Assignment 5

Due date: 26th May

1. The Gamma matrices satisfy the anticommutation relation,

$$\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}.$$

a) Show that the matrices $S^{\mu\nu} = \frac{i}{4}[\gamma^\mu, \gamma^\nu]$ satisfy the algebra of $M^{\mu\nu}$:

$$[S^{\mu\nu}, S^{\rho\sigma}] = i\left(g^{\nu\rho}S^{\mu\sigma} - g^{\mu\rho}S^{\nu\sigma} - g^{\nu\sigma}S^{\mu\rho} + g^{\mu\sigma}S^{\nu\rho}\right).$$

b) Show that in d dimensions, such that $\delta^\mu_\mu = d$, the γ^μ satisfy the trace relations

$$\mathrm{Tr}\,\gamma^\mu\gamma^\nu = f(d)g^{\mu\nu}, \qquad \mathrm{Tr}\,\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma = f(d)\left(g^{\mu\nu}g^{\rho\sigma} - g^{\mu\rho}g^{\nu\sigma} + g^{\mu\sigma}g^{\nu\rho}\right).$$

Here $f(d) = \mathrm{Tr}\ I$ is an arbitrary, well-behaved function with $f(4) = 4$.

2. Derive the 'Gordon identity'

$$\bar{u}(p')\gamma^\mu u(p) = \frac{1}{2m}\bar{u}(p')\left[(p'^\mu + p^\mu) + \frac{i}{2}S^{\mu\nu}q_\nu\right]u(p),$$

where $q = p' - p$.

3. Take the Yukawa lagrangian,

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu\partial_\mu - m)\psi - \frac{1}{2}\phi(\partial^\mu\partial_\mu + m^2)\phi - g\bar{\psi}\psi\phi.$$

Show that a ϕ^4 vertex would appear in the renormalized perturbation theory as a one-loop counterterm. Estimate the counterterm.

Will any other counterterm be generated which is not there in $\mathcal{L}$ above?

4. For the above Lagrangian, find the β function for g, $\beta(g) = \mu\ \partial g(\mu)/\partial\mu$.