Limits on Lorentz Invariance in Electrodynamics

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A test theory in Special Relativity is a generalization of the Lorentz transformations of Special Relativity using additional parameters. One can then analyse experiments using the test theory (rather than Special Relativity itself) and fit parameters of the test theory to the experimental results. If the fitted parameter values differ significantly from the values corresponding to Special Relativity, then the experiment is inconsistent with Special Relativity. But more normally, such fits can show how well a given experiment confirms or disagrees with Special Relativity, and what the experimental accuracy is for doing so. This gives a general and tractable method of analysis which can be common to multiple experiments. Different test theories differ in their assumptions about what form the transformation equations could reasonably take. There are at present two main test theories of Special Relativity:

(1) Test theory of Robertson, Mansouri and Sexl (see ref. [1, 2]).
(2) Test theory of Edwards (see ref. [3]).

These test theories can also be used to examine potential alternate theories to Special Relativity. Such alternate theories predict particular values of the parameters of the test theory, which can easily be compared to values determined by experiments analyzed with the test theory.
We concentrate on the framework given by Robertson, Mansouri and Sexl. Here we assume that there exists a frame Σ in which the velocity of light is constant and isotropic in all directions and equals $c = 3 \times 10^8 m/s$. We investigate the behaviour of velocity of light in a frame S, which is moving with constant velocity $\mathbf{v}$ with respect to Σ. We start with transformation equations in differential form (for a full discussion see ref.[2]) for the case of where the velocity of S as measured in Σ is along the positive X axis and assuming Einstein synchronisation in S:

$$\begin{aligned} dT =& \frac{1}{a}\left(dt + \frac{v dx}{c^2}\right);\ dX = \frac{dx}{b} + \frac{v}{a}\left(dt + \frac{v dx}{c^2}\right); \\ dY =& \frac{dy}{d};\ dZ = \frac{dz}{d} \end{aligned} \tag{1}$$

with c the velocity of light in vaccuum in Σ. Here $a = a(v); b = b(v); d = d(v)$. Using the usual expansion of the three parameters [$a \approx 1 + \alpha v^2/c^2 +$ (terms of order c^{-4}); $b \approx 1 + \beta v^2/c^2 +$ (terms of order c^{-4}); $d \approx 1 + \delta v^2/c^2 +$ (terms of order c^{-4})], setting $c^2 dT^2 = dX^2 + dY^2 + dZ^2$ in Σ, and transforming according to (1), we find the coordinate travel time of a light signal in S:

$$dt = \frac{dl}{c}[1 - (\beta - \alpha - 1)\frac{v^2}{c^2} - (\frac{1}{2} - \beta + \delta)sin^2\theta\frac{v^2}{c^2}] + (\text{terms of order } c^{-4}) \quad (2)$$

α = time dilation parameter
β = length contraction parameter parallel to **v**
δ = length contraction parameter perpendicular to **v**

where dl = $\sqrt{dx^2 + dy^2 + dz^2}$ and θ is the angle between the direction of light propagation and the velocity **v** of S in Σ. In Special Relativity $\alpha = -\frac{1}{2}$; $\beta = \frac{1}{2}$; $\delta = 0$, and (1) reduces to usual Lorentz transformations. Generally, the best candidate for Σ is taken to be the frame of the cosmic microwave background (CMB) with velocity of solar system in that frame taken as 377 km/s, declination $\approx -6.4^o$, and right ascension (RA) $\approx 11.2h$.

References

[1] H.P.Robertson, Rev. Mod. phys. **21** 378 (1949).

[2] R.Mansouri *et al.*, Gen. Relativ. Gravit. **8** 497 (1977); **8** 515 (1977); **8** 809 (1977).

[3] Edwards, Am. J. Phys. **31** 482 (1963).