Probing $U_A(1)$ Restoration with Domain-Wall Fermions

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The Symmetries of QCD

- QCD with N_f flavors of massless fermions is invariant under

$$G(N_f) \equiv SU_V(N_f) \otimes SU_A(N_f) \otimes U_V(1) \otimes U_A(1).$$

- $SU_V(N_f)$ conserves isospin while $U_V(1)$ conserves baryon number.
- What about $SU_A(N_f)$ and $U_A(1)$?
  - If N_f is small, $SU_A(N_f)$ spontaneously broken; the light mesons are the (pseudo-)Goldstone bosons.
  - $U_A(1)$ lost when the theory is quantized — Chiral anomaly.
- $SU_A(N_f)$ restored above a certain temperature T_c; what about $U_A(1)$?

The Fate of $U_A(1)$ at Finite Temperature

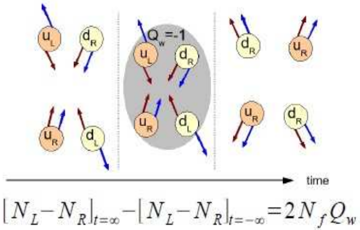
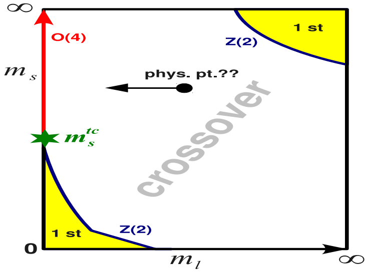


Figure: From the talk by H. Warringa, Strong and Electroweak Matter 2008.

- Gauge fields with non-trivial topology ($Q_{\text{top}} \neq 0$) can change the axial charge by $2N_f Q_{\text{top}}$.
- Effect quantum-mechanical; its probability decreases with increasing temperature T (Pisarski,Yaffe).



- If the effects of the anomaly are small around T_c, the 2-flavor transition will be first-order rather than second-order (Pisarski,Wilczek).
- Similarly, the phase diagram in the $T - \mu$ plane can change drastically (Fukushima (PRD 2008), P. Deb *et al.*(PRC 2009)).

Looking for $U_A(1)$ Restoration on the Lattice

- The restoration of symmetries affects the particle spectrum viz.

$$\begin{array}{ccc} \pi^{\pm}(\gamma_5 \otimes \tau^{\pm}) & \xrightarrow{U_A(1)} & \delta(I \otimes \tau^{\pm}) \\ SU_A(N_f)\downarrow & & \downarrow SU_A(N_f) \\ \sigma(I \otimes I) & \xleftarrow[U_A(1)]{} & \eta(\gamma_5 \otimes I) \end{array}$$

- On the lattice, observe $\pi^{\pm}$, etc. by looking at appropriate correlators viz.

$$C(t) = \sum_{x,y,z} \left\langle \bar{\psi}\Gamma_T\psi(0,0,0,0)\,\bar{\psi}\Gamma_T\psi(x,y,z,t)\right\rangle ,$$

where Γ_T is a Dirac $\otimes$ flavor matrix ($\pi^{\pm} \sim \gamma_5\tau^{\pm}$, etc.)

- Stronger Statement: The correlators themselves become equal (upto a sign) when the symmetry is restored.

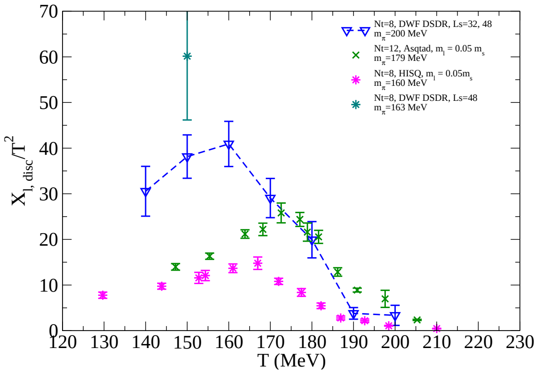
$U_A(1)$ Restoration: A Review

- Is $U_A(1)$ restored at $T = T_c$ (Shuryak)? **Negative** for $N_f = 2$ **light** flavors, **affirmative** for $N_f \geqslant 3$ (Cohen, Evans *et al.*, Hatsuda and Lee).
- Studies of the 2- or 2+1-flavor theory with staggered fermions find that $U_A(1)$ is not restored at $T = T_c$ (Karsch and Laermann, Bernard *et al.*, Chandrasekharan and Christ, Kogut *et al.*, Christ and Wu, Cheng *et al.*[RBC-Bielefeld]).
- However theoretical issues in extrapolating staggered studies to the chiral limit(Vink, Vink and Smit).
- Also difficult to connect to topology since an index theorem for staggered quarks was not known (until recently).

Domain-Wall Fermions

- Five-dimensional fermions with a low-energy spectrum that is (i) four-dimensional, and (ii) chiral.
- Exact chiral symmetry for infinite fifth dimension. For $L_s < \infty$, massless fermions acquire “residual mass” m_{res}:
  1. Weak coupling: $m_{\text{res}} \propto \exp(-AL_s)$.
  2. Stronger coupling: New contributions from gauge fields, $m_{\text{res}} \propto L_s^{-1}$. Use smoother gauge fields (Iwasaki) and an improved action (DSDR).
- Satisfy an index theorem for $L_s = \infty$; Dirac spectrum will be QCD-like.

Thermodynamics with a Chiral Action



- DSDR+Iwasaki lattices of size $16^3 \times 8 \times L_s$ ($L_s = 32$ or 48) (Note – volume still small). $m_\pi = 200$ MeV throughout.
- $T_c \approx 160$ MeV. Vector/axial vector correlators also become degenerate at this temperature.

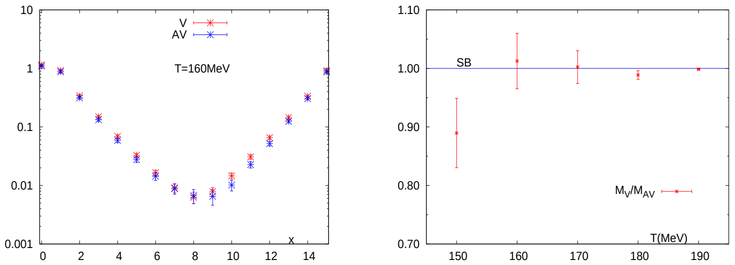
Screening Correlators and Masses

- Screening correlators are defined by (Γ_T = Dirac $\otimes$ Flavor matrix)

$$C(x) = \sum_{y,z,t} \left\langle \bar{\psi}\Gamma_T\psi(0,0,0,0)\,\bar{\psi}\Gamma_T\psi(x,y,z,t)\right\rangle .$$

- We only looked at *connected* correlators *i.e.* bilinears with different quark flavors ($\bar{u}\gamma_\mu d$, etc.).
- The vector and axial vector correlators become degenerate when $SU_A(N_f)$ is restored. Similarly, if $U_A(1)$ is restored the scalar and pseudoscalar correlators should become degenerate.
- From the long-distance behavior of the correlators, $C(x) \sim \exp(-M_\Gamma x)$, we can extract screening masses M_Γ.

The V/AV Channels



The vector and axial vector channels become degenerate at $T \approx 160$ MeV. This behavior implies that $M_V/M_{AV} \approx 1$ and is consistent with $\chi_{\rm disc}$ peaking around the same temperature.

$U_A(1)$ Symmetry and the S/PS Correlators

- The scalar and pseudoscalar correlators that we measured were

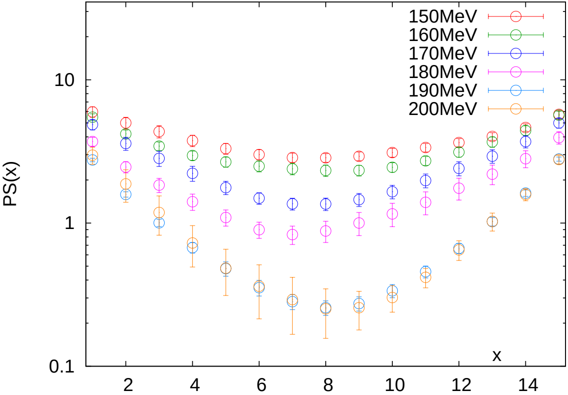
$$C_S(x) = \langle \bar{u}d(x)\bar{u}d(0) \rangle, \quad (1)$$
$$C_{PS}(x) = \langle \bar{u}\gamma^5 d(x)\bar{u}\gamma^5 d(0) \rangle. \quad (2)$$

- In terms of LH and RH components, these are

$$C_{S/PS}(x) = \langle \bar{u}_L d_R(x)\bar{u}_L d_R(0) + \bar{u}_R d_L(x)\bar{u}_R d_L(0) \rangle$$
$$\pm \langle \bar{u}_L d_L(x)\bar{u}_L d_L(0) + \bar{u}_R d_R(x)\bar{u}_R d_R(0) \rangle. \quad (3)$$

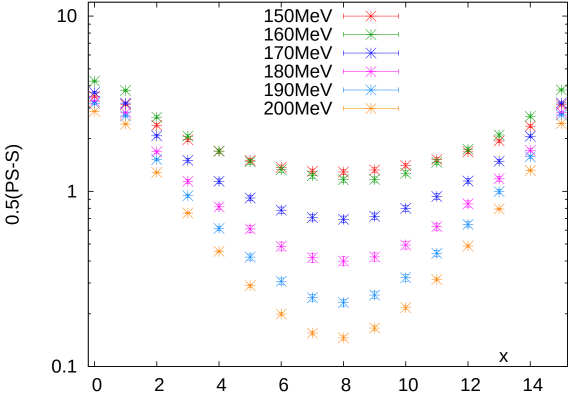
- The terms on the first line of eq. (3) break $U_A(1)$ symmetry while the terms on the second line preserve it.
- These terms may be isolated by looking at the sum and difference of C_S and C_{PS} respectively.

Pseudoscalar: All T



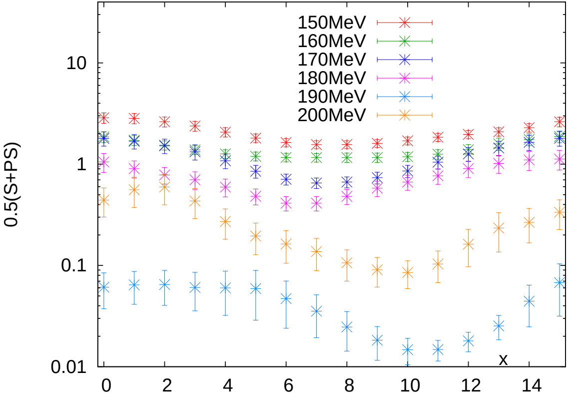
- Good signal for PS; S significantly noisier.
- However sum and difference separately give clean signals. So why is the scalar so noisy?

The $U_A(1)$-Respecting Sector



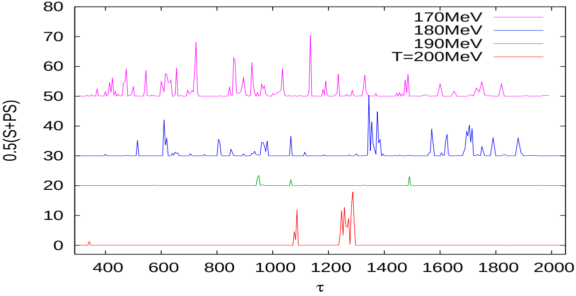
Difference $\equiv PS$ ($\equiv -S$) in the absence of $U_A(1)$ violation.
Changes smoothly with T.

The $U_A(1)$-Violating Sector



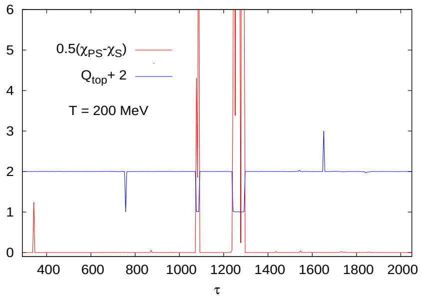
- Non-zero even above T_c.
- Sum comparable to the difference at each temperature. This is why the scalar is so noisy.

Where does $U_A(1)$-Violation Come From?



- The sum is zero *except* for specific configurations.
- Fewer and fewer such configurations at greater T. $U_A(1)$-breaking decreases because the number of spikes (rather than their magnitude) decreases.
- The correlation between $U_A(1)$-violation and non-trivial configurations has been observed before by Gavai, Gupta and Lacaze (PRD 2002).

The Mechanism of $U_A(1)$ Violation

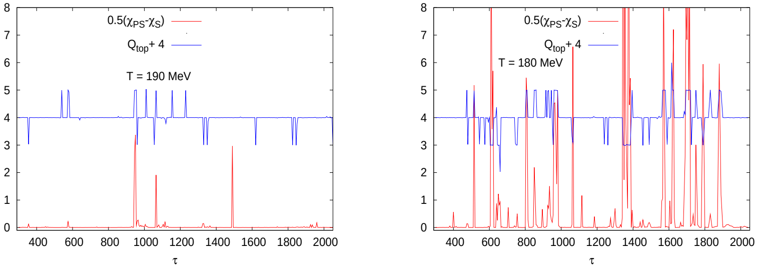


- In QCD, gauge fields with non-trivial winding number Q_{top} can change the net axial charge viz.

$$\Delta\left(N_L - N_R\right) = 2N_f Q_{\text{top}}. \qquad (4)$$

- On the lattice, Q_{top} is well-defined for a chiral Dirac action. Hence we should expect the spikes to be correlated with fluctuations in Q_{top}.

Spikes and Topology



- The correlation between the spikes and Q_{top} is good but not perfect. There are fluctuations in Q_{top} that do not produce spikes and vice-versa. Need to understand this better.

$SU_A(N_f)$ Versus $U_A(1)$ Restoration

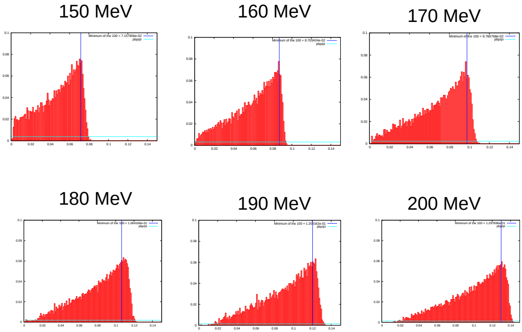
- Corresponding to each correlator is a susceptibility viz. $\chi_\Gamma = \left|\sum_x C_\Gamma(x)\right|$.
- $SU_A(N_f)$ breaking implies non-zero condensate $\Sigma \equiv \langle\psi\bar{\psi}\rangle$.
- $U_A(1)$ breaking: No order parameter, look for $(\chi_\pi - \chi_\delta) \to 0$.
- The chiral condensate Σ and the S/PS susceptibilities both depend on $\rho(\lambda)$ as

$$\Sigma = \int d\lambda \rho(\lambda) \frac{2m}{m^2 + \lambda^2}, \tag{5}$$

$$\chi_\pi - \chi_\delta = \int d\lambda \rho(\lambda) \frac{4m^2}{(m^2 + \lambda^2)^2}. \tag{6}$$

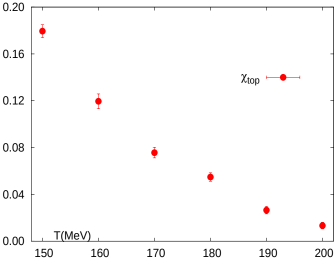
Is $U_A(1)$ restored at the same time that $SU_A(N_f)$ is?

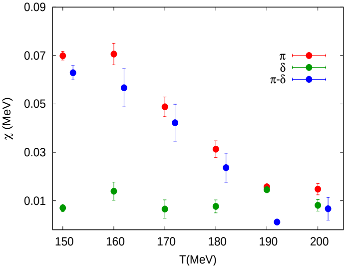
The Eigenvalue Density



- $\Sigma = \pi\rho(0)$ (Banks and Casher). Sharp decrease in $\rho(0)$ for $T \geqslant T_c$.
- Some evidence for $\rho(\lambda) \sim \lambda^z$ with $z > 1$.

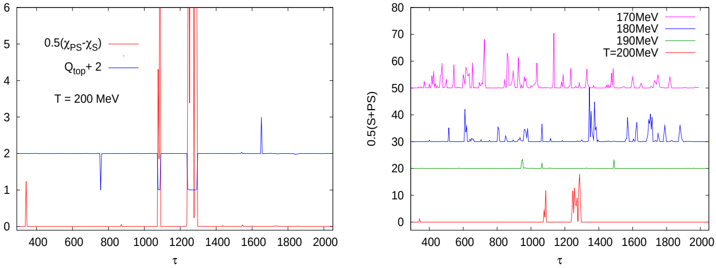
Susceptibilities versus Temperature





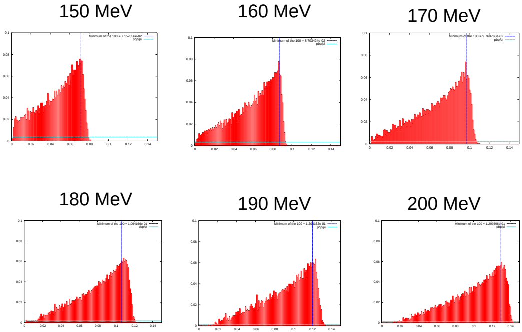
- χ_{top} is seen to decrease smoothly; this is also evident from the Q_{top} time history.
- $\chi_\pi - \chi_\delta \neq 0$. This had been observed before by N. Christ and L. Wu (Lattice 2001).

Conclusions



- We see a strong correlation between non-zero topological charge and $U_A(1)$ violation in the scalar and pseudoscalar correlators.
- $U_A(1)$-breaking is nonzero at all temperatures studies; however decreases with increasing temperature.

Conclusions (contd)



- Should be possible to relate this to the spectrum of low-lying eigenvalues.
- This requires a better understanding of the volume and quark mass effects.