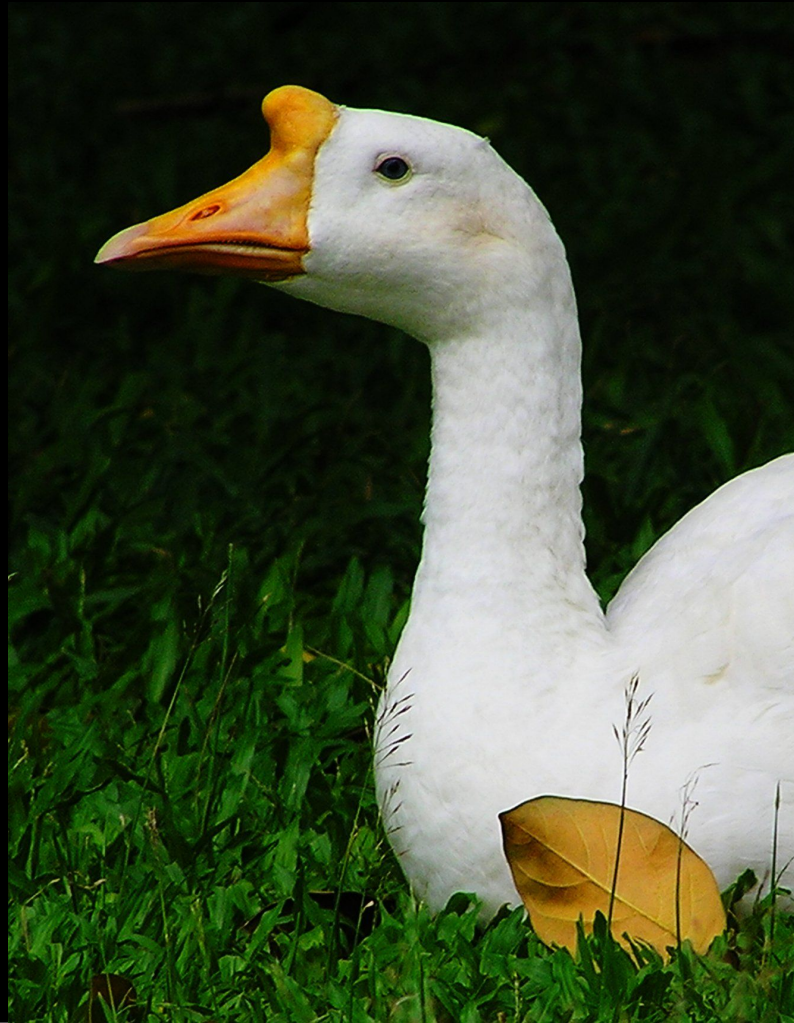


Phases of QCD

- Normal points and phase transitions; types of transitions; order parameters and susceptibilities; phase diagrams; how transitions are found; uncertainties in lattice computations (1)
- The QCD phase diagram: finite temperature, finite chemical potential, the full phase diagram (1)
- What do we know about the properties of QCD matter? (2)

Lecture 3



Properties of QCD Matter

- At high T and low μ ? At high μ ? Is it fluid?
- What particles make up the fluid?
- What is the energy density? Baryon density? Pressure? Specific heat? Compressibility? Susceptibilities?
- What is the speed of sound?
- What are the shear and bulk viscosities?
- What are the relaxation times?

Is QCD matter fluid?

- If inter-particle potentials have a minimum as a function of distance, then at small enough temperature matter may form a solid. Otherwise matter is fluid.
- QCD forces decrease with temperature (asymptotic freedom), hence at large enough temperature it is fluid.
- No evidence for local minimum in any lattice measurement to date.
- No observational evidence for solid-liquid phase transition(s).

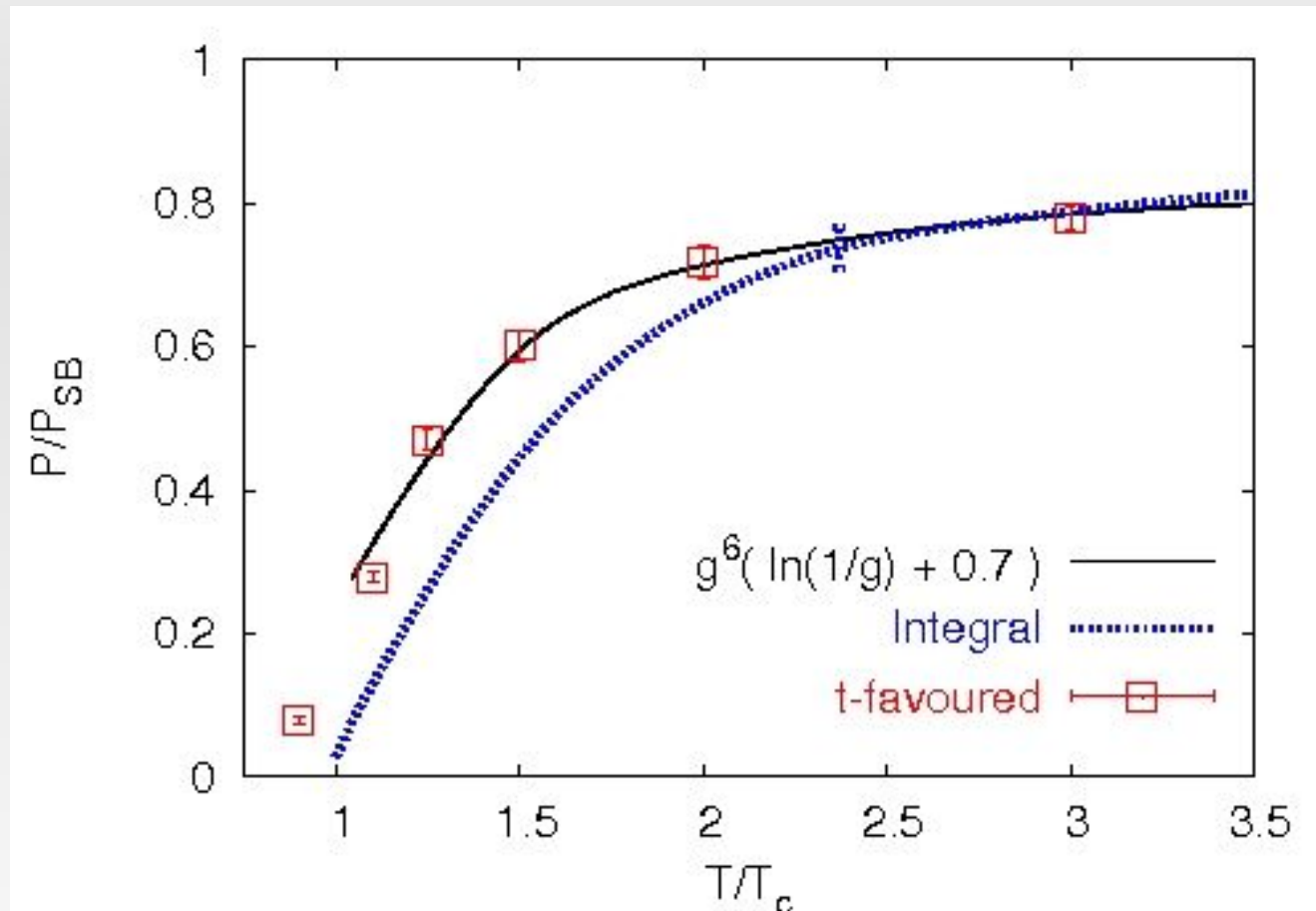
What makes up QCD matter?

- At low enough T and μ , hadrons. Experimental evidence all around us.
- At large T and small m , free quarks. Clear recent evidence from lattice (to be discussed later).
- At large μ , hadron-like objects. Symmetry argument: at $m=0$, in the colour superconducting phases, $s=\langle\psi\psi\rangle$ is non-zero; hence massless pions needed

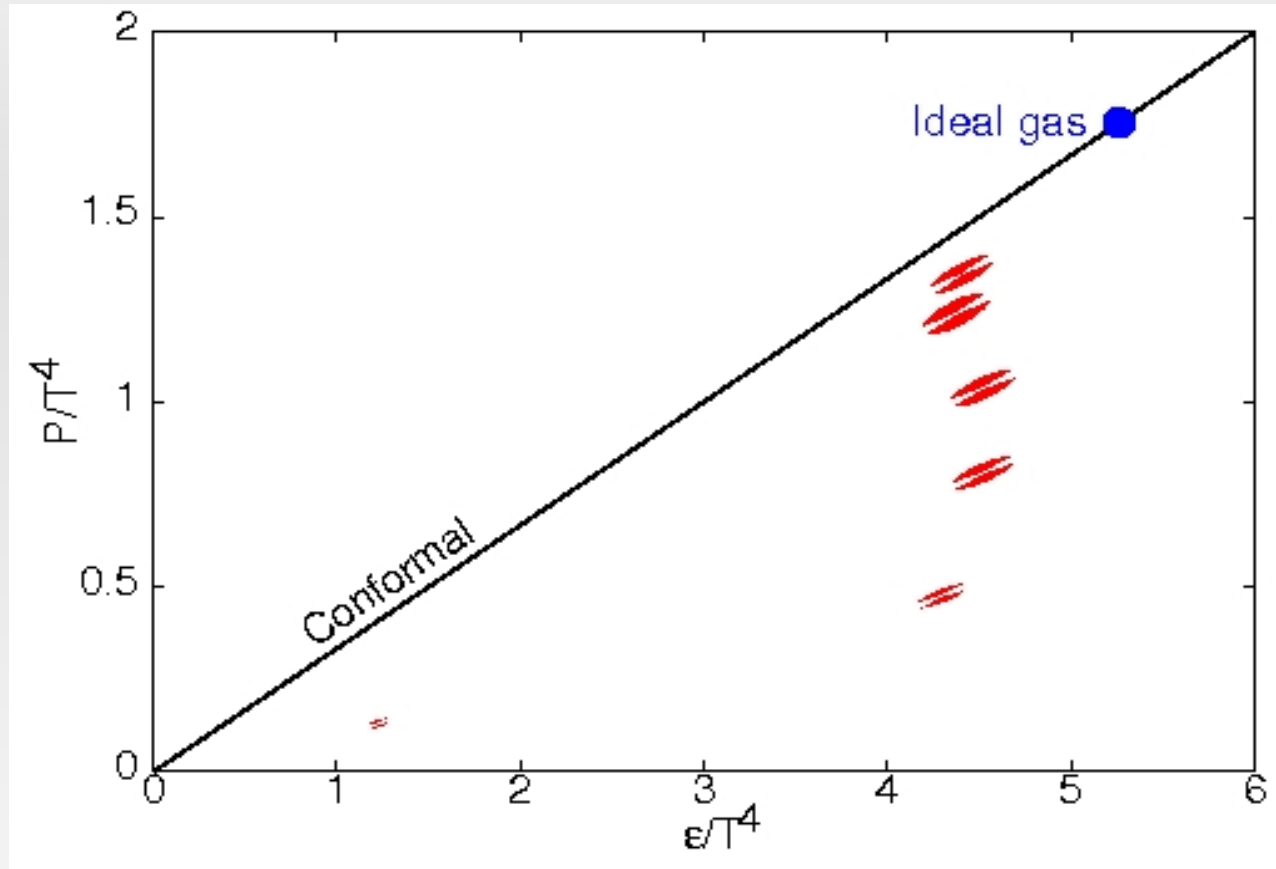
The equation of state

- Gluon matter well studied on lattice, full QCD less so.
- Weak coupling expansions for P badly behaved. Resummed in dimensional reduction and through “ Φ -derivable” methods
- Strong coupling methods use toy models in the AdS/CFT correspondence and the large N_c approximation. All models have high supersymmetry, unlike QCD.

The pressure of gluon matter

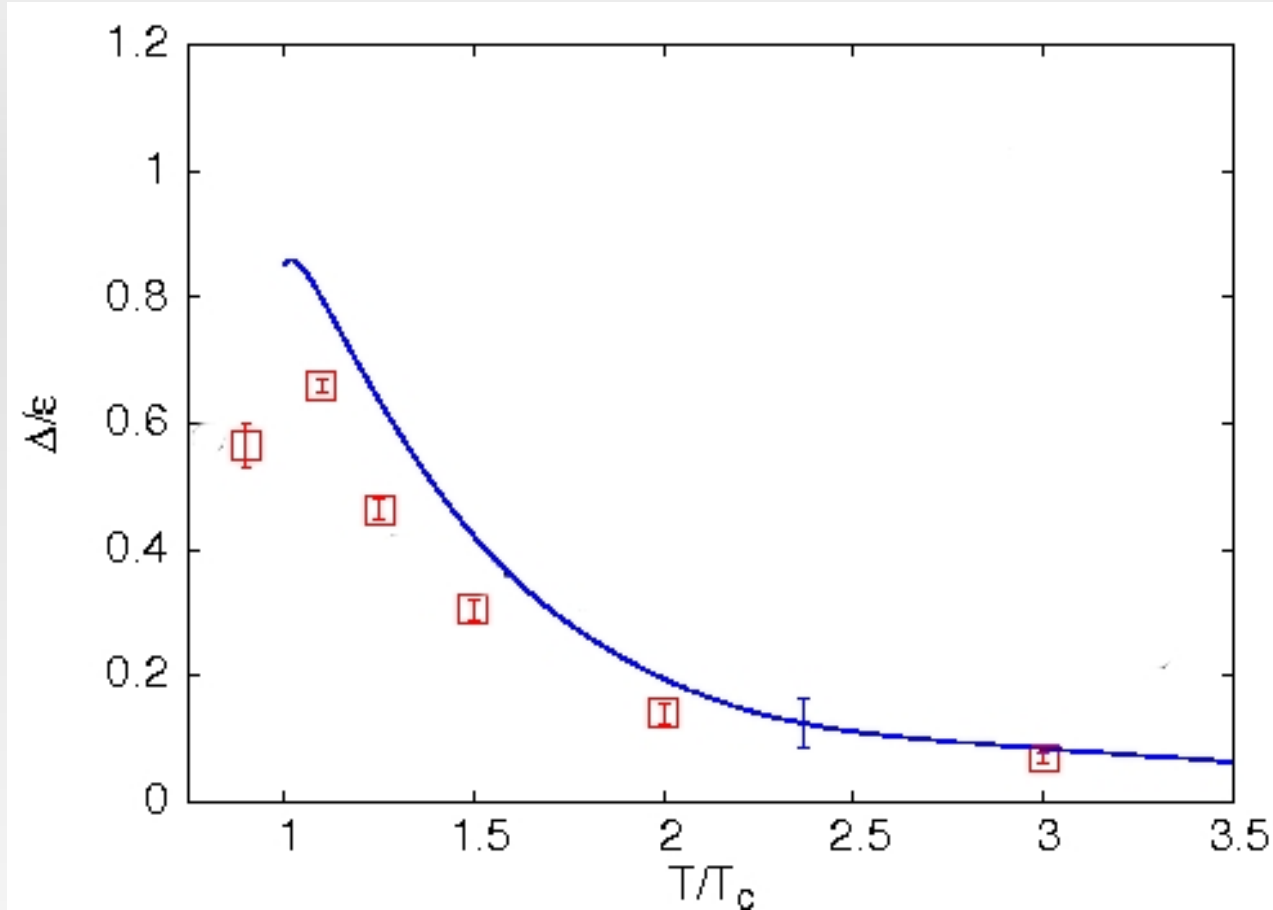


The EOS of gluon matter



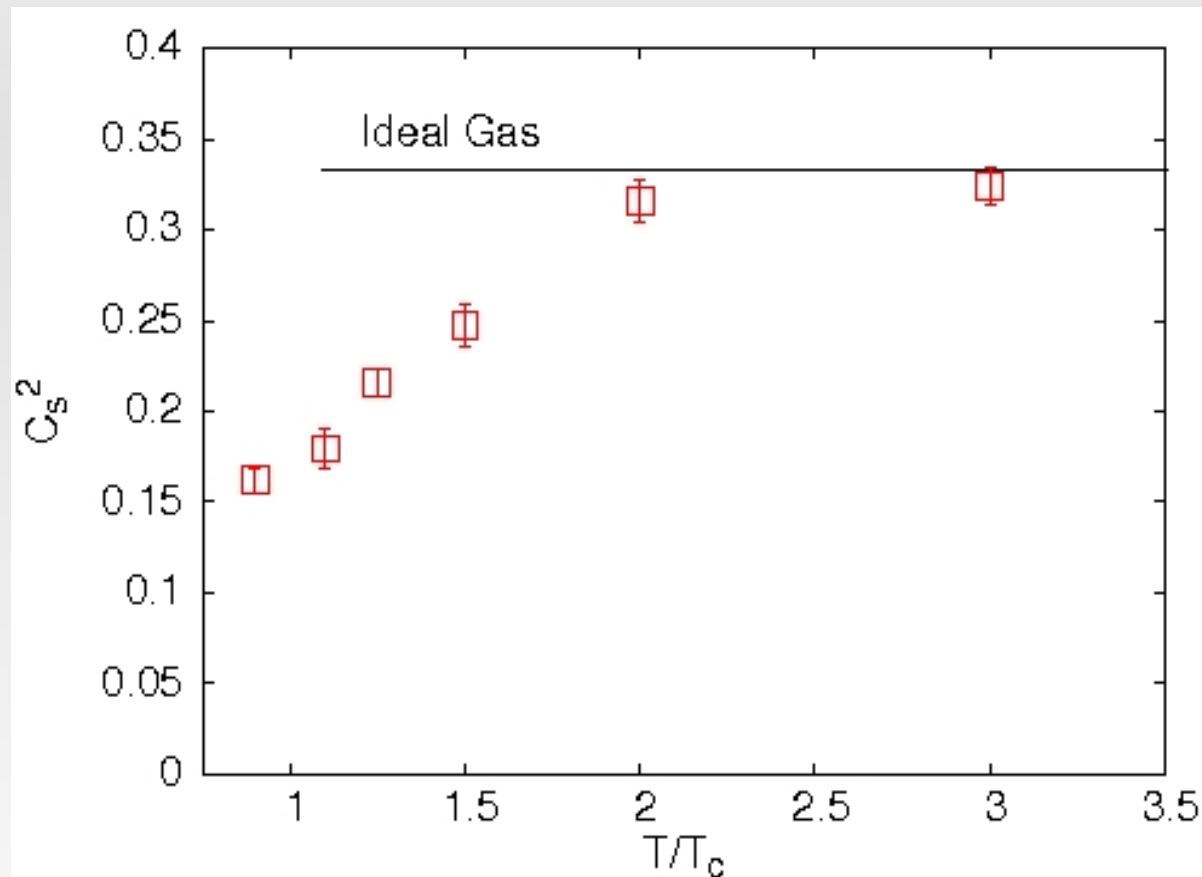
Conformal symmetry and large N_c expansion used in AdS/CFT approach: rules out running of coupling, implies $c_s^2 = 1/3$, $c_V/T^3 = 4\epsilon/T^4$

Conformal symmetry?

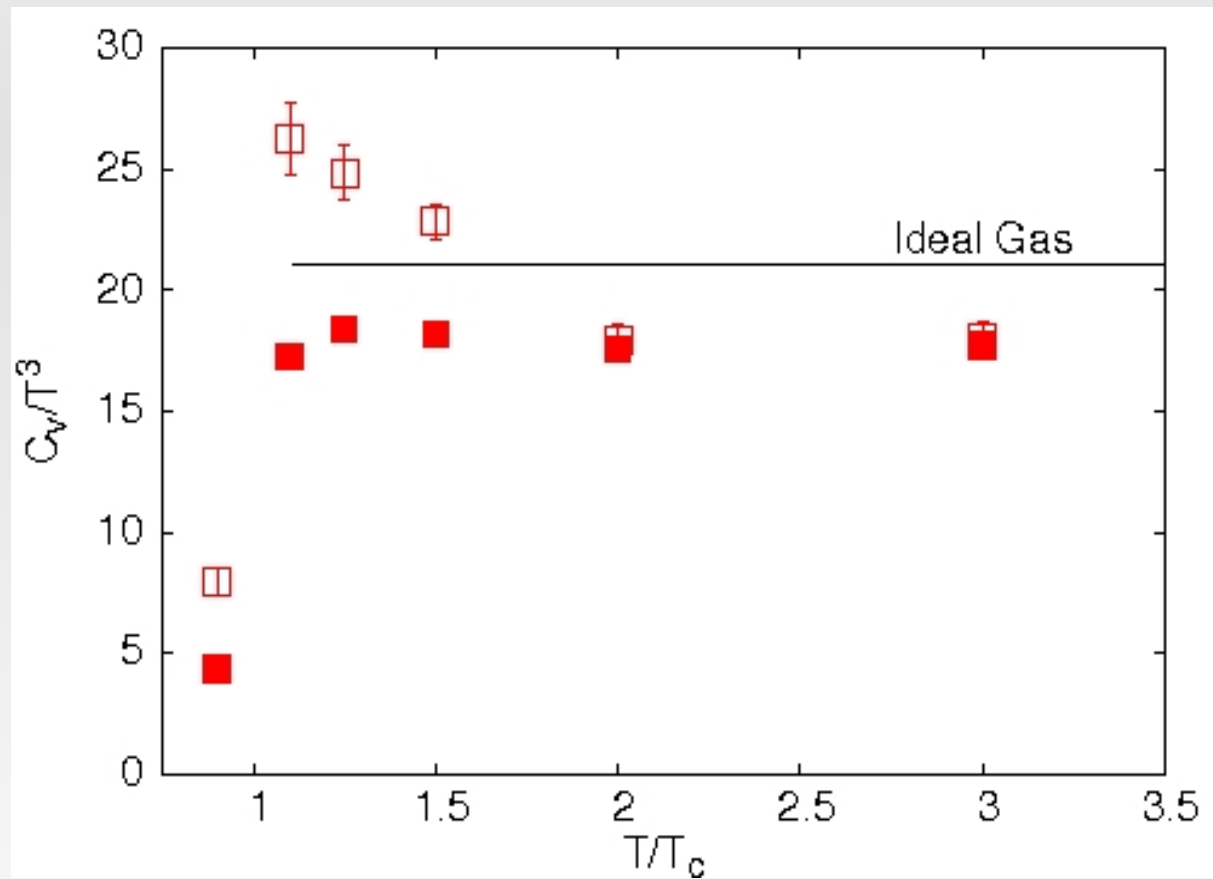


$\Delta = \epsilon - 3P$. Conformal symmetry means $\Delta = 0$. The only long-distance scale in the problem is the energy density, ϵ . Hence Δ/ϵ is a measure of conformal symmetry breaking.

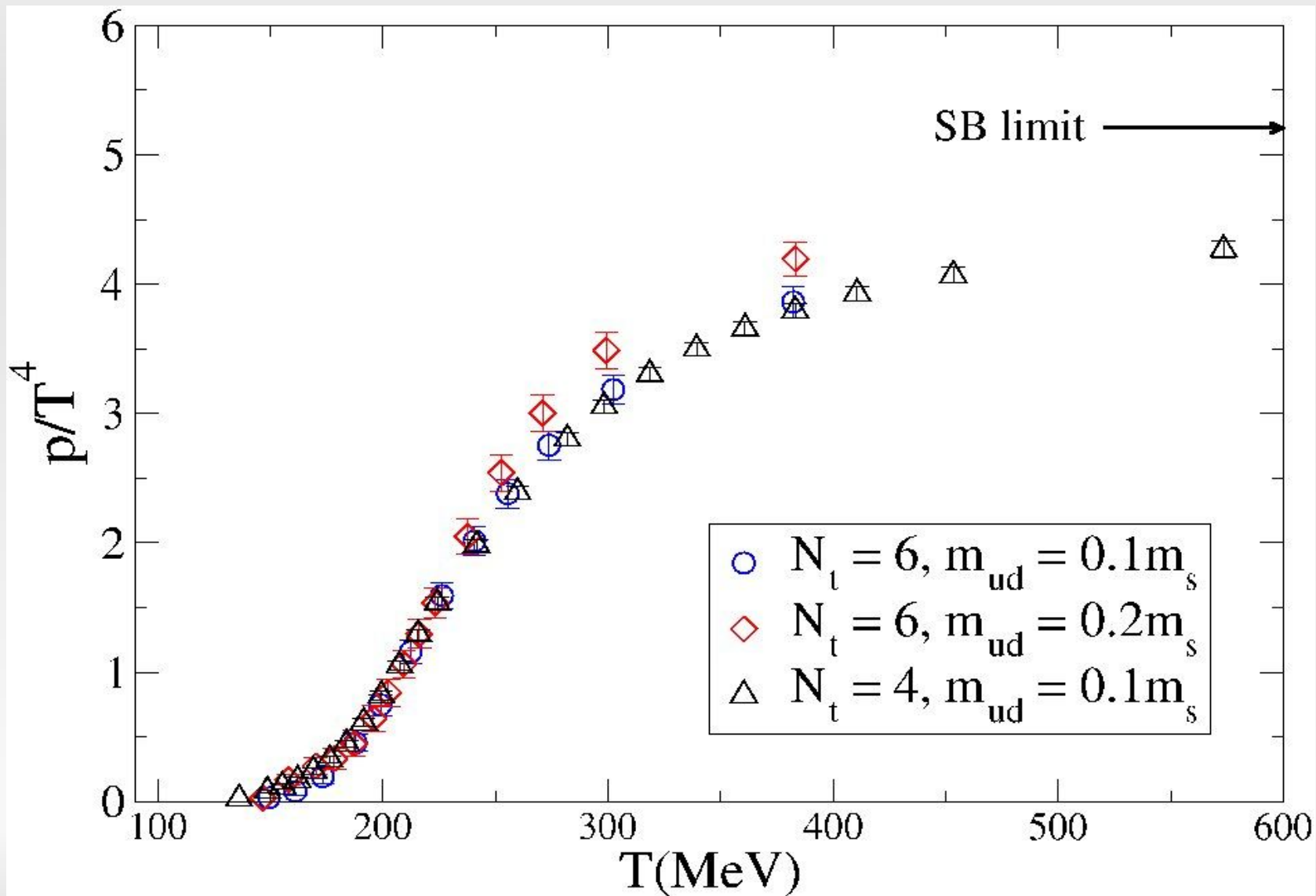
c_s in gluon matter



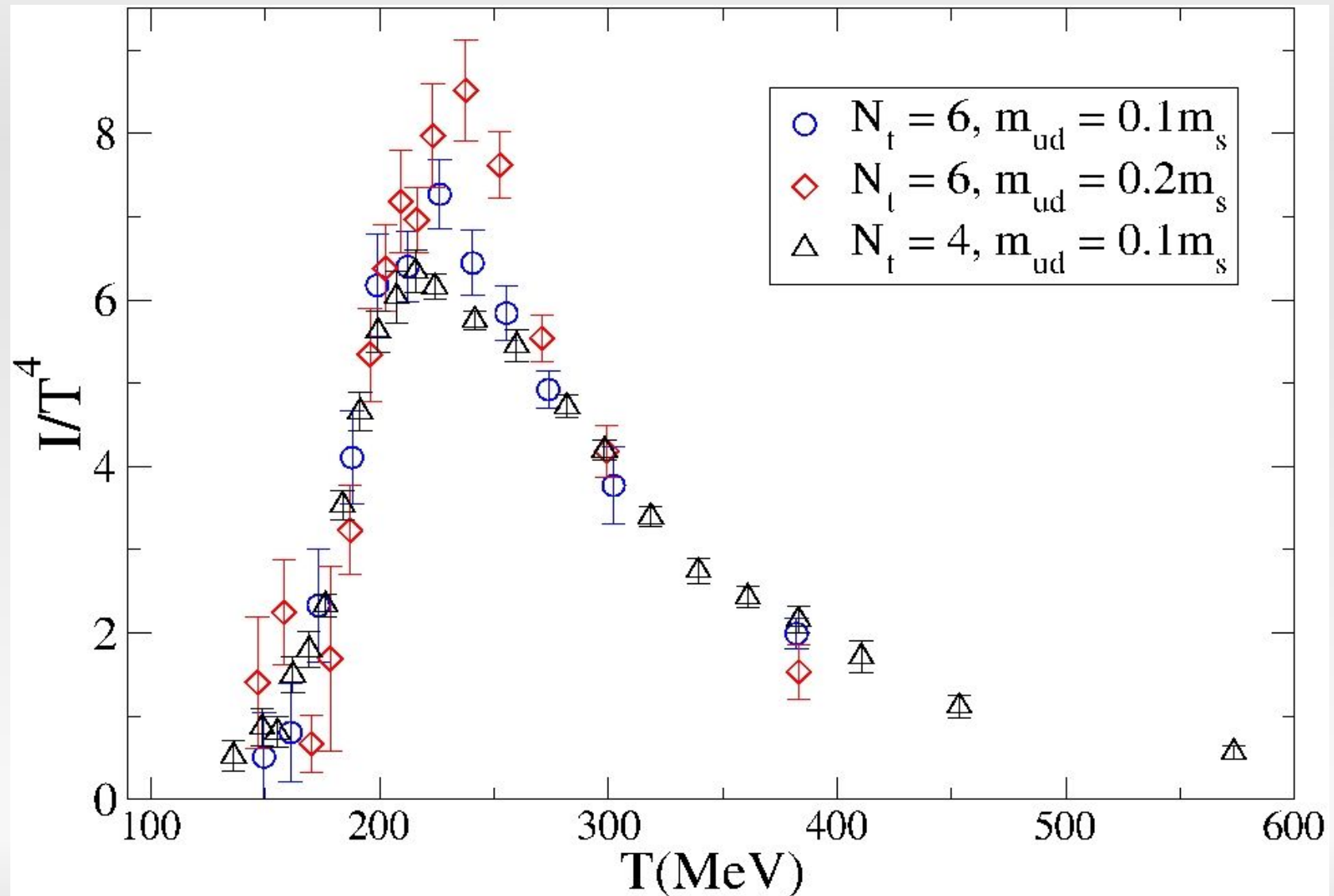
c_v in gluon matter



Pressure in QCD



Conformal measure in QCD



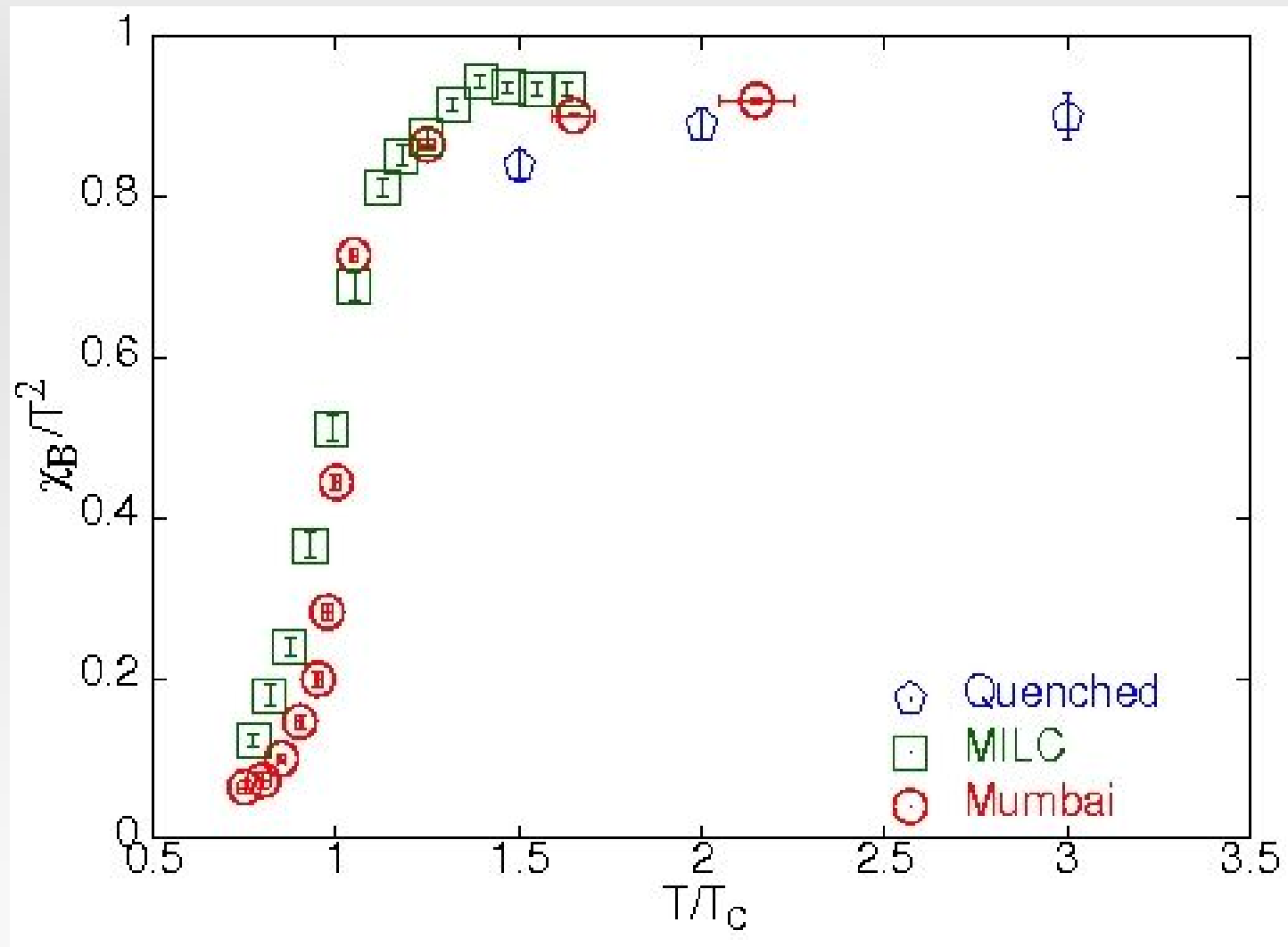
Susceptibilities

$$P(T,\mu) = P(T,0) + \sum (d^n P/d\mu^n) \mu^n/n!$$

In the expansion around $\mu=0$, the series is even. In the expansion around other μ , the $n=1$ coefficient is the **quark number**. The $n=2$ coefficient is the quark number susceptibility (**QNS**). Higher coefficients are called generalized **susceptibilities**.

The Taylor series for the second derivative **diverges at a critical point**. (Find CEP)

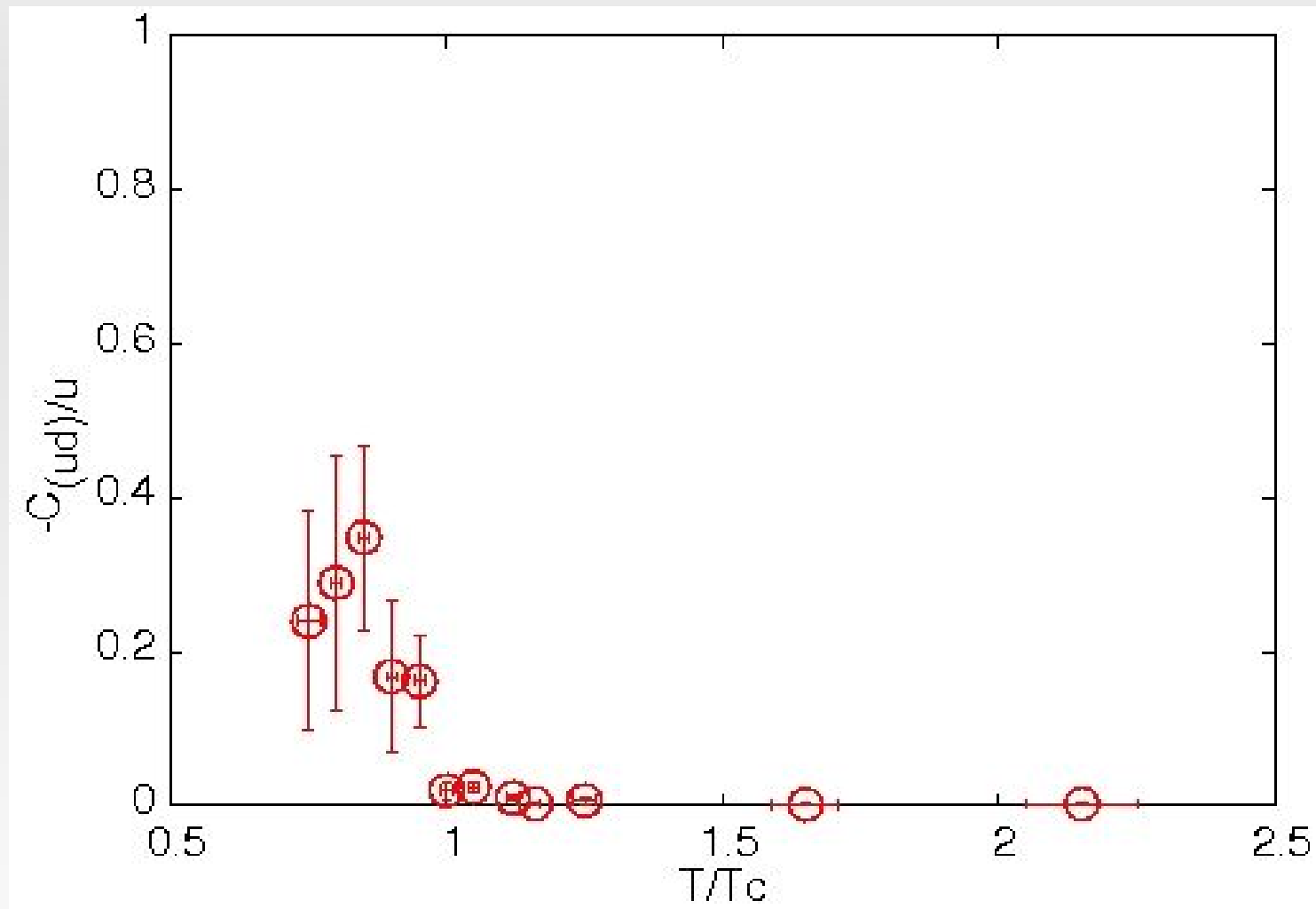
QNS in full QCD



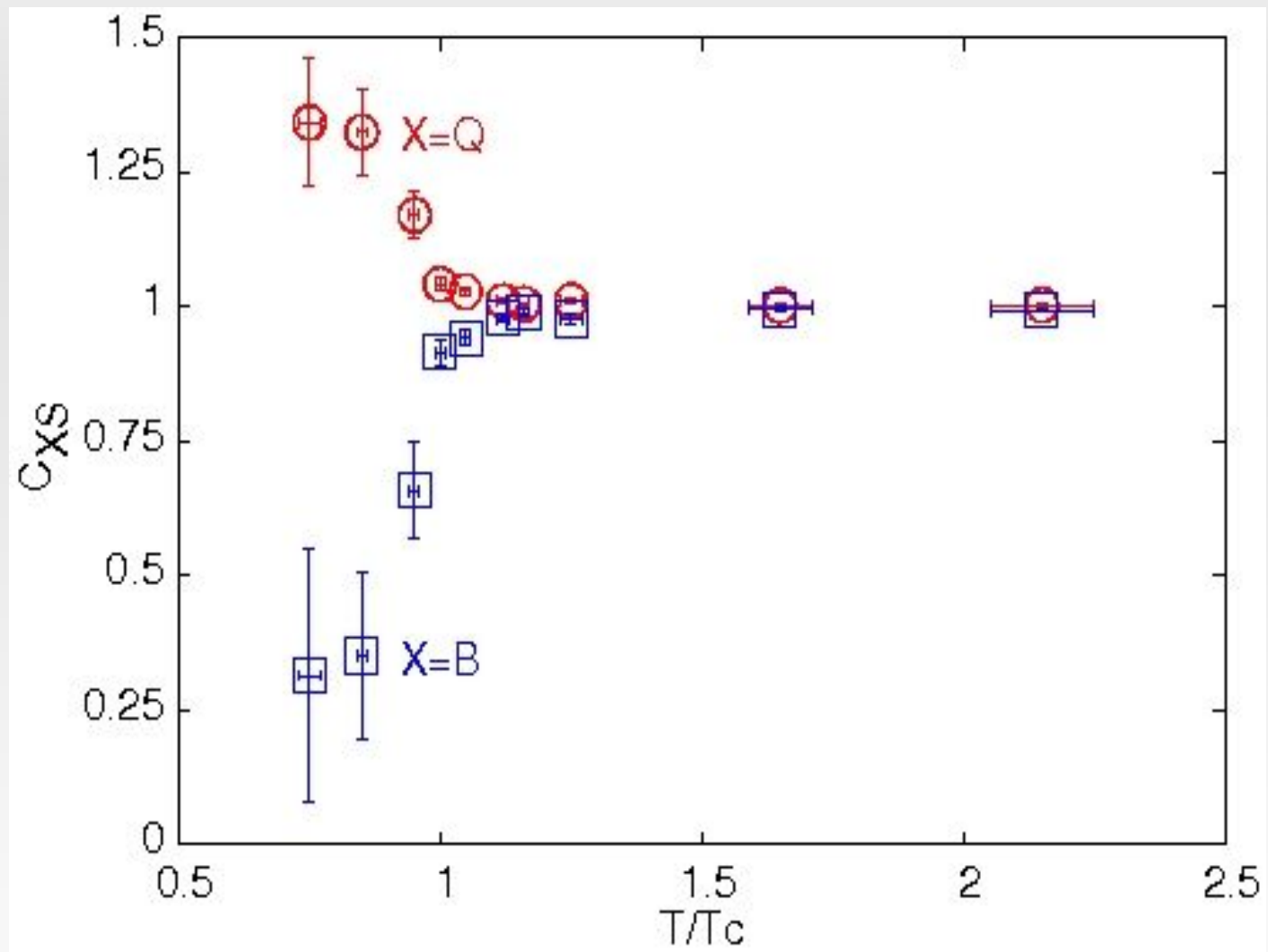
Linkage

- In $N_f=2+1$, three conserved quantities (B, Q and S), so three chemical potentials. 3X3 **matrix of QNS** (off-diagonal, χ_{AB} , are mixed 2nd derivatives).
- QNS are the response of a system to external perturbation.
- **Linkage** = χ_{AB}/χ_{AA} , says if unit quantity of A is excited by external perturbation then how much B is excited. Identifies light excitations in a system.

Linkage between u and d



Linkage between S, B, Q



Viscosity

- Shear viscosity: velocity differences are evened out. Particles do not travel in straight lines; they are scattered. Conformal symmetry plays little direct role.
- Bulk viscosity: compression/rarefaction leads to energy transport. Particles cannot be created or destroyed at will; fluid feels an “intrinsic scale”. Conformal symmetry breaking is essential.

Other transport

- Every conserved quantity has to be transported from one place to another for it to change. Relations between transport due to linkage.
- Viscosity is transport of energy momentum.
- Diffusion is transport of particles (here, baryon number)
- Electrical conductivity is transport of charge (related to isospin in strong interactions)

Computations

- Shear viscosity proportional to mean-free path: weak coupling implies large mean-free path implies breakdown of hydro?
- Weak coupling theory: η at LO is $g^4 \ln g$. First complete computation in QCD available.
- Strong coupling theory: done assuming conformal symmetry and large N_c in a toy model called $N=4$ SYM using AdS/CFT (string theory). Gives the result: $\eta/S \geq 1/4\pi$

Lattice results

- Technical problems in making the connection numerically between Euclidean and Minkowski time. Not fully solved.
- Two measurements of shear viscosity available on the lattice.
- Electrical conductivity is easier to measure. Lattice result available.
- Lattice results compatible with bounds, but do not saturate them.

Summary of Lecture 3

- QCD matter is a fluid. It contains quarks, but the interaction between them is not necessarily weak.
- Conformal symmetry is good to about 10% above $2T_c$, but completely broken near T_c . Expected, since $T_c \sim \Lambda_{\text{QCD}}$ and Λ_{QCD} parametrizes scale anomaly.
- EOS, c_s fairly well understood. Transport coefficients not well determined as yet.