Causality and Dispersion Relations - 3 : A physical example

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Abstract

Titchmarsh's theorem [1] provides an intimate connection between causality and the dispersion relations first discovered in optics by Kramer and Kronig. Any physical system characterised by a square integrable function which is the Fourier transform of a causal function, will exhibit dispersive and absorptive phenomena. We shall concretise this statement mathematically, and discuss in detail the example of dispersion and absorption in dielectric media having a complex and frequency dependent electric susceptibility $\chi_e(\omega)$. In particular, we shall consider a simple model of a dielectric medium, and demonstrate how a causal connection between the electric field $\mathbf{E}$ and the electric displacement $\mathbf{D}$ in the medium, leads to the phenomena of anomolous dispersion and resonant absorption [2].

References

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