Towards faster computation of higher order quark no. susceptibilities in QCD

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Outline

Introduction

- Quark number susceptibilities(QNS) determine the fluctuations of conserved charges like electric charge, Baryon number, strangeness in the QCD medium.
- These help us in understanding the properties of the QCD medium specially the nature and the correlations amongst the quasi-particles for $T > T_c$. Recent experiments suggest that the QGP phase consists of interacting quasiparticles rather than free gas consisting of quarks and gluons.
- Determining these quantities accurately from first principle lattice calculations is important for understanding the QCD medium.
- Also important for estimating the location of the critical end point of QCD by Taylor series method [Gavai& Gupta('03)].

Current results

- There are about 10-5% differences between the lattice and the HTL resummed results for second order diagonal susceptibility in the temperature range $2-5T_c$ [Rebhan('05)].
- We need to separate the cut-off effects from the effects due to interactions $\rightarrow$ reduce cut-off effects in the fermion operators.
- Improved staggered fermion operators like the $p4$ and Asqtad are currently used for computing susceptibilities $\rightarrow$ have small cut-off effects but expensive to compute.
- Can we reduce the cut-off effects in the existing results? Motivation from the continuum theory for introducing μ in lattice fermion operators.

Basic set up

- The discretized version of the Dirac operator on the lattice is of the form

$$D(0)_{x,x'} = \left(3 + \frac{a}{a_4} + M\right)\delta_{x,x'} -$$
$$\sum_{j=1}^{3}\left(U_j^\dagger(x-\hat{j})\frac{1+\gamma_j}{2}\delta_{x,x'+\hat{j}} + U_j(x)\frac{1-\gamma_j}{2}\delta_{x,x'-\hat{j}}\right)$$
$$-\frac{a}{a_4}\left(U_4^\dagger(x-\hat{4})\frac{1+\gamma_4}{2}\delta_{x,x'+\hat{4}} + U_4(x)\frac{1-\gamma_4}{2}\delta_{x,x'-\hat{4}}\right).$$

- Introduce chemical potential μ as a Lagrange multiplier corresponding to a "number density" defined in a point-split form.

$$D(\mu)_{xy} = D(0)_{xy} - \frac{\mu}{2}\left[(\gamma_4+1)U_4^\dagger(y)\delta_{x,y+\hat{4}} - (1-\gamma_4)U_4(x)\delta_{x,y-\hat{4}}\right] .$$

Basic set up

- This operator was motivated for the Overlap fermions [Gavai & Sharma(10)] but this formalism can be applied for all fermion operators on the lattice. We use staggered fermions for our computations.
- Potentially divergent $\mu^2/a^2(1/a^2)$ terms present in the lattice expression of energy density (quark no. susceptibility)$\rightarrow$ have to perform a zero-temperature subtraction to remove such terms.
- Conventional method: $\mathrm{e}^{\pm\mu\mathrm{a}_4}$ is multiplied with U_4 , $U_4^\dagger$ respectively in $D_W(0)$ leads to a $D_W(\mu)$ [Hasenfratz-Karsch (83)] $\rightarrow$ do not give the divergences. In general functions, $f(\mu a_4),\ g(\mu a_4)$ multiplying U_4 , $U_4^\dagger$ respectively and satisfying $f.g = 1$, $f - g \approx \mu a_4$ lead to cancellation of the divergences[Gavai(85)].

Basic set up

- The quark number susceptibilities(QNS) are defined as

$$\chi_{ij}(\mu_i, \mu_j) = \frac{T}{V} \frac{\partial^{i+j} \ln Z(T, \mu_{i,j}, m)}{\partial \mu_i \mu_j}$$

- On the lattice, $T = 1/(N_T a_4)$ and $V = N^3 a^3$.
- The partition function in terms of the lattice Dirac operator is

$$Z(T, \mu_{i,j}, m) = \langle DetD \rangle = \int \mathcal{D}U \mathrm{e}^{-\mathrm{S_G}} \mathrm{DetD}.$$

- For 2 flavour QCD and conserved isospin symmetry $\mu_u = \mu_d$.
- The baryon no. susceptibilities can be expressed in terms of QNS by noting that $\mu_B = 3\mu_u = 3\mu_d$.
- The χ_{ij}'s can be written as derivatives of the Dirac operator. Example :$\chi_{20} = \frac{T}{V} \langle \mathrm{Tr}(D^{-1}D'' + (D^{-1}D')^2) \rangle$.
- On the lattice these are computed at $\mu_B = 0$ to avoid the fermion sign problem.

Basic set up

- The computation of higher order QNS is much faster using our operator

$$D' = \sum_{x,y} N(x,y), \qquad \text{and} \qquad D'' = D''' = D''''... = 0 \ ,$$

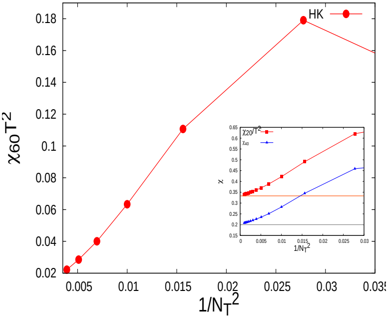
incontrast to the to the $\exp(\pm\mu a_4)$-prescription, where,

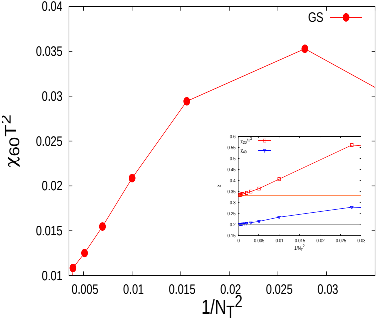
$$D' = D'''... = \sum_{x,y} N(x,y) \qquad \text{and} \qquad D'' = D'''' = D''''''... \neq 0 \ .$$

Each derivative term comes with a inverse of fermion operator D^{-1}. Inversion is the most expensive step on the lattice so need to cut down on fermion matrix inversions $\rightarrow$ achieved for our operator.

Free fermi gas results

- We compute the susceptibilities for free fermions ($U = 1$). The additional lattice artifacts in the second and higher order QNS are estimated on a 24^3 lattice with infinite temporal extent and subtracted from our computations.
- The cut-off effects are different in both the methods. For higher order susceptibilities cut-off effects are smaller in our method.



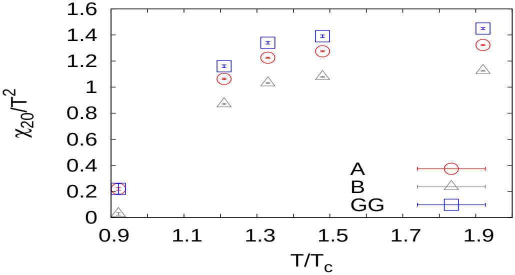


Simulation details

- The configurations used were generated for $N_f = 2$ flavour QCD with Wilson action for gauge part and naive staggered fermions. The details mentioned in [Gavai& Gupta ,PRD 78,114503('08)]
- R-algorithm was used to generate the configurations. Traces were computed using 500 random vectors.
- The input pion mass: $M_\pi \approx$230MeV.
- Lattice size was $24^3 \times 6$. The susceptibilities were measured for temperatures upto $1.92T_c$.

Results for χ_{20}

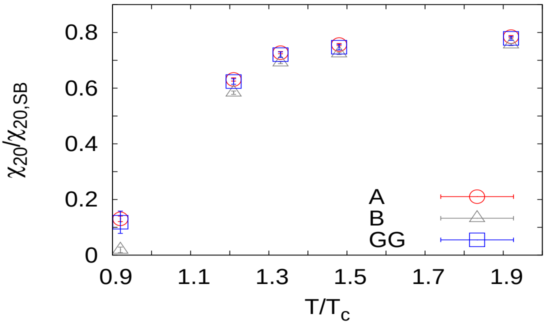
- χ_{20}/T^2 computed using our fermion operator has $\mathcal{O}(1/a^2)$ terms on the lattice. We compared two different prescriptions to remove them:
  A) subtract the free quark gas $T = 0$ contribution computed on a 24^3 lattice with infinite temporal direction.
  B)subtract the free theory contribution obtained from a 24^4 lattice.
  The GG results were for χ_{20}/T^2 using HK operator. [Gavai& Gupta('08)]



- The subtraction scheme B seems to give results close to the free fermion results at $T > 1.5\,T_c$: consistent with the RBC & MILC results which were computed using different improved fermions.

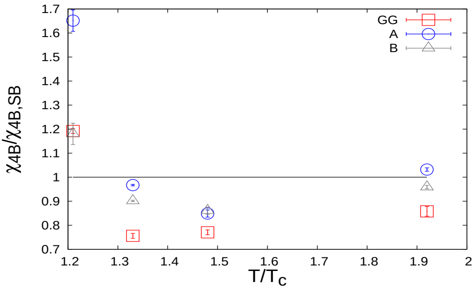
Results for χ_{20}

- How dominant are the cut-off effects?
- At $T < T_c$ the cut-off effects are huge and both these subtraction procedures do not work out.
- At high temperatures the ratio is independent of the method.

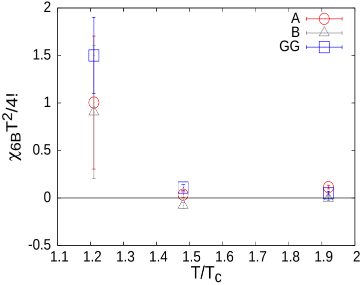


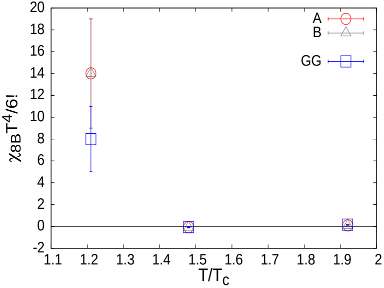
Fourth order susceptibility

- χ_n have a leading order cutoff dependent term of the form a^{n-4} in our method.
- The cut-off effects are similar for χ_{20} and χ_{4B} for $T > T_c$.



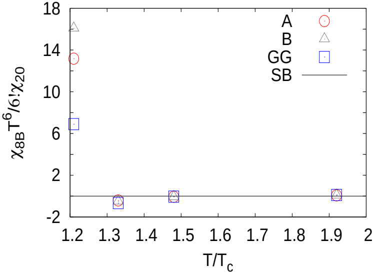
Higher order susceptibilities

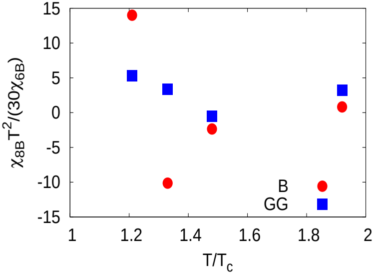




- Higher order susceptibilities $\chi_{6,8}$ have an additional cutoff dependent which are irrelevant in the continuum limit.
- The results are not very sensitive to the subtraction procedure.

Ratios of susceptibilities

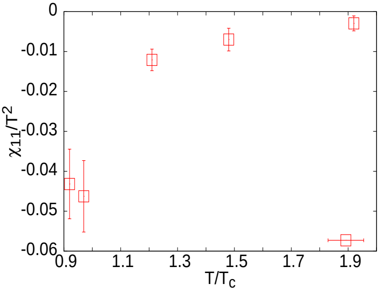


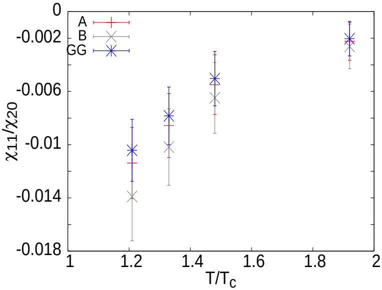


- Ratio of higher order susceptibilities are independent of the method. These ratios appear in the expression for the radius of convergence.
- These are sensitive indicators of the location of critical point and allow for bracketing the critical region.

Off-diagonal susceptibilities

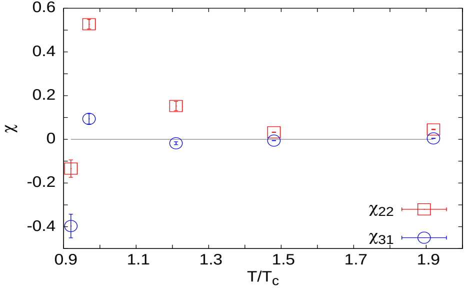
- Zero for the free theory$\rightarrow$ gives a qualitative estimate of the interactions of the QCD medium.
- The lowest order finite off-diagonal susceptibility is χ_{11}/T^2-should be same in both the methods. Gives us a consistency check.
- χ_{11}/T^2 falls to zero rapidly for $T > 1.5T_c \rightarrow$ indicates that the medium consists of very weakly interacting quasiparticles at $T \sim 2T_c$ which carry the quantum numbers of quarks.
- The data seems to rule out the colour di-quark model of Shuryak and Zahed.





Off-diagonal susceptibilities

- Do not require any subtractions because χ_{11} data suggest that the correlations between u and d quarks are vanishingly small at $T > T_c$.
- The χ_{22} peak near T_c and fall to zero rapidly for $T > T_c \to$ gives an estimate of the T_c. Previous estimate of T_c from this peak is consistent with the peak in χ_L. [Gavai& Gupta ,PRD 78('08)]
- χ_{31} falls to zero $\to$ indicative of deconfinement at $T > T_c$.



Conclusions

- Computing higher order quark number susceptibilities(QNS) from existing lattice fermion operators is very expensive.
- We suggest a lattice fermion operator which would allow us to reduce computation time for higher order QNS. Also has reduced lattice cut-off effects.
- There would be additional lattice artifacts appearing in the QNS computed using our method. We have proposed a method to eliminate these additional terms in the QGP phase without much extra computational effort.

Outlook

- The proposed subtraction scheme fails for $T < T_c$. One has to do a full QCD simulation on a symmetric lattice to estimate the zero temperature values of χ_n's as done for pressure and number density computation on the lattice.
- Continuum extrapolation of higher order susceptibilities is desirable for estimating the location of the critical end point in QCD phase diagram $\rightarrow$ important for experimental searches. This method could still be efficient for continuum extrapolation of χ_n's for $n \geq 8$. We would like to address these issues in future studies.

Acknowledgements

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