Spinor representations of Lorentz Group

Suryanarayan Dash

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Vectors,tensors and spinors are differing on the basis of their transformation properties.There is a special kind of subgroup SL(2,C) of Homogeneous Lorentz group under which spinors are transformed .
The real homogeneous Lorentz transformation is the set of 4 x 4 real matrices that satisfy the equation Λ^{Tr} G Λ = G where G define the metric of diagonal entry(+1,-1,-1,-1) and Λ the matrix of transformation.We find detΛ=$\pm$1. Then L(IR) splits into 4 disjoint classes depending upon the value of det Λ and the zeroth component, as proper orthochronous denoted as $L_+^\uparrow$ with $\Lambda_0^0 \geq 1$ and improper orthochronous $L_-^\uparrow$ with $\Lambda_0^0 \geq 1$ and corresponding nonorthochronous entry.Where only $L_+^\uparrow$ is a subgroup of L(IR).
Then to understand the correspondence between the proper three-dimensional rotation group $O_+(3)$ and SU(2,C) the subgroup of SL(2C), we impose the condition like det = +1 and unitarity of the complex transformation matrix.We found the minimum parameter to describe these two group are same.We discussed SL(2C)$\rightarrow$ O(3) as two-to-one homomorphism and claim that it is not more than two-to-one.The correspondence between the proper orthochronous Lorentz group $L_+^\uparrow$ and SL(2C) on the same procedure as SL(2C) with O(3).
We know as in case of L.T.,the transformation matrices acts on 4-vector,similarly the transformation matrices of SL(2C) (also called covering group) act on 2 dimensional complex vector space where the vectors are called spinors.We discussed how a two component spinor transform under SL(2C) imposing the condition that the determinant of the transformation matrix should be +1 for proper transformation.Where the the metric of transformation can be chosen as antisymmetric.
In this case the representation expressed in a reduced dimensional form as compared to vectors.So a detailed understanding of the theory of spinors can help to understand the Lorentz covariance of Dirac's equation for electron .

References:

- Lecture notes on 'Introduction to Lorentz Spinors' by D.N.Williams URL:http://www.personal.umich.edu/williams/notes.
- Electrodynamics and Classical theory of Fields and Particles by A.O.Barut.