Strongly Interacting matter under charge neutrality and beta equilibrium

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Motivation

- Strongly interacting matter under extreme conditions: under active investigation.
  Having a model that can be used for:
  High temperature and low chemical potential: Heavy Ion Collision
  Low temperature and high chemical potential: Neutron star

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  Having a model that can be used for:
  High temperature and low chemical potential: Heavy Ion Collision
  Low temperature and high chemical potential: Neutron star
- Ultimate goal is to understand the whole phase diagram of strongly interacting matter.

Neutron star:phase transiton

- Mass $\rightarrow$ M $\sim$ 1 - 2M_0: M_0 is solar mass.
  Radius $\rightarrow$ R $\sim$ 10 km.
  Density $\rightarrow \rho \sim 10^{15}$ g/cm^3
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- Possible observational signatures: Gamma Ray Burst. [Bhattacharya et al, PRC 71(2005)]

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- conversion is two step process.
  1. nuclear to two flavour quark matter.
  2. two flavour to three flavour quark matter.

- Study of three flavour quark matter, matter in bulk should remain in beta equilibrium.

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- Charge neutrality condition: $\frac{2}{3}n_u - \frac{1}{3}n_d - \frac{1}{3}n_s - n_e = 0$
- Study is done within the framework of 2+1 flavour PNJL model.

previous works

1.The phase structure of charge neutral quark matter under β equilibrium is studied for a wide range of quark-quark coupling strengh within a four fermion model. [Abuki, Kunihiro, Nucl. Phys.A 768, 2006 118-159]
2.The study of phase diagram and pion modes of electrically neutral 2 flavour quark matter within PNJL model.[Abuki et al,PRD 78 014002,2008]
3.Phase diagram of 2 flavour quark matter under neutron star constraints for a non local covariant quark model. [Dumm et al, Eur. Phys J.A 31, 2007]
4.The study of dense charge neutral 3 flavour quark matter within NJL model. [Buballa, PRD 72 034004, 2005]

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- Use of effective models to understand the phases of Quantum Chromo Dynamics is a popular tool.
- Lattice Gauge Theory gives wealth of information in μ=0 limit. But computation at high μ region is a non-trivial task.
- Instead of actual QCD Lagrangian, a model Lagrangian is constructed, keeping in mind that it should describes the global features of QCD and also mathematically tractable.

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- The thermodynamic potential for NJL model:

$$
\begin{aligned}
\Omega = & 2g_S \sum_{f=u,d,s} \sigma_f^2 - \frac{g_D}{2}\sigma_u\sigma_d\sigma_s \\
& - 6 \sum_{f=u,d,s} \int_0^{\Lambda} \frac{d^3p}{(2\pi)^3} E_f \Theta(\Lambda - |\vec{p}|) \\
& - 2T \sum_{f=u,d,s} \int_0^{\infty} \frac{d^3p}{(2\pi)^3} \ln\left[1 + e^{-\frac{(E_f-\mu)}{T}}\right] \\
& - 2T \sum_{f=u,d,s} \int_0^{\infty} \frac{d^3p}{(2\pi)^3} \ln\left[1 + e^{-\frac{(E_f+\mu)}{T}}\right]
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- In presence of dynamical quarks: indicator of phase transition.

- The thermodynamic potential for PNJL model:

$$\Omega = \mathcal{U}'[\Phi, \bar{\Phi}, T] + 2g_S \sum_{f=u,d,s} \sigma_f^2 - \frac{g_D}{2}\sigma_u\sigma_d\sigma_s$$
$$-2T\sum_{f=u,d,s}\int_0^\infty \frac{d^3p}{(2\pi)^3} \ln\left[1 + 3(\Phi + \bar{\Phi}e^{-\frac{(E_f-\mu)}{T}})e^{-\frac{(E_f-\mu)}{T}} + e^{-3\frac{(E_f-\mu)}{T}}\right]$$
$$-2T\sum_{f=u,d,s}\int_0^\infty \frac{d^3p}{(2\pi)^3} \ln\left[1 + 3(\bar{\Phi} + \Phi e^{-\frac{(E_f+\mu)}{T}})e^{-\frac{(E_f+\mu)}{T}} + e^{-3\frac{(E_f+\mu)}{T}}\right]$$

where, $\sigma_f = \langle\bar{\psi}_f\psi_f\rangle$ $E_f = \sqrt{p^2 + M_f^2}$ with,

$$M_f = m_f - 2g_S\sigma_f + \frac{g_D}{2}\sigma_{f+1}\sigma_{f+2}$$

For the Polyakov loop part we have,

$$\frac{\mathcal{U}'(\Phi,\bar{\Phi},T)}{T^4} = \frac{\mathcal{U}(\Phi,\bar{\Phi},T)}{T^4} - \kappa \ln[J(\Phi,\bar{\Phi})]$$

with,

$$\frac{\mathcal{U}(\Phi,\bar{\Phi},T)}{T^4} = -\frac{b_2(T)}{2}\bar{\Phi}\Phi - \frac{b_3}{6}(\Phi^3+\bar{\Phi}^3) + \frac{b_4}{4}(\bar{\Phi}\Phi)^2$$

$$b_2(T) = a_0 + a_1(\frac{T_0}{T}) + a_2(\frac{T_0}{T})^2 + a_3(\frac{T_0}{T})^3$$

b_3 and b_4 is constant. [Pisarski et al.PRD 62 111501(R),2000]

$J[\Phi, \bar{\Phi}]$ is the Jacobian of transformation from Wilson line L to $(\phi, \bar{\phi})$

$$J[\Phi, \bar{\Phi}] = (27/24\pi^2)(1 - 6\Phi\bar{\Phi} + 4(\Phi^3 + \bar{\Phi}^3) - 3(\Phi\bar{\Phi})^2)$$

$J(\Phi, \bar{\Phi}) \Longrightarrow$ Vander Monde determinant. [Ghosh et al, PRD 77, 094024, 2008]

- Electrons are described by the free non-interacting gas of fermions. Correspponding thermodynamic potential is:

$$\Omega_e = -(\frac{{\mu_e}^4}{12\pi^2} + \frac{{\mu_e}^2 T^2}{6} + \frac{7\pi^2 T^4}{180})$$

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- The thermodynamic potential considered here: $\Omega + \Omega_e$
- The number densities of u, d, s quarks and electrons are given as: $n_u = -\frac{\partial\Omega}{\partial\mu_u}$, $n_d = -\frac{\partial\Omega}{\partial\mu_d}$, $n_s = -\frac{\partial\Omega}{\partial\mu_s}$ and, $n_e = -\frac{\partial\Omega}{\partial\mu_e}$

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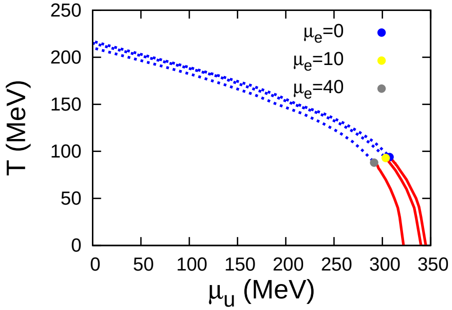
- Look for the contour where total charge n_Q goes to zero.
- Study the phase diagram considering the constraint of beta equilibrium.

Phase diagram

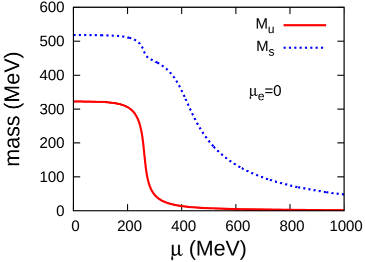
The QCD phase diagram for matter under beta equilibrium is obtained for different electron chemical potentials.



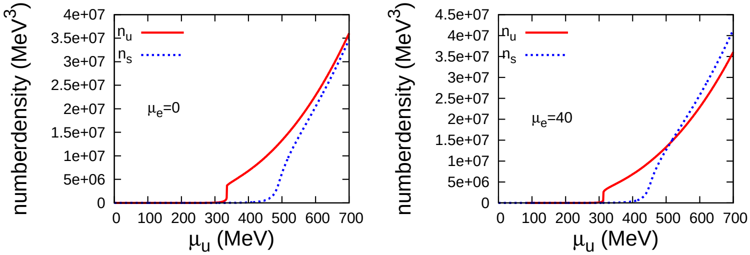
Critical End Point:

$\mu_e = 0 \rightarrow (307,94)$; $\mu_e = 10 \rightarrow (303,93)$; $\mu_e = 40 \rightarrow (291,88)$

The Consituent quark masses are plotted against chemical potential.



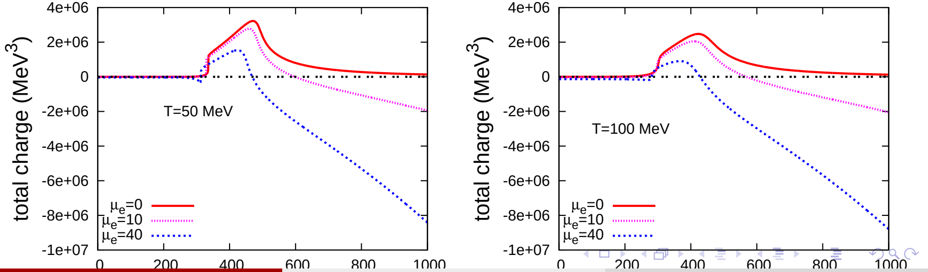
The numberdensity of u and s quark is plotted against chemical potential.



For μ_e=0, $n_u > n_s$ due to heavy strange mass, at high μ the mass effect is negligible.
For $\mu_e \neq 0$, at low μ similar situation. At high μ regime $\mu_s > \mu_u$ condition dominates; $n_s > n_u$.

search for charge neutrality

- The net charge density is plotted with chemical potential at different temperatures.



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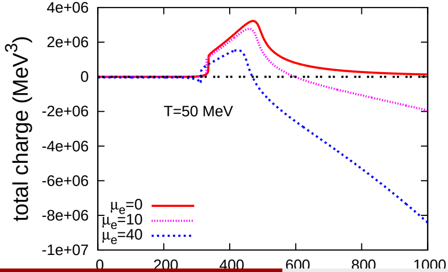

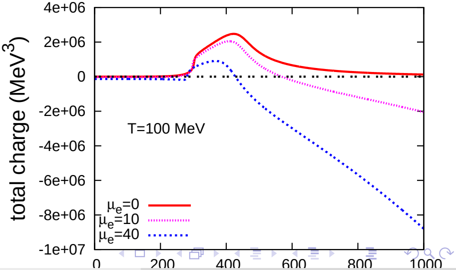

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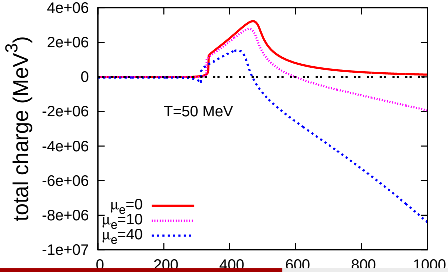
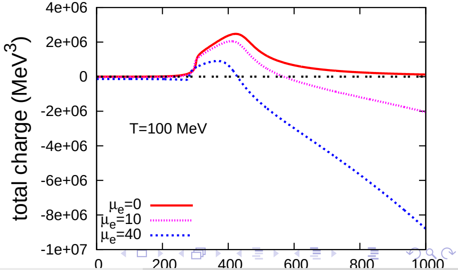
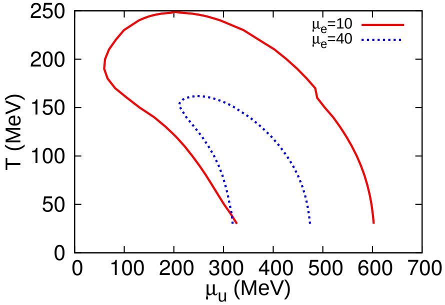
- The net charge density is plotted with chemical potential at different temperatures.
- When μ_e is zero, total charge always positive, goes to 0 asymptotically at high μ.
- for μ_e non zero, we get two charge neutral points, one at low μ, other at high μ region.

  $\mu_d = \mu_u + \mu_e$; $\mu_s = \mu_d$

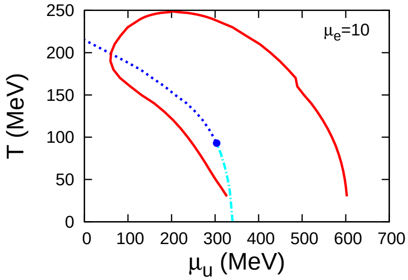




The charge neutral contour for different electron chemical potential is obtained.

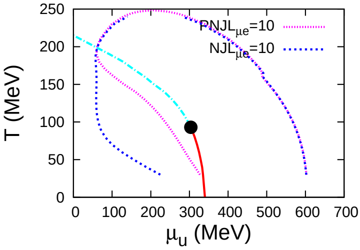


The location of charge neutral contour with respect to the phase diagram is obtained.

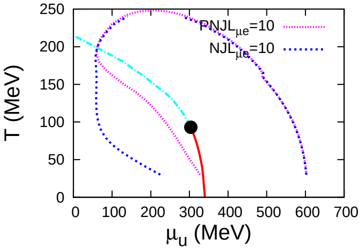


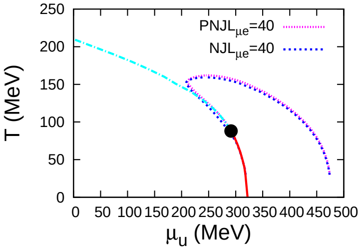
- At μ_e=40 MeV, charge neutral contour lies on the phase boundary. Charge neutrality is satisfied in this region.

- The charge neutral trajectory for both NJL and PNJL model is compared.

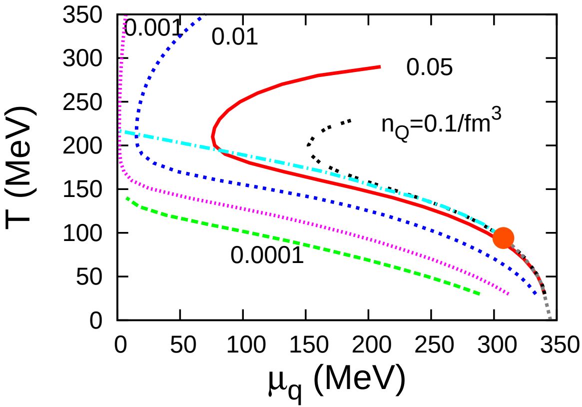


- The charge neutral trajectory for both NJL and PNJL model is compared.
- They should be same in the deconfined phase.

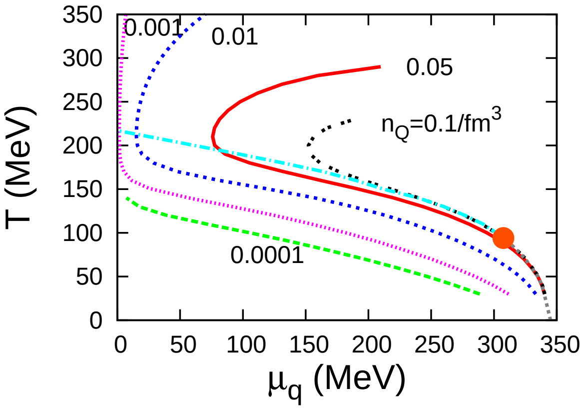




- For μ_e=0: We have μ_u=μ_d=μ_s



- For μ_e=0: We have $\mu_u = \mu_d = \mu_s$
- Charge conservation trajectory is studied under this condition.



Summary

Quark-hadron phase transition in neutron star: implication of beta equilibrium on the phase diagram.

The study is done within the frame work of 2+1 flavour PNJL model.

Charge neutral contour in the T - μ plane, with respect to the phase diagram is obtained.

The total charge going to negative as increasing chemical potential.

Collaborators

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Rajarshi Ray, Bose Institute, Kolkata
Abhijit Bhattacharya, University of Calcutta