

# Phases of QCD

- Normal points and phase transitions; types of transitions; order parameters and susceptibilities; phase diagrams; how transitions are found; uncertainties in lattice computations (1)
- The QCD phase diagram: finite temperature, finite chemical potential, the full phase diagram (1)
- What do we know about the properties of QCD matter? (2)

# Lecture

# 1

# Phase transitions and lattice

- Thermodynamics and the phase diagram
- Some special points on the phase diagram: first order transition, critical points (simple system), tricritical points, higher order multicritical points (complex systems).
- Order parameters and susceptibilities show phase transitions
- Lattice computations have statistical and systematic errors: need control

# A quick overview

- Thermodynamics: **gross properties** of matter; total energy in a glass of water, its volume, the concentration of iron in it, its entropy, etc
- Statistical mechanics: properties of particles which lead to thermodynamics; correlation functions, **fluctuations**, etc
- Field theory: treat **particle production** and other inherently quantum processes that contribute to thermodynamics.

# Entropy

- Every system in equilibrium is described by a certain number of extensive quantities (**state variable**). 2<sup>nd</sup> law says entropy ( $S$ ) is one of these. They together define **Gibbs space**.
- For each equilibrium system,  $S$  is the maximum possible for fixed values of other extensive variable.  $S(E, V, \dots)$  is called the **entropy surface**.
- This surface has continuous derivatives (**smooth**) and always lies on one side of a tangent (**convex**)

# Examples

- Important special case of three dimensional Gibbs space.
- Example 1: chemically pure water (or any other substance) described by internal energy ( $E$ ), molar volume ( $V$ ) and the entropy ( $S$ ).
- Example 2: QCD described by internal energy ( $E$ ), baryon number ( $B$ ) and the entropy ( $S$ ).
- Ferromagnets, superconductivity...

# Examples (continued)

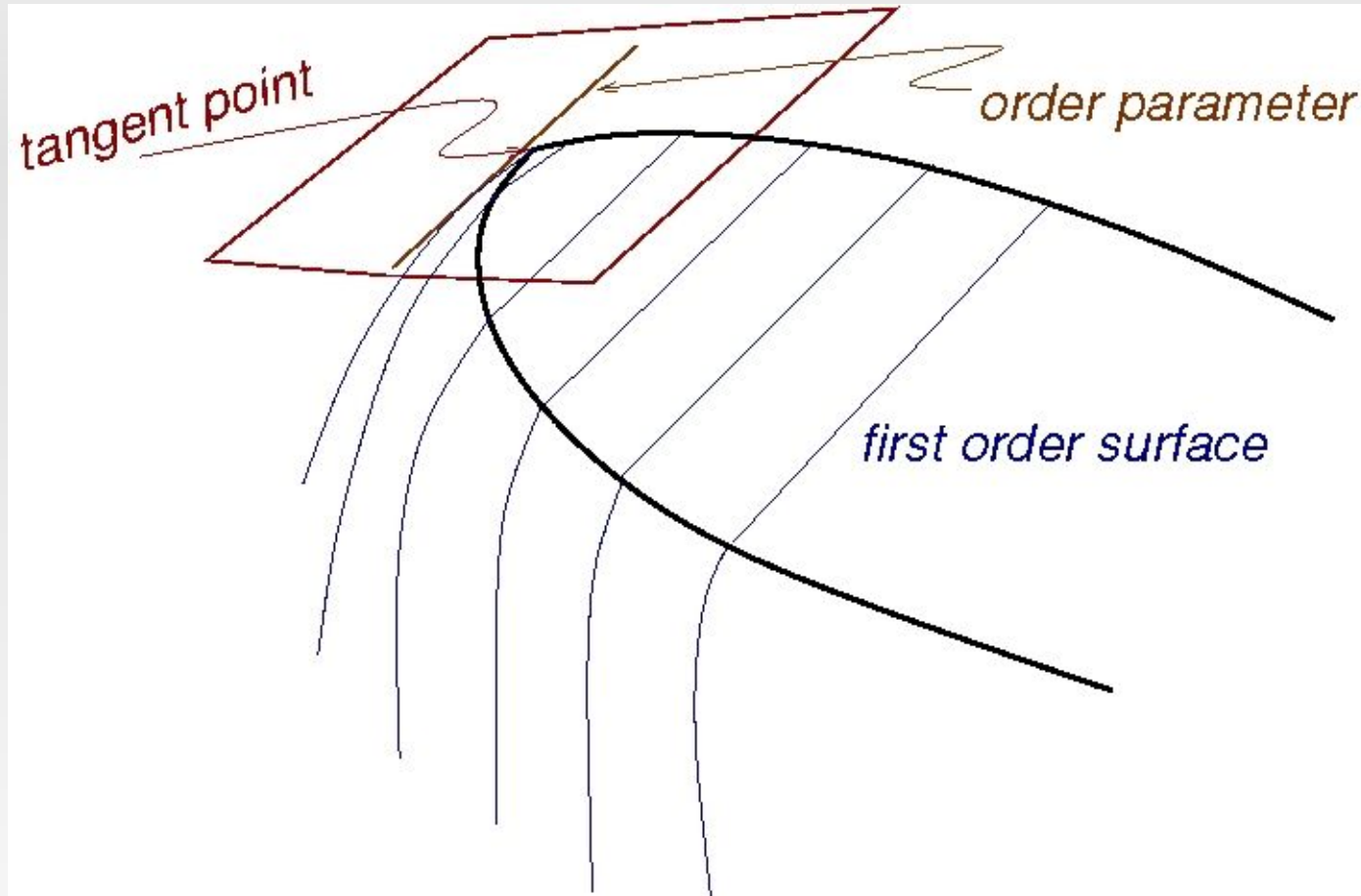
- **Couplings**:  $\beta = dS/dE$  and  $\beta P = -dS/dV$  are continuous
- **Convexity**:  $d^2S/dE^2$ ,  $d^2S/dV^2$  are negative. Closely related to positivity of specific heat ( $c_v$ ) and compressibility ( $K$ )
- **Energy representation**: Instead of writing  $S(E,V)$ , one may write  $E(S,V)$ . Same surface, same properties.
- **Intensive variables**: closely related to couplings:  $T = dE/dS$  and  $P = -dE/dV$

# Phase transitions

- Usually a tangent touches any convex surface at exactly one point. On entropy surface, these are **normal points**.
- Exceptions are flat portions of a surface; these are called **1<sup>st</sup> order phase transitions**.
- Special points on the boundaries of these flat portions will turn out to be 2<sup>nd</sup> order phase transitions (**critical points**)



# A generic entropy surface



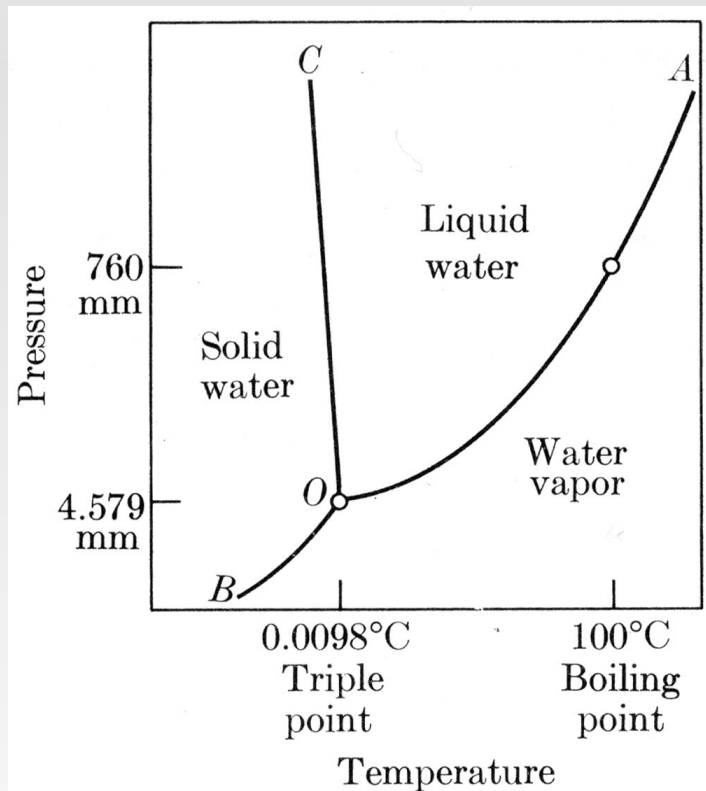
# First order transitions

- At a **1<sup>st</sup> order phase transition**, the entropy surface coincides exactly with the tangent.
- Example: the entropy of one mole of water at 100 C is  $S$ , of steam at 100 C is  $S$ . As one mole of water is heated to 100 C, it begins to turn to steam without change in T. When a fraction  $f$  has become steam, the entropy is linear  $S(f) = f S + (1-f) S$ . Similarly for  $E$  and  $V$ , hence surface is flat.

# Phase diagrams

- Thermodynamic systems plotted in terms of the intensive quantities are called **phase diagrams**.
- Normally each point on the phase diagram corresponds to exactly one phase.
- On the flat portion of  $E(S,V)$ , there is a coexistence of two phases for some  $T$  and  $P$ . This is a **1<sup>st</sup> order line**.
- How does such a line end?

# Phase diagram of water

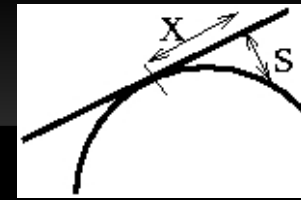


- No end to the line: (B) always a phase transition
- **Triple point:** (O) 2 lines meet, 3 phases coexist
- **Critical point:** (A) 2 phases no longer distinct

# Special points

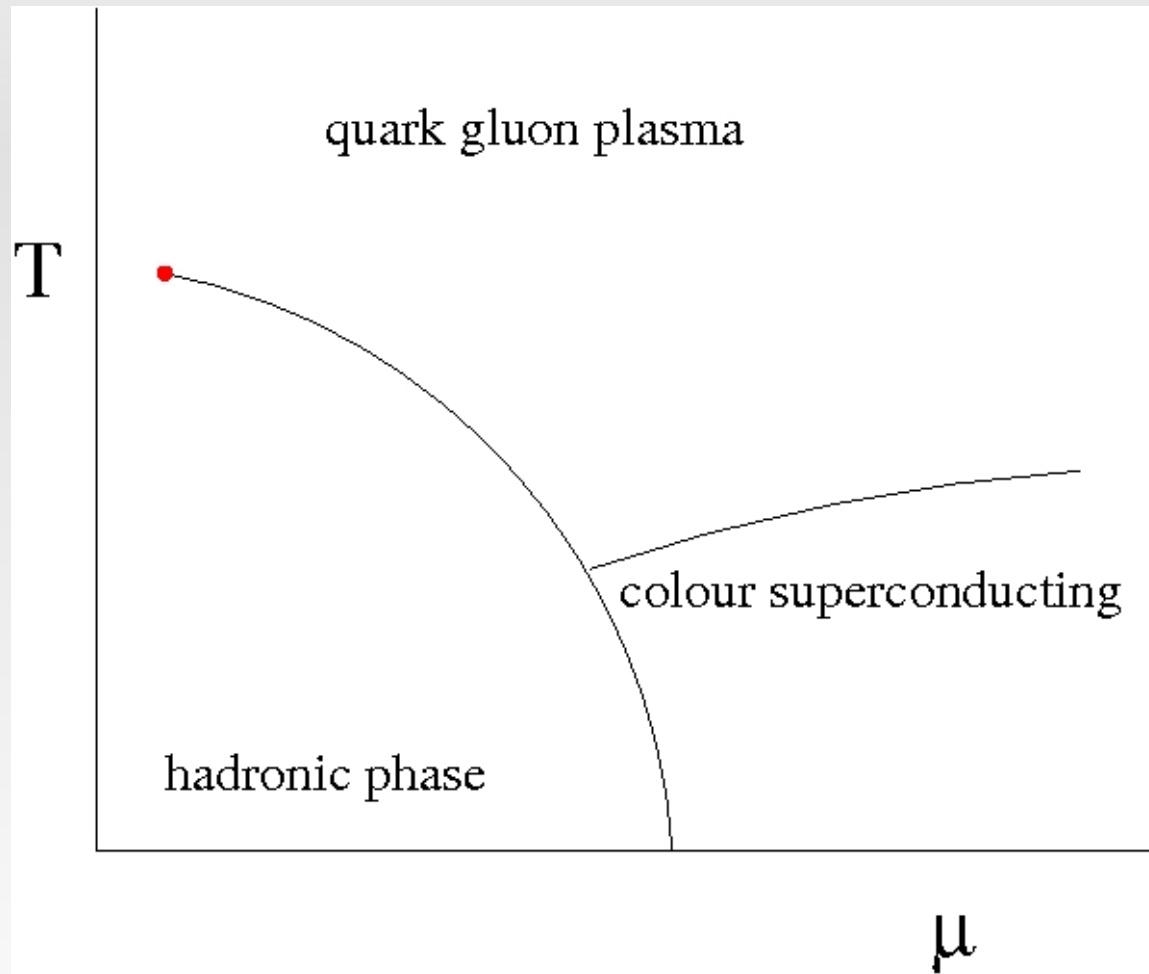
- At a triple point three different phases coexist. As thermodynamics conditions (ie, extensive variables) are changed slightly, the balance of the three phases shift.
- If the latent heat changes as some parameter is tuned, and the latent heat eventually becomes zero, then we have a critical point: the two phases are no longer distinct.
- Critical points are spectacular [movies]

# Critical points



- $x$  is distance from the point of tangency,  $s$  the difference between tangent and entropy surface. If  $s(x)=x^{1+n}$  then at a normal point  $n=1$ .
- If  $q$  is the intensive variable, then legendre transformed surface recedes from the tangent as  $f(q)=q^{1+1/n}$
- As a result the susceptibilities,  $f''(q)$ , diverge as fractional powers of  $q$ . **Critical indices.**
- Happens where 1<sup>st</sup> order surface ends.

# Phase diagram of QCD



# Field theory

Except the kinetic term, each term in a Lagrangian defines both an **extensive** and an **intensive** variable. Example:

$$L = (d\phi)^2 + m^2 \phi^2 + \lambda \phi^4$$

The extensive quantities are constructed from field operators, and their expectation values are proportional to the volume. The intensive quantities are called couplings.



# Lattice field theory

- Field theories have divergences. Large momenta have to be cutoff and couplings changed as cutoff changes so as to keep physical observables fixed.
- Usually done in weak-coupling expansion. But this is not necessary. For many computations, it is better to do this in some other way: lattice.
- Lattice spacing  $a$ , implies Brillouin zone boundary  $2\pi/a$ , implies UV cutoff.

# Lattice thermodynamics

- In any quantum theory, evolution operator is  $\exp(iHt)$ .
- The thermodynamics of the same quantum theory requires the density matrix  $\exp(-H/T)$ . Formally equivalent to evolving by time  $t=i/T$ . (**Euclidean formalism**)
- Lattice thermodynamics uses this formulation of quantum statistical mechanics for the field theory.

# Numerical treatment

- UV cutoff removed (renormalization performed) by taking **continuum limit**  $a=0$ . Extrapolation needed
- In QCD the gauge coupling ( $g$ ) is changed as  $a$  changes, while keeping quark mass ( $m$ ) fixed. Making  $m$  physical means getting the right **value of  $m_\pi/m_\rho$** .
- Computer memory is limited, so finite volume is used. Thermodynamics requires extrapolation to **“infinite” volume**, ie, large  $V/m_\pi^3$

# What can be studied?

- The **phase diagram**: CEP, 1<sup>st</sup> order line.  
(Due to earlier misunderstanding  $T_c$  refers to crossover temperature)
- The extensive variables and their dependence on intensive variables, ie, the equation of state (**EOS**)
- Second derivatives:  $c_s$ ,  $c_v$ ,  $K$  etc
- Correlation functions, **screening masses**
- **Critical exponents**
- Transport coefficients (?)

# The state of the art

Collab	Year	a	$m_{\pi}/m_{\rho}$	$Lm_{\pi}$	Reported
Wuppertal	2004	1/4 fm	0.185	2–3	CEP
Mumbai	2005	1/4 fm	0.3	3–10	QNS/CEP
Bielefeld	2005	1/4 fm	0.7	16	QNS
Muenster	2002	1/4 fm	0.3	3–4	F
RBC	2006	1/6 fm	?	?	Xov

# Summary of lecture 1

- Phase transitions are identified using order parameters and susceptibilities.
- The constraints on the QCD phase diagram arise from the number of extensive quantities required to describe the thermodynamics of QCD matter.
- Lattice computations for QCD matter require control over  $a$ ,  $m$  and  $V$ .
- Next lecture: QCD phase diagram in detail