Assignment 3

Due date: 16th April

1. Start from the path integral expression for the quantum mechanical propagator,

$$\langle x_f; t_f | x_i; t_i \rangle \;=\; \int_{x(t_f)=x_f, x(t_i)=x_i} \mathcal{D}x[t]\; e^{iS[x]}.$$

a) Evaluate $\langle x_f; t_f | x_i; t_i \rangle$ for the simple harmonic oscillator:

$$L \;=\; \frac{1}{2} m\dot{x}^2 \;-\; \frac{1}{2} m\omega^2 x^2.$$

b) Show that the wave function satisfies the Schrödinger equation:

$$i\dot{\psi}(x,t) \;=\; -\frac{1}{2m}\frac{\partial^2 \psi(x,t)}{\partial x^2} \;+\; \frac{1}{2} m\omega^2 x^2\; \psi(x,t).$$

[Use $\psi(x,t) = \int dy \langle x; t | y; t' \rangle \psi(y; t')$.]

2. Evaluate the integral

$$Z(\lambda) \;=\; \int \frac{dx}{2\pi}\; e^{-x^2/2 - \lambda x^4/4}$$

for small λ by expanding the integrand in λ:

$$Z(\lambda) \;=\; \sum_n \frac{1}{n!}\; \lambda^n Z_n.$$

Investigate the convergence properties of the series. (Does the series converge? What is the radius of convergence? Is it an asymptotic series?)

3. The spectral function, $\rho(p_0, \vec{p})$, was defined in the previous assignment.

   a) Show that $\rho(p)$ is odd function of p_0, and that on the forward lightcone it can be written as $\rho_+(p^2)$, where $\rho_+(p^2)$ is real and positive semidefinite.

   b) Show that the Feynman propagator in momentum space is given by

$$D_F(p) \;=\; \int_0^\infty \frac{dm^2}{\pi} \rho_+(m^2)\; D_F^0(p, m)$$

   where $D_F^0(p, m) = i/(p^2 - m^2 + i\epsilon)$ is the Feynman propagator for the free theory.