Phases of QCD

- Normal points and phase transitions; types of transitions; order parameters and susceptibilities; phase diagrams; how transitions are found; uncertainties in lattice computations (1)
- The QCD phase diagram: finite temperature, finite chemical potential, the full phase diagram (1)
- What do we know about the properties of QCD matter? (2)

Lecture 4

Plan

- QCD matter
  - What are the dimensions of physical parameters and what are the scales?
  - How large is the relaxation time?
- Identifying the CEP
  - What is the spectrum of fluctuations?
  - What are the hydrodynamic modes?

Dimensions and scales

- T is the only external scale in the thermodynamics of QCD at $\mu=0$, so $\varepsilon=bT^4$, $P=cT^4$ and $S=dT^3$. If there are internal scales in the problem, m, then b(m/T), c(m/T), d(m/T).
- If b, c, d are constant, then c_s^2=constant. Otherwise, function of T through m/T.
- $\eta=fT^3$ so η/S is a function of m/T. Constant if f and d are constant.
- Conformal symmetry gives b=3c, $d=4f\pi$.

Mean free path

- S is like number density of particles; so $T=(S/d)^{1/3}$ is like the inverse of interparticle spacing.
- The mean free path $\lambda=\ell/T$, so ℓ is the mean free path in units of the interparticle separation.
- In gases, the λ is large compared to the interparticle spacing. In liquids, they are comparable. So ℓ distinguishes gases from liquids.Call it the fluidity
- No interactions: mean free path is infinite, S is finite, so ℓ is infinite. Ideal gas

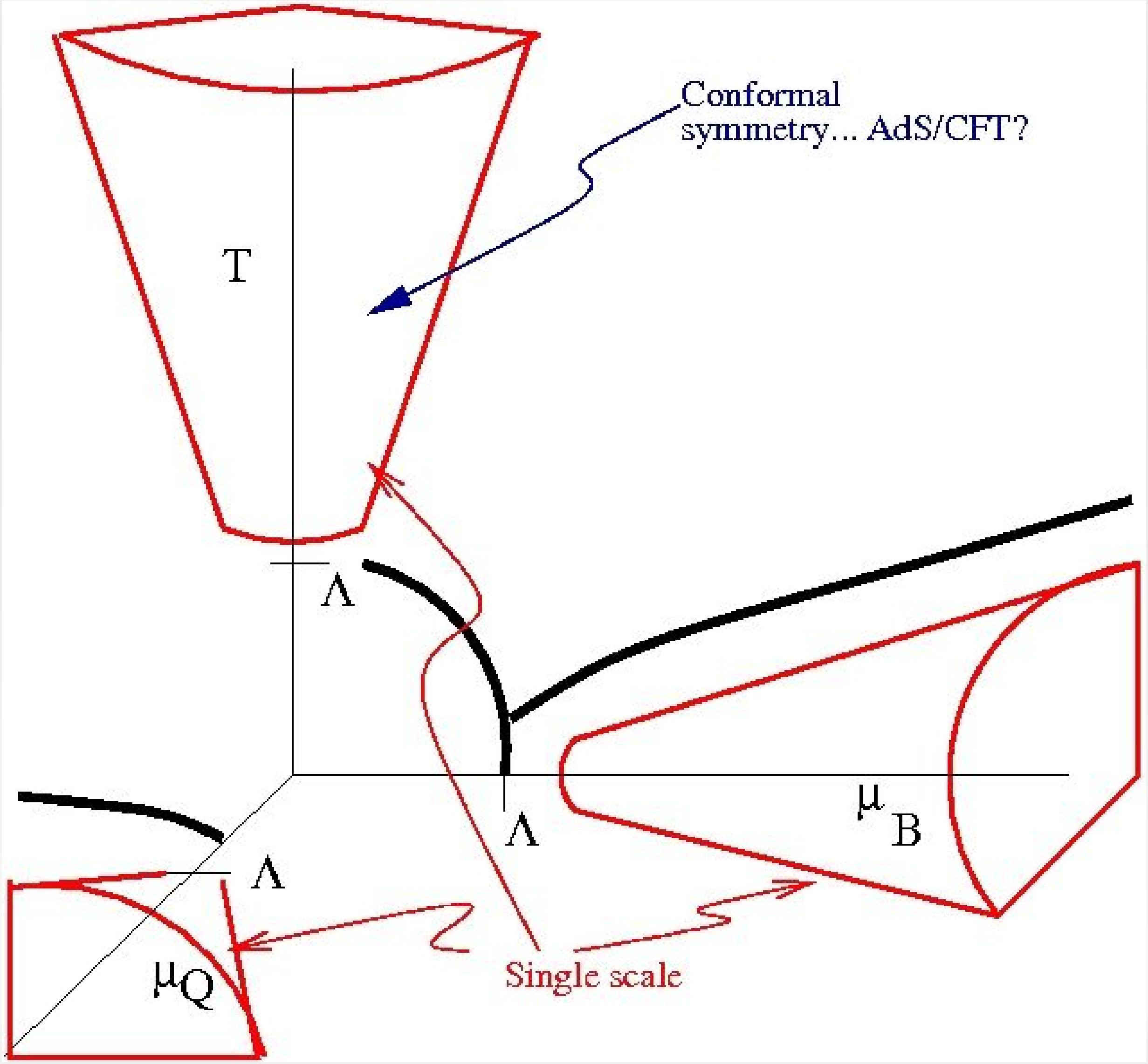
Relaxation time

- In weak coupling, the computation of every transport coefficient is made by first computing a mean-free path. Relaxation time follows in weak-coupling theory.
- In strong-coupling, there is an toy N=4 SYM computation of the relaxation time.
- No estimate of relaxation time directly on the lattice as yet: analytic continuation from Euclidean to Minkowski needed

Dimensions and scales

- When $T \gg \Lambda, m$, then approximate conformal symmetry is not hard to understand.
- Since T_c, T^E, $\mu^E \approx \Lambda$, in this range of parameters, $c_s^2 \neq 1/3$, $\zeta > 0$.
- When $\mu \approx T$, then conformal symmetry is clearly broken, and all previous simplifications must be given up.
- When $\mu \gg T, \Lambda, m$ then single scale again, but large N_c arguments fail. [Caution]

Single scale regions



Lattice methods

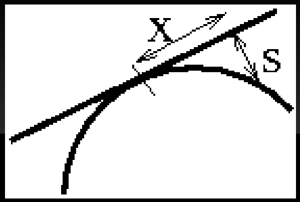
- A numerical lattice computation is a Monte Carlo integration. Works only if the integrand is real and non-negative
- In the T-μ_I plane this property holds and computations can be done.
- Any non-zero μ_B spoils this property. Problem arises in many branches of physics involving fermions, and is called “sign-problem”. QCD, high-T superconductivity, etc.

Solving the sign-problem

- Simplest robust method is to expand any observable in a Taylor series. Measure coefficients at $\mu=0$.
- $O(\mu) = O(0) + \sum O^n(0)\ \mu^n/n!$
- Fails if the series expansion does not converge. But that is exactly a point at which a phase transition happens!
- Lattice computations give results up till the nearest phase transition only.

Searching for the CEP

Fluctuations



- At equilibrium the fluctuations of a small part of system depends on $\Delta S(\Delta X)$. Since the linear term in the Taylor expansion vanishes, probability of fluctuation is $\exp[-\chi(\Delta X)^2]$
- At normal point fluctuations are Gaussian
- At critical point, susceptibilities diverge, and hence fluctuations are strongly non-Gaussian

Series Extrapolation

- Because lattice computations impossible at $\mu \neq 0$, QNS at the CEP is built up by Taylor expansion from $\mu = 0$
- The CEP is identified by divergence of the series
- As a result, the series cannot be continued beyond the CEP
- One way is to resum the series by Pade approximants.

Experimental signature

- All measurements of event-to-event fluctuations of conserved quantities till now are consistent with Gaussian fluctuations.
- This is consistent with the idea that the CEP has not yet been seen.
- Look for deviations from Gaussian to look for CEP.

Hydrodynamic modes

- Hydrodynamics is the dynamics of conserved quantities. Hence at the CEP one must take into account the baryon number density, n_B, as well as the stress-energy tensor $T_{\mu\nu}$, and possibly s.
- Since $T_E \approx \Lambda_{QCD}$, conformal symmetry is badly broken, and shear and bulk viscosity needed. Also, diffusion constant for B, thermal conductivity and electrical conductivity.
- At criticality these diverge.

Kinetic thory

- The correlation length becomes infinite at criticality, and consequently relaxation times diverge.
- As a result, the derivative expansion used to obtain hydrodynamics from kinetic theory may break down.
- Possibility: no hydrodynamics at the CEP, and full kinetic theory may be needed.
- System <u>may</u> re-thermalize after this.

Many unknowns

- Non-Gaussian fluctuations are the essential signal of the CEP
- They are deeply connected with the turbulent (non-hydro) dynamics of fluids at criticality
- How do these appear in observables? Hard to predict, as yet.
- But look for non-Gaussian behaviour.

Summary

Summary (1)

- In the “real world” there is a first order phase transition line in QCD with a critical end point. The CEP may be open to experimental study through energy scans at a collider.
- The only signal of the CEP which is known at present is non-Gaussian fluctuations of conserved quantities.
- The influence of flow on these fluctuations is not known at all.

Summary (2)

- At finite T in the “real world” the high temperature phase is a fluid of quarks and (possibly) gluons.
- The EOS of this matter is reasonably well understood. There are no major technical barriers to a better computation of the EOS.
- The domain of validity of weak-coupling theory is reasonably well constrained.

Summary (3)

- Toy models treated in the AdS/CFT framework are applicable to a small domain of interest in the phase diagram of QCD. This could extend a little the region covered by weak-coupling theory.
- Other regions of the phase diagram need a more complete method for non-perturbative computations.
- At present lattice computations can only be extended to the vicinity of the CEP.

Summary (4)

- Apart from the EOS, c_s, c_V, QNS and screening masses are known in QCD or its pure-glue cousin.
- Unresolved technical problems in the continuation from Euclidean lattice simulations to real-time, currently present a barrier to accurate quantitative estimates of transport coefficients and relaxation times.
- Weak coupling and AdS/CFT results available.