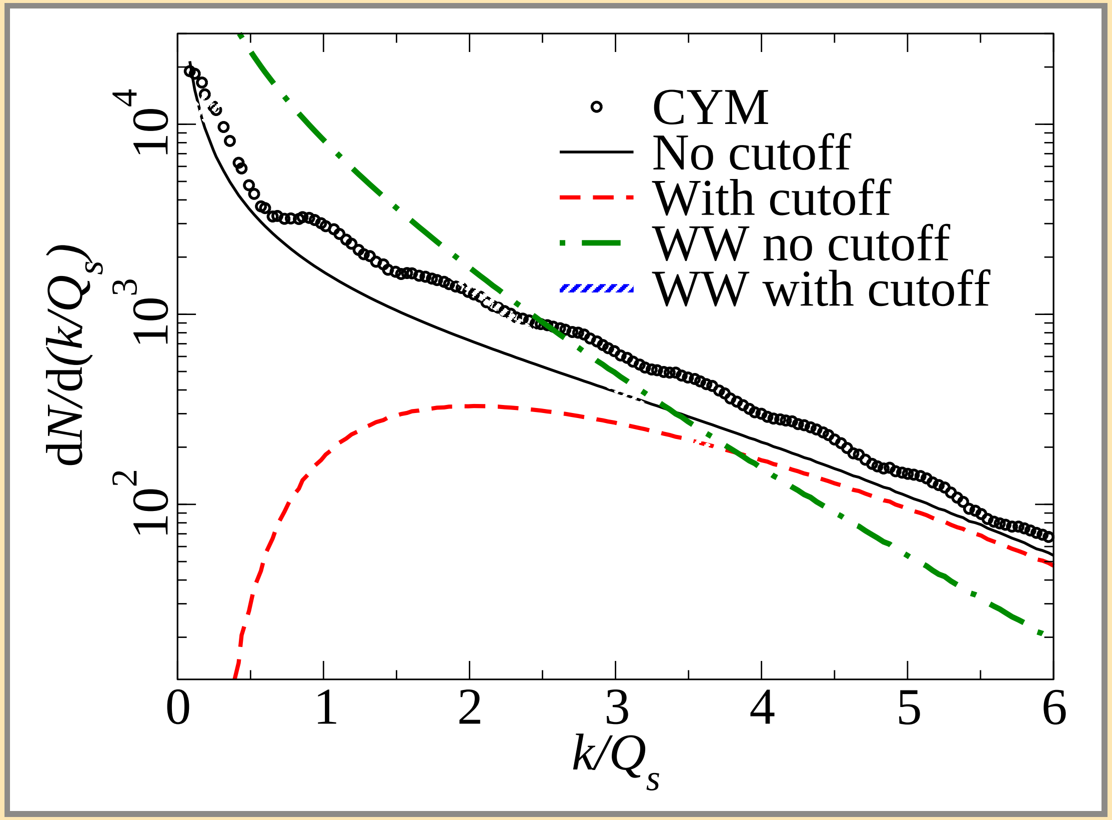


The differential multiplicity has a perturbative tail $\sim \frac{1}{\underset{\sim}{k}^4}$ but is **infrared finite**.
The spectrum is not thermal (at these timescales).

How good is k_T-factorization?

$$\frac{\mathrm{d}N}{\mathrm{d}y\,\mathrm{d}^2\boldsymbol{p}} = \frac{1}{\alpha_s}\frac{1}{\boldsymbol{p}^2}\int\frac{\mathrm{d}^2\boldsymbol{k}}{(2\pi)^2}\phi(\boldsymbol{k})\phi(\boldsymbol{p}-\boldsymbol{k})$$

- Use $\phi^{\mathrm{WW}}(\boldsymbol{k}) \sim \int \frac{\mathrm{d}^2\mathbf{r}_T}{\mathbf{r}_T{}^2} e^{i\boldsymbol{k}\cdot\mathbf{r}_T}\,\mathrm{Tr}\,U^\dagger(-\mathbf{r}_T/2)U(\mathbf{r}_T/2)$
  in stead of $\phi(\boldsymbol{k}) \sim \boldsymbol{k}^2 \int \mathrm{d}^2\mathbf{r}_T e^{i\boldsymbol{k}\cdot\mathbf{r}_T}\,\mathrm{Tr}\,U^\dagger(-\mathbf{r}_T/2)U(\mathbf{r}_T/2)$
- Add cutoff $|\boldsymbol{k}| < |\boldsymbol{p}|$



Phenomenology, what is Q_s at RHIC

RHIC @ 130/200 GeV: [1] $\frac{dN_{tot}}{d\eta} \approx 1000 \quad \frac{dE_T}{d\eta} \approx 600\text{GeV}$

Relating initial state (calculated) to final state (measured), different scenarios:

Ideal hydro, entropy conservation	Free streaming, energy conservation
▼	▼
$N_{init} \approx N_{final}$	$E_T^{init} \approx E_T^{final}$
▼	▼
$g^2\mu \sim 1.9\text{GeV}$	$g^2\mu \sim 1.4\text{GeV}$
▼	▼
Lot of energy down the beampipe in $p\,dV$ work.	Multiplicity grows, 1 gluon $\rightarrow\sim$ 2 pions.

First scenario agrees to within 10% with detailed comparison to HERA fits+nuclear geometry [8,9]

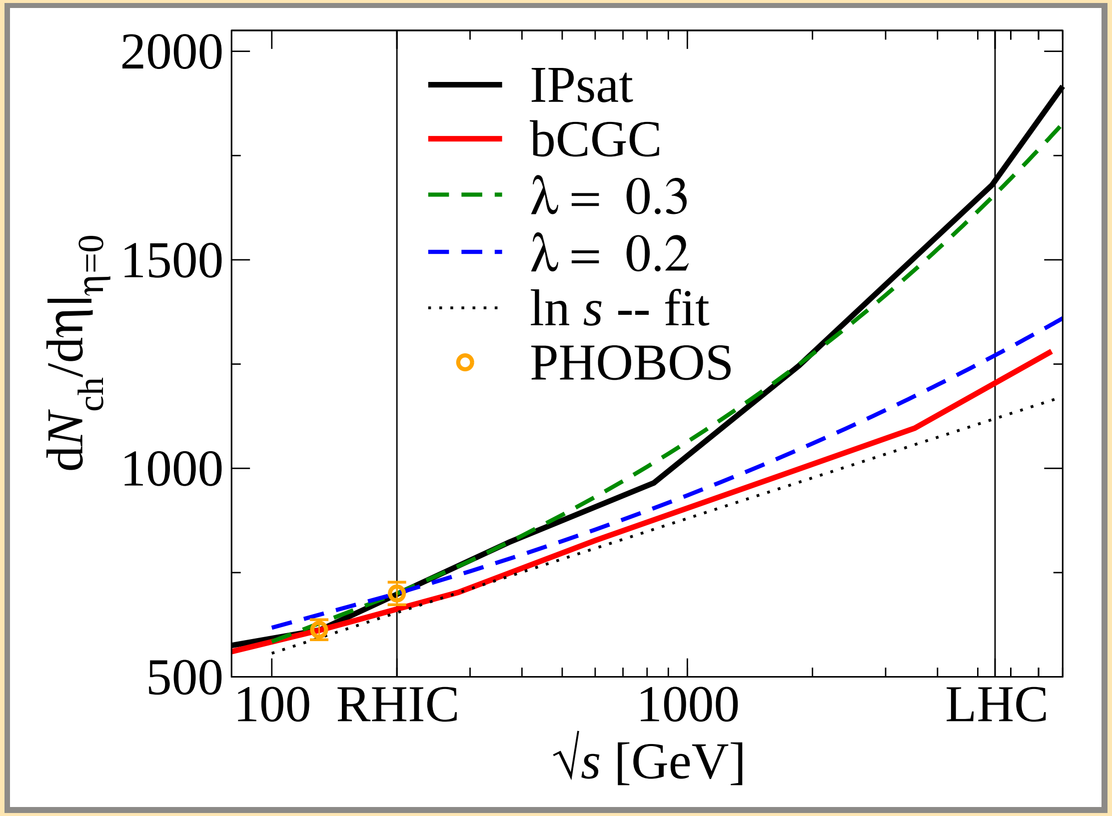
[1] Total, including neutral particles. At this level of approximation, $N_{ch} \approx \frac{2}{3}N_{tot}$

[8] H. Kowalski, T. Lappi and R. Venugopalan, *Phys. Rev. Lett.* **100** (2008) 022303 [0705.3047].
[9] T. Lappi, *Eur. Phys. J.* **C55** (2008) 285 [0711.3039].

LHC multiplicity

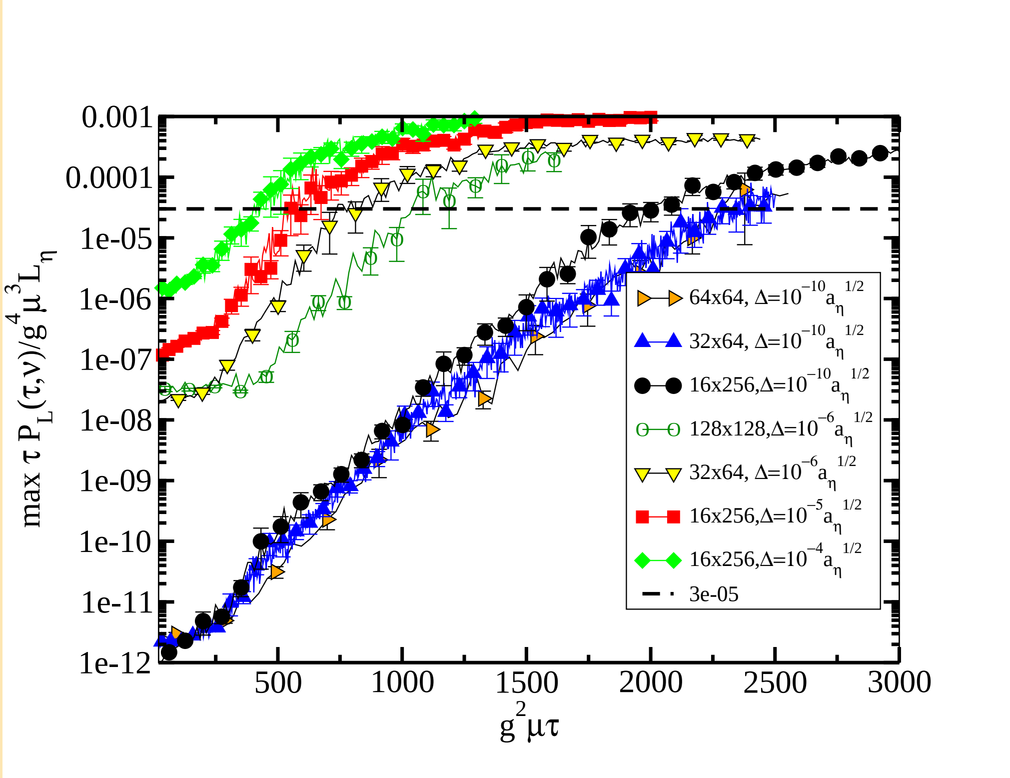
The prediction for LHC depends (almost entirely) on energy dependence of Q_{s}.
LO: $Q_{\mathrm{s}}^2 \sim x^{-\lambda} = e^{\lambda \ln 1/x}$, NLO: $Q_{\mathrm{s}}^2 \sim e^{C\sqrt{\ln 1/x}}$; DIS fits vary between

$$Q_{\mathrm{s}}^2 \sim x^{-0.2} \cdots x^{-0.3} \quad \blacktriangleright \quad \frac{\mathrm{d}N}{\mathrm{d}\eta} \sim Q_{\mathrm{s}}^2 \pi R_A^2 \sim \sqrt{s}^{0.2} \cdots \sqrt{s}^{0.3}$$



Giving up boost invariance: plasma instabilities

Romatschke & Venugopalan[10,11]



Allow for η-dependence of modes: plasma instability.
Growth rate related to the "plasmon mass" $\Gamma \sim \sqrt{g^2\mu/\tau}$

François' lecture IV

[10] P. Romatschke and R. Venugopalan, *Phys. Rev. Lett.* **96** (2006) 062302 [hep-ph/0510121].
[11] P. Romatschke and R. Venugopalan, *Phys. Rev.* **D74** (2006) 045011 [hep-ph/0605045].

Conclusions

- Leading order classical field, 1 nucleus
- 2 colliding nuclei
  - Initial condition
  - Weak field limit
  - Some numerical results: RHIC numbers work out well so far

Next lecture: going beyong LO.