Fluctuations & Correlations of conserved charges in PNJL model

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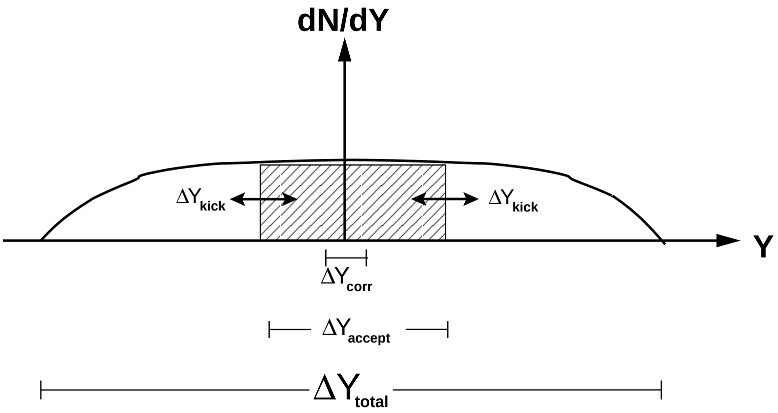
- The most efficient way to address fluctuations of a system created in a heavy-ion collision is via the study of event-by-event fluctuations.
- In addition, the study of fluctuations may reveal information beyond its thermodynamic properties.



Charge fluctuations will be able to tell us about the properties of the early stage of the system, the QGP, if the following criteria are met:

$$\Delta Y_{accept} \gg \Delta Y_{corr} \quad \text{and} \quad \Delta \mathrm{Y}_{\mathrm{total}} \gg \Delta \mathrm{Y}_{\mathrm{accept}} \gg \Delta \mathrm{Y}_{\mathrm{kick}}$$

V. Koch, arXiv : 0810.2520

Motivation behind PNJL Model

- Nambu-Jona-Lasinio (NJL) model : Originally proposed for studying hadronic d.o.f. Later extended to quark d.o.f.
  Reproduces chiral symmetry breaking of QCD suscessfully through a non-vanishing chiral condensate.

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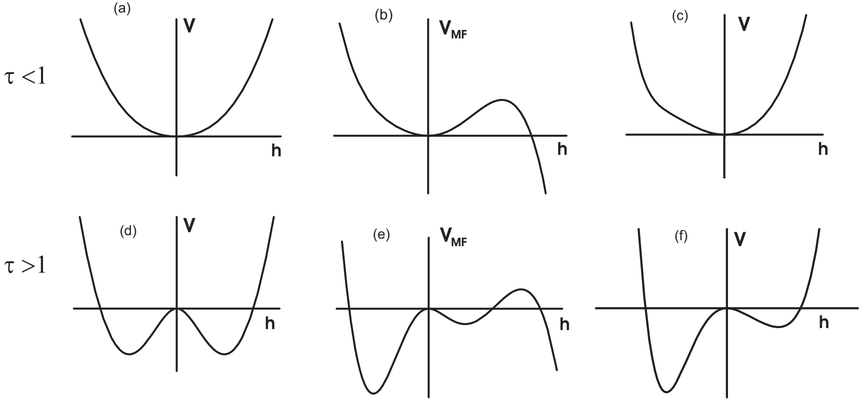
- Polyakov loop model : Originally proposed for pure gauge system. Reproduces confinement-deconfinement transition of QCD.
- Polyakov loop-Nambu-Jona-Lasinio (PNJL) model tied together these two aspects of QCD.



Here $\tau = \frac{N_c G \Lambda^2}{2\pi^2} > 1$ in order to have chiral symmetry broken.

A.A. Osipov *et. al.* Annals of Physics 322 (2007) 2021.

Thermodynamic Potential I

$$\Omega = \mathcal{U}'[\Phi, \bar{\Phi}, T] + 2g_S \sum_{f=u,d,s} \sigma_f^2 - \frac{g_D}{2}\sigma_u\sigma_d\sigma_s + 3\frac{g_1}{2}(\sum_{f=u,d,s} {\sigma_f}^2)^2$$

$$+3g_2 \sum_{f=u,d,s} \sigma_f^4 - 6 \sum_{f=u,d,s} \int_0^\Lambda \frac{d^3p}{(2\pi)^3} E_f \Theta(\Lambda - |\vec{p}|)$$

$$-2T \sum_{f=u,d,s} \int_0^\infty \frac{d^3p}{(2\pi)^3} \ln\left[1 + 3\Phi e^{-\frac{(E_f-\mu_f)}{T}} + 3\bar{\Phi} e^{-2\frac{(E_f-\mu_f)}{T}} + e^{-3\frac{(E_f-\mu_f)}{T}}\right]$$

$$-2T \sum_{f=u,d,s} \int_0^\infty \frac{d^3p}{(2\pi)^3} \ln\left[1 + 3\bar{\Phi} e^{-\frac{(E_f+\mu_f)}{T}} + \Phi e^{-2\frac{(E_f+\mu_f)}{T}} + e^{-3\frac{(E_f+\mu_f)}{T}}\right]$$

Thermodynamic Potential II

where, $\sigma_f = \langle\bar{\psi}_f\psi_f\rangle$ $E_f = \sqrt{p^2 + M_f^2}$ with,

$$M_f = m_f - 2g_S\sigma_f + \frac{g_D}{2}\sigma_{f+1}\sigma_{f+2} - 2g_1\sigma_f(\sigma_u^2 + \sigma_d^2 + \sigma_s^2) - 4g_2\sigma_f^3$$

A. Bhattacharyya *et. al.*, Phys. Rev. D 82, 014021 (2010).

For the Polyakov loop part we have,

$$\frac{\mathcal{U}'(\Phi, \bar{\Phi}, T)}{T^4} = \frac{\mathcal{U}(\Phi, \bar{\Phi}, T)}{T^4} - \kappa \ln[J(\Phi, \bar{\Phi})]$$

S. K. Ghosh *et. al.* Phys. Rev. D 77, 094024 (2008).

$$\frac{\mathcal{U}(\Phi, \bar{\Phi}, T)}{T^4} = -\frac{b_2(T)}{2}\bar{\Phi}\Phi - \frac{b_3}{6}(\Phi^3 + \bar{\Phi}^3) + \frac{b_4}{4}(\bar{\Phi}\Phi)^2$$

and,

$$J[\Phi, \bar{\Phi}] = (27/24\pi^2)(1 - 6\Phi\bar{\Phi} + 4(\Phi^3 + \bar{\Phi}^3) - 3(\Phi\bar{\Phi})^2)$$

$J(\Phi, \bar{\Phi}) \Longrightarrow$ VdM determinant.

$$P(T,\mu_q,\mu_Q,\mu_S) = -\Omega(T,\mu_q,\mu_Q,\mu_S),$$

$$\frac{p(T,\mu_q,\mu_Q,\mu_S)}{T^4} = \sum_{n=i+j+k} c_{i,j,k}^{q,Q,S}(T)\left(\frac{\mu_q}{T}\right)^i\left(\frac{\mu_Q}{T}\right)^j\left(\frac{\mu_S}{T}\right)^k$$

where,

$$c_{i,j,k}^{q,Q,S}(T) = \frac{1}{i!j!k!}\frac{\partial^i}{\partial(\frac{\mu_q}{T})^i}\frac{\partial^j}{\partial(\frac{\mu_Q}{T})^j}\frac{\partial^k(P/T^4)}{\partial(\frac{\mu_S}{T})^k}\Big|_{\mu_{q,Q,S}=0}$$

$$\mu_u = \mu_q + \frac{2}{3}\mu_Q, \quad \mu_d = \mu_q - \frac{1}{3}\mu_Q, \quad \mu_s = \mu_q - \frac{1}{3}\mu_Q - \mu_S$$

For diagonal Taylor coefficients we have used,

$$c_n^X = \frac{1}{n!}\frac{\partial^n\left(P/T^4\right)}{\partial\left(\frac{\mu_X}{T}\right)^n}; \qquad n = i + j$$

For off-diagonal Taylor coefficients we have used,

$$c_{i,j}^{X,Y} = \frac{1}{i!j!}\frac{\partial^{i+j}\left(P/T^4\right)}{\partial\left(\frac{\mu_X}{T}\right)^i \partial\left(\frac{\mu_Y}{T}\right)^j}$$

Diagonal and off-diagonal susceptibilities are respectively defined as,

$$\chi_{XY} = \frac{\partial^2(P/T^4)}{\partial(\mu_X/T)\partial(\mu_Y/T)} \qquad \chi_{XX} = \frac{\partial^2(P/T^4)}{\partial(\mu_X/T)^2}$$

- Pressure consists of two parts; one regular part and one non-analytic part.

$$P(T,\mu_u,\mu_d) = P_r(T,\mu_u,\mu_d) + P_s(\bar{t},\bar{\mu}_u,\bar{\mu}_d)$$

with $\bar{t} = (T - T_C)/T_C$ and $\bar{\mu}_{u,d} = \mu_{u,d}/T$.

- $$t \equiv \bar{t} + A\mu_q^2 + B\mu_I^2$$

- From universal scaling behaviour;

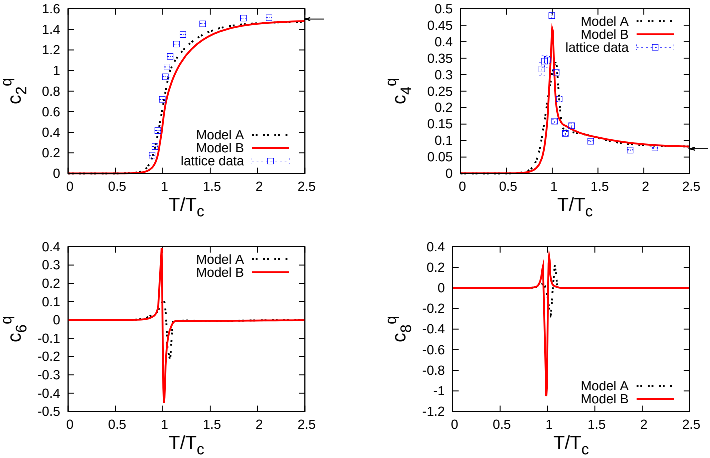
$$P_s(\bar{t},\bar{\mu}_u,\bar{\mu}_d) \sim t^{2-\alpha}$$

Then second and forth cumulant get contribution like;

$$(\partial^2 P_s/\partial\mu_X^2) \sim t^{1-\alpha} + \text{regular} \quad \text{and} \quad (\partial^4 P_s/\partial\mu_X^4) \sim t^{-\alpha} + \text{regular}$$

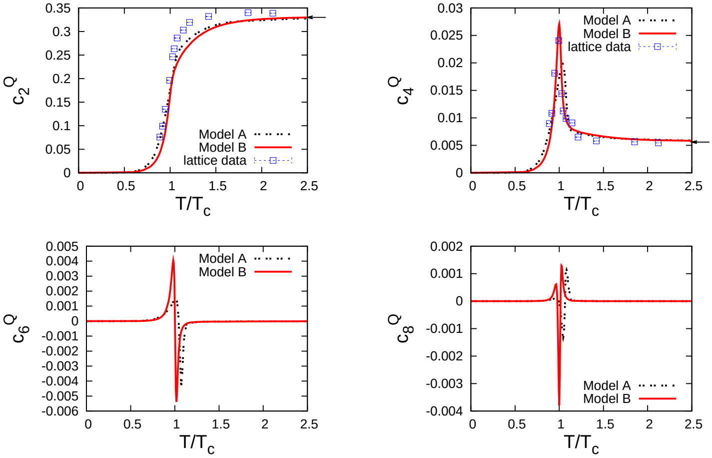
S. Ejiri *et. al.* Phys. Lett. B 633 (2006) 275.

Taylor Coefficents for μ_q.....



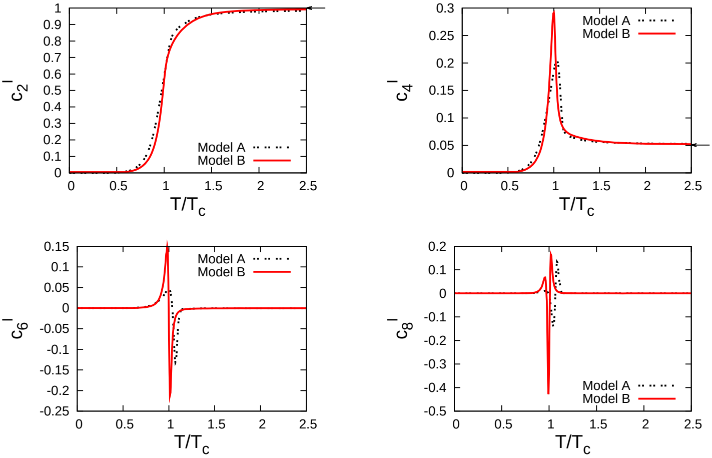
Lattice data taken from M. Cheng *et. al.* Phys. Rev. D 79, 074505 (2009).

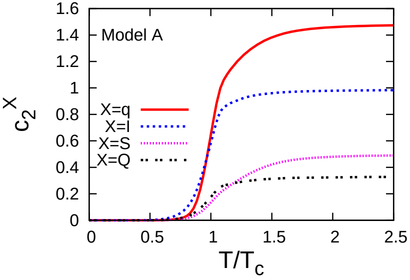
Taylor Coefficents for μ_Q.....



Lattice data taken from M. Cheng *et. al.* Phys. Rev. D 79, 074505 (2009).

Taylor Coefficents for μ_I.....

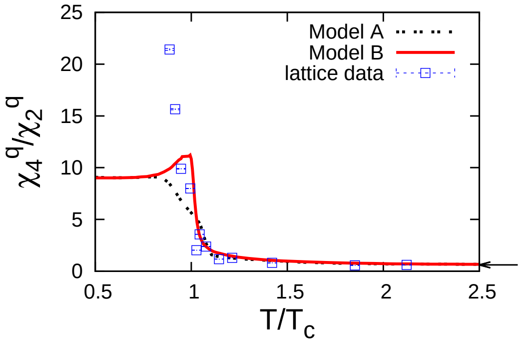




All diagonal Taylor coefficients show charecteristic crossover $\Rightarrow$ QCD phase transition liberates quarks.

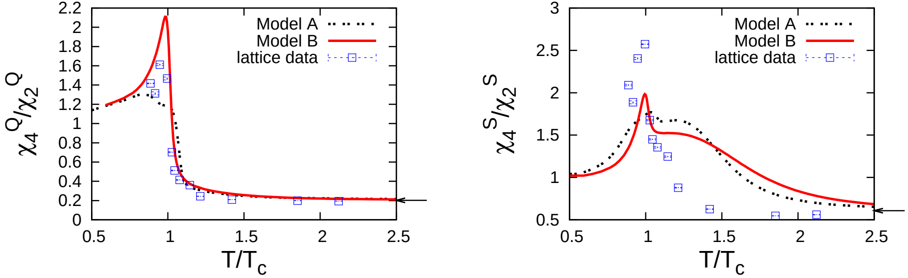
S. Gottlieb *et al.*, Phys. Rev. Lett. 59, 2247 (1987); R. V. Gavai *et al.*, Phys. Rev. D 40, 2743 (1989).

Kurtosis I



Kurtosis is a sensetive probe of deconfinement.
At low T kurtosis $R_q = (N_c B)^2 = 9$ and at high T it becomes unity in classical consideration and if corrected by quantum statistics $R_q = (6/\pi^2)$.

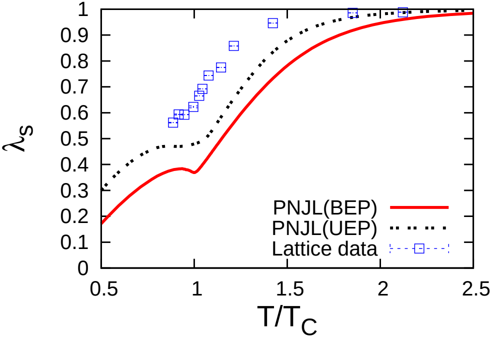
Kurtosis II



At low T, R_Q is dominated by charge fluctuations in pion sector resulting $R_Q = 1$. At high T, $R_Q = 2/\pi^2$ which is its SB limit.

Kurtosis for strange sector shows a peak at T_c. Model shows enhanced fluctuations after T_c and then converges to its SB limit.

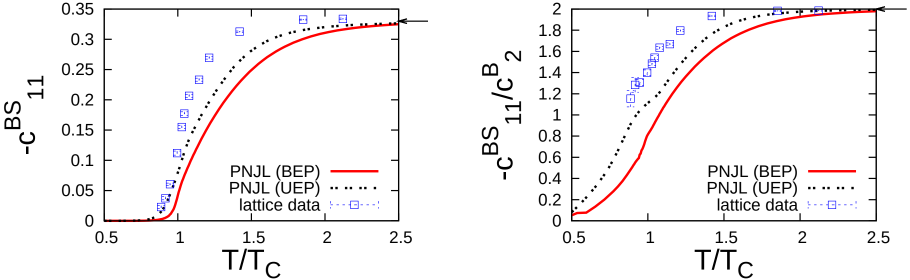
Wróblewski parameter



$$\lambda_s = \frac{2\langle s\bar{s}\rangle}{\langle u\bar{u} + d\bar{d}\rangle} \approx \frac{\chi_2^s}{\chi_2^u + \chi_2^d} = \frac{\chi_2^s}{\chi_2^u}$$

$\lambda_s^{8q}(T_c) \approx 0.37$ and $\lambda_s^{6q}(T_c) \approx 0.48$ with experimental bound $\lambda_s^{RHIC}(T_c) \approx 0.47 \pm 0.04$.

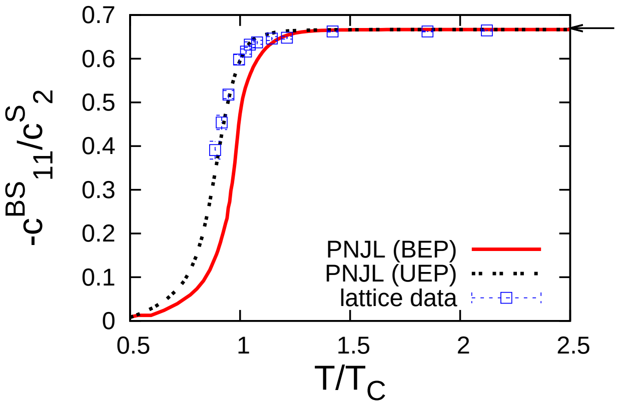
B-S Correlation I



In the high T phase B and S is highly correlated resulting high value of c_{11}^{BS}.

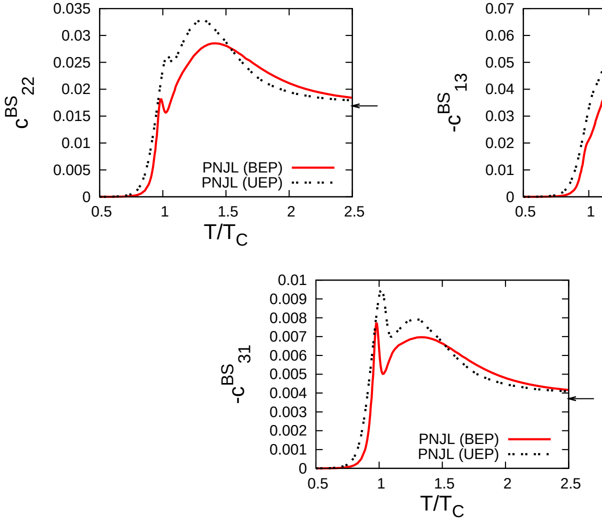
Lowest lying baryons do not carry strangeness $\Rightarrow$ Ratio of the right pannel goes to zero at low temperature.

B-S Correlation II

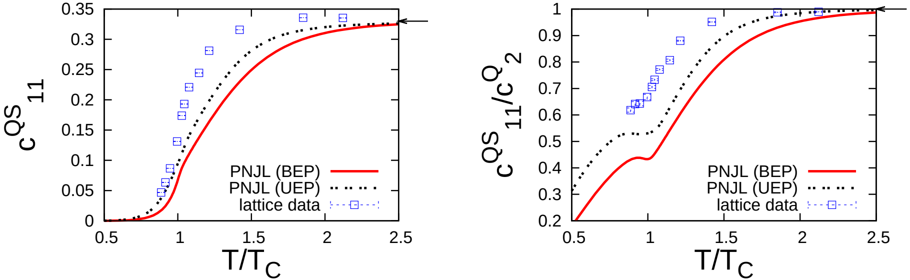


$$\mathrm{C_{BS}} = -3\frac{\langle BS\rangle - \langle B\rangle\langle S\rangle}{\langle S^2\rangle - \langle S\rangle^2} = -3\frac{\chi_{BS}}{\chi_{SS}} = -\frac{3}{2}\frac{c_{11}^{BS}}{c_2^{S}}$$

B-S Correlation III



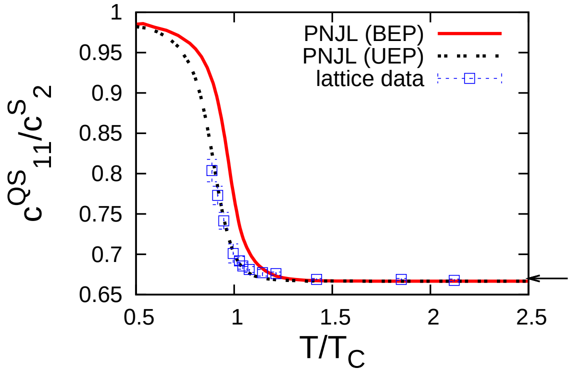
Q-S Correlation I



At high T, Q and S are related by strange quasiparticle which leads to high value of c_{11}^{QS}.

Lowest lying charged particle do not carry strangeness ⇒ Ratio in the right pannel vanishes at low T.

Q-S Correlation II



$$\mathrm{C_{QS}} = -3\frac{\langle QS\rangle - \langle Q\rangle\langle S\rangle}{\langle S^2\rangle - \langle S\rangle^2} = 3\frac{\chi_{QS}}{\chi_{SS}} = \frac{3}{2}\frac{c_{11}^{QS}}{c_2^S}$$

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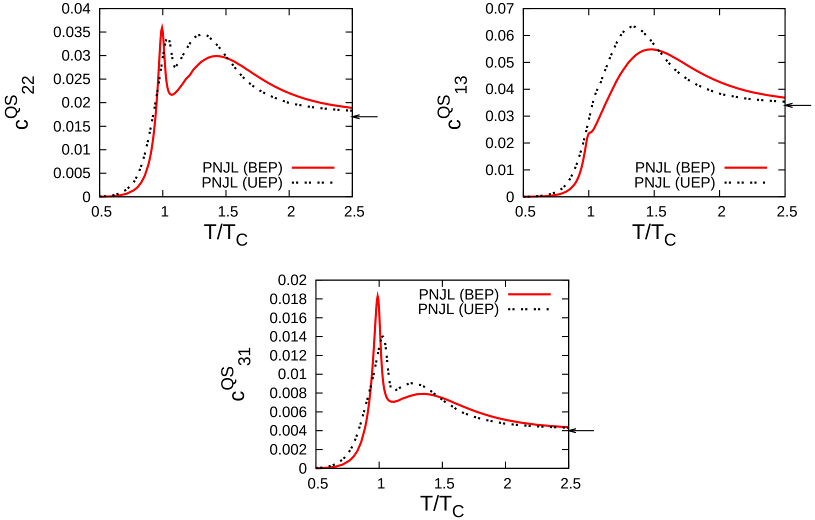
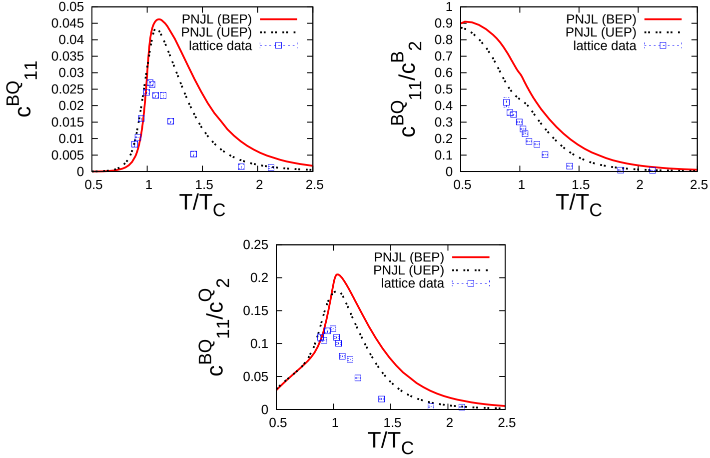


B-Q Correlation I



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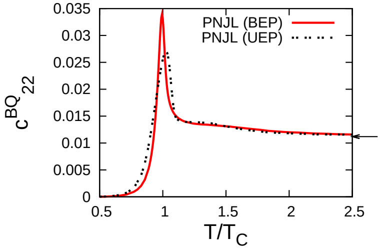
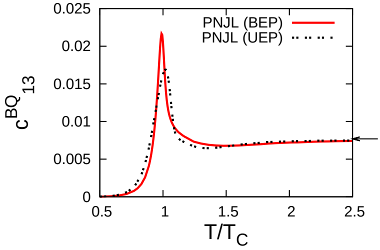
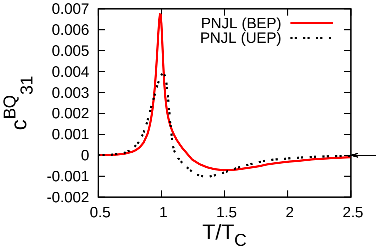






Strangeness Carriers I

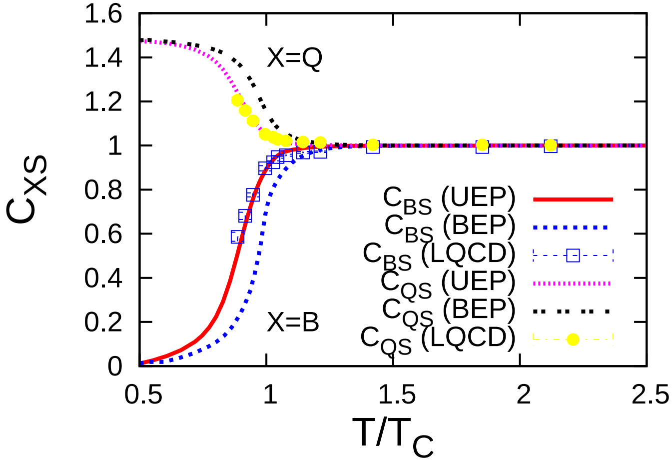
- In QGP phase $B_s = -\frac{1}{3} S_s$ and $Q_s = \frac{1}{3} S_s$
- No such direct relation for hadron gas.
-

$$\mathrm{C_{BS}} = -3\frac{\chi_{BS}}{\chi_{SS}} = 1 + \frac{\chi_{us} + \chi_{ds}}{\chi_{ss}} = 1 + \frac{c_{11}^{us}}{c_2^s}$$

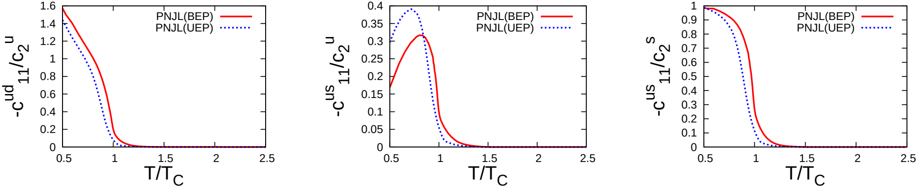
$$\mathrm{C_{QS}} = 3\frac{\chi_{QS}}{\chi_{SS}} = 1 - \frac{2\chi_{us} - \chi_{ds}}{\chi_{ss}} = 1 - \frac{1}{2}\frac{c_{11}^{us}}{c_2^s}$$

V. Koch *et. al.*, PRL 95, 182301 (2005); R. V. Gavai and S. Gupta, Phys Rev D 73, 014004 (2006).

Strangeness Carriers II



Light Flavor I



$$\frac{\chi_{BU}}{\chi_{UU}} = \frac{1}{3}\left(1 + \frac{\chi_{ud} + \chi_{us}}{\chi_{uu}}\right) = \frac{\chi_{BD}}{\chi_{DD}}$$

$$\frac{\chi_{QU}}{\chi_{UU}} = \frac{1}{3}\left(2 - \frac{\chi_{ud} + \chi_{us}}{\chi_{uu}}\right)$$

$$\frac{\chi_{QD}}{\chi_{DD}} = -\frac{1}{3}\left(1 - \frac{2\chi_{ud} - \chi_{us}}{\chi_{uu}}\right)$$

R. V. Gavai and S. Gupta, Phys Rev D 73, 014004 (2006).

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- Light flavor sector also shows that u flavor is carried along with B=1/3 and Q=2/3 and d flavor is carried along with B=1/3 and Q=-1/3.
- Non-zero extremely small values of flavor off-diagonal susceptibilities give a conception of quark quasiparticles which are dressed by interaction.

List of collaborators

- Rajarshi Ray (Bose Institute)
- Sanjay K. Ghosh (Bose Institute)
- Sibaji Raha (Bose Institute)
- Abhijit Bhattacharayya (Univ. of Calcutta)
- Paramita Deb (Univ. of Calcutta)

Thank You.