Assignment 1

Due date: 10th March

1. The action for the free scalar field,

$$S = \int d^4x \, \frac{1}{2} \, \left[\partial_\mu \phi \partial^\mu \phi \; - \; m^2 \phi^2\right],$$

   is invariant under the Lorentz transformations

$$x^\mu \;\rightarrow\; x'^\mu = \Lambda^\mu_\nu x^\nu, \qquad \phi(x) \;\rightarrow \phi'(x) = \phi(\Lambda^{-1} x).$$

   Find the Noether currents and conserved charges associated with these transformations.

2. The complex Klein-Gordon field, given by the Lagrangian density $\mathcal{L} = \partial^\mu \phi^* \partial_\mu \phi - m^2 \phi^* \phi$, should, in the proper non-relativistic limit, give the Schrödinger field. To see that, you'll need to isolate the rest mass part of the energy by defining $\phi(t) \sim \exp(-imt)\psi(t)$. Show that the nonrelativistic lagrangian corresponds to a classical field satisfying Schrödinger equation. Also find the nonrelativistic limit of the conserved current, $j_\mu = i(\phi^*.\partial_\mu \phi - \partial_\mu \phi^*.\phi)$.

3. Find the action of the creation and annihilation operators on the Fock space states,

$$a^\dagger_{\vec{q}} | n_{\vec{p_1}}, ..., n_{\vec{p_n}}, ... \rangle, \, a_{\vec{q}} | n_{\vec{p_1}}, ..., n_{\vec{p_n}}, ... \rangle.$$