The phase diagram of QCD

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Symmetries and conserved quantities

Two flavour phase diagram

- Flavour symmetry is exact
- Flavour symmetry explicitly broken

Three flavours

- Columbia plot
- Crossover temperature

Physics at finite μ_B

Summary

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Phase transitions

QFTs contain many symmetries:

1. Exact gauge symmetries
2. Higgsed gauge symmetries
3. Global flavour symmetries
4. Global chiral symmetries

Any of these can be broken or restored at finite temperature. For every global symmetry there can be chemical potentials; large and interesting phase diagrams.

Since colliders may create thermalized strongly interacting matter, phase diagram of QCD is being mapped out. Tools of the trade: effective field theories, lattice gauge theory.

How many flavours

- When quarks are massless, chiral symmetry, *i.e.*, independent flavour transformations of the two chiralities. Chiral symmetry broken by the Goldstone mechanism: massless pions and a quark condensate. Symmetry restored at a temperature T_c.
- Remaining flavour symmetries not exact: difference in quark masses breaks flavour symmetry. When $m_u \neq m_d$, isospin symmetry broken. When flavour symmetry completely broken, then independent chemical potential, μ, for each flavour.
- Since $m_{\pi^0} \simeq m_{\pi^\pm}$, flavour SU(2) is a good approximate symmetry of the hadron world. Flavour SU(3) is not useful without symmetry breaking terms (Gell-Mann and Nishijima).
- If some $m \gg \Lambda_{QCD}$ then that quark is not approximately chiral. In QCD two flavours are light ($m_{u,d} \ll \Lambda_{QCD}$) and one is medium heavy ($m_s \simeq \Lambda_{QCD}$). Recall that $m_\pi = 0.2 m_\rho$ but $m_K = 0.7 m_\rho$.

Extensive thermodynamical quantities

Symmetries determine the number of conserved quantities and therefore the number of extensive thermodynamic variables.

1. Gluon gas: only energy is conserved, so internal energy, U, and entropy, S, are the extensive quantities. Only T is an intensive quantity. There is either no phase transition (in U(1) of EM) or a single second order transition (in unphysical SU(2) colour) or a first order transition (in SU(3) colour).
2. Two flavours of quarks: energy, baryon number and isospin are conserved, so S, U, net baryon number, B, and net charge, Q, are extensive. T, and chemical potentials, μ_B and μ_Q are intensive quantities. Complicated phase diagrams are possible.
3. Three flavours of quarks: energy, baryon number, isospin and strangeness are conserved. So S, U, B, Q, and net strangeness, $\mathcal{S}$, are extensive thermodynamic variables. T, μ_B, μ_Q, and μ_S are intensive variables. Phase diagram more complicated.

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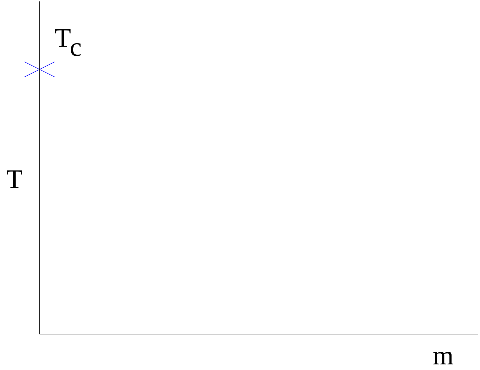
The two flavour world

- What distinguishes the phases? Exact answer only for massless quarks. In the vacuum chiral symmetry is broken; $\langle\overline{\psi}\psi\rangle$ chiral condensate is non-vanishing. Pions are the massless fluctuations around the vacuum. At high temperature $\langle\overline{\psi}\psi\rangle = 0$.
- When correlation lengths finite then susceptibility always finite:

$$\chi = \int d^3x C(x), \quad C(x) \simeq \exp(-mx), \quad \xi = 1/m_\pi.$$

  At critical points correlation lengths diverge; integral diverges, so susceptibility diverges.
- When quarks are massive, then at transition scalar mass degenerate with $m_\pi \neq 0$. No vanishing masses, so all susceptibilities finite. May still have a maximum as T changes: cross over for massive quarks.

The two flavour phase diagram



Pisarski, Wilczek, 1980s; Rajagopal, Stephanov, Shuryak hep-ph/9806219 and hep-ph/9903292

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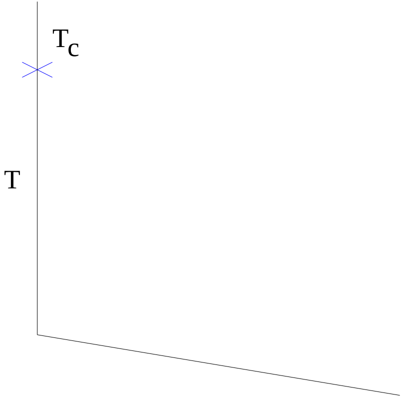

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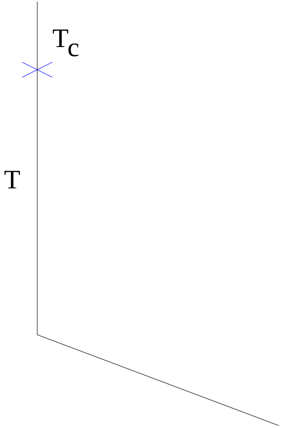

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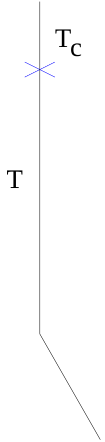

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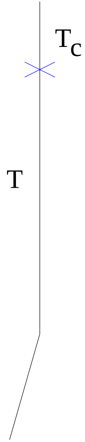

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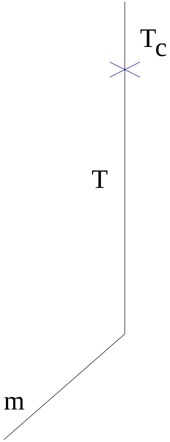

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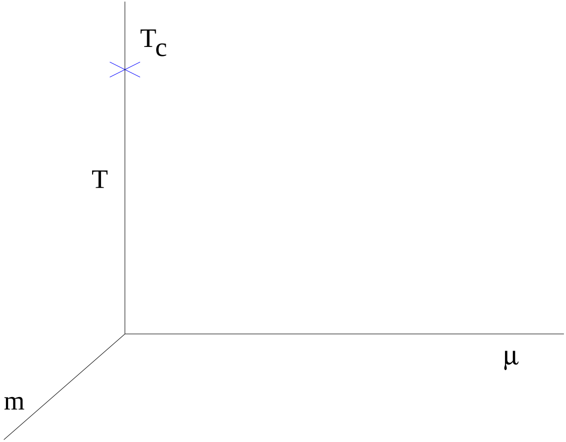

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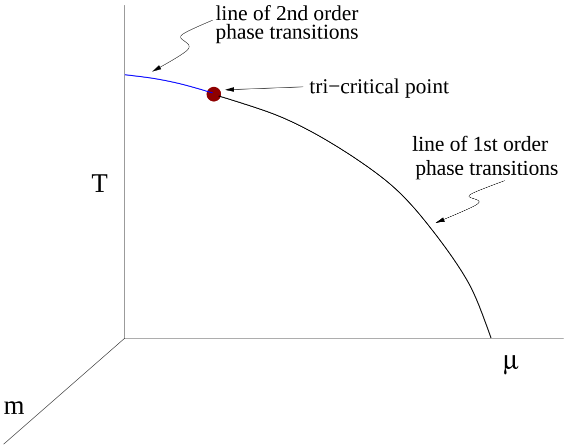

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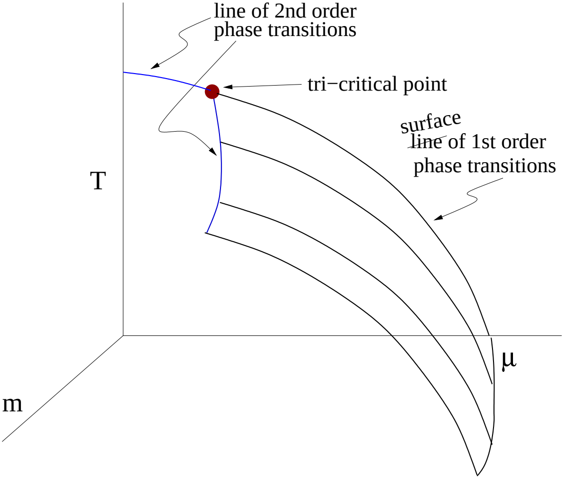

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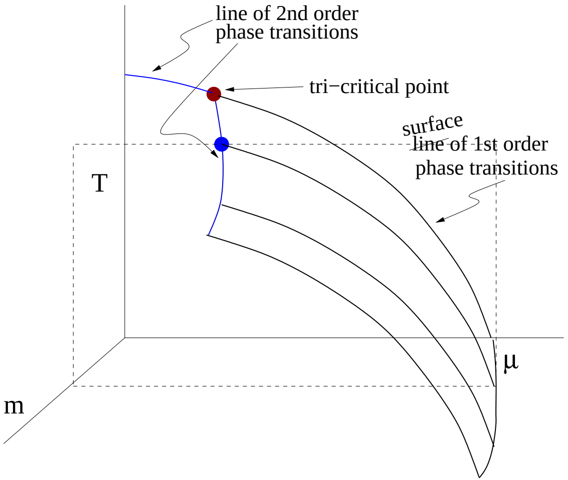

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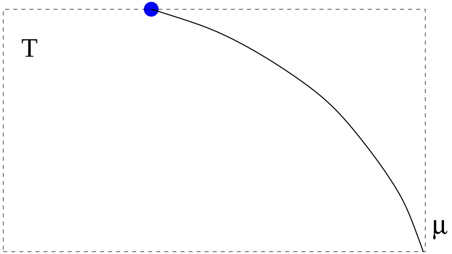

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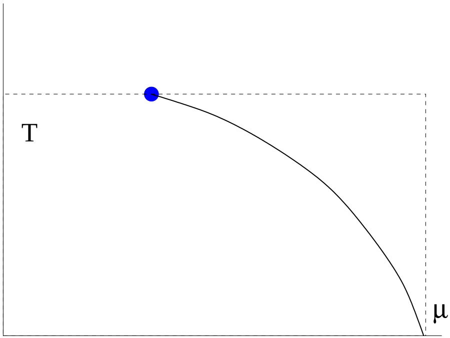

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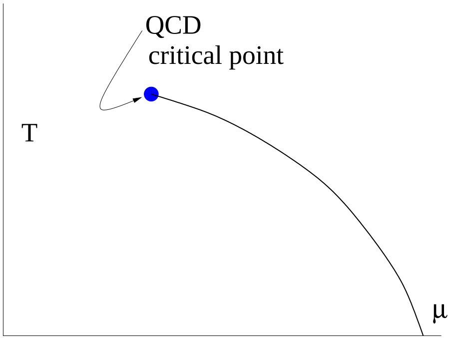

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Lattice results for broken isospin

Only one study; $m_{\pi^0}/m_{\pi^\pm}$ is changed from 1 to 0.78 (physical value bracketed).

$$\frac{T_c}{\Lambda_{\overline{MS}}} = \begin{cases} 0.49 \pm 0.02 & (m^2_{\pi^0}/m^2_{\pi^\pm} = 1) \\ 0.49 \pm 0.02 & (m^2_{\pi^0}/m^2_{\pi^\pm} = 0.78) \end{cases}$$

Both results extrapolated to the physical value of m_π/m_ρ.
Gavai, SG, hep-lat/0208019 (2002)

Caveats:

- lattice spacings are coarse by today's standards
- quark masses can be now be more realistic
- finite volume effects yet to be quantified.

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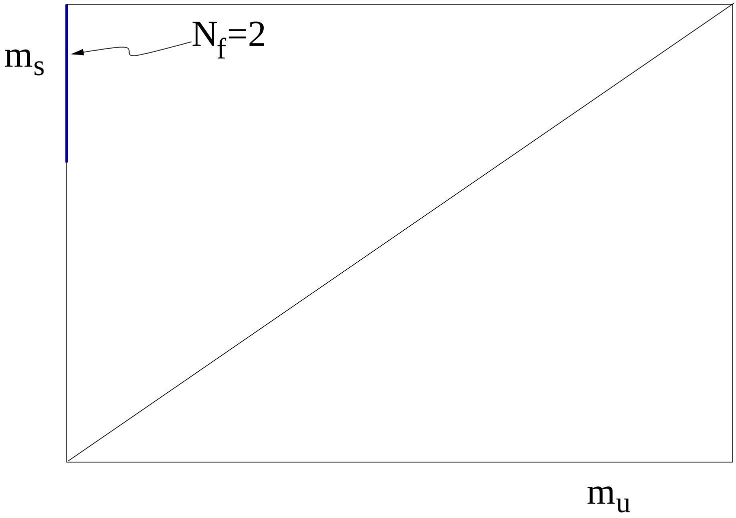

Columbia plot: Brown et al, PRL 65, 2491 (1990)

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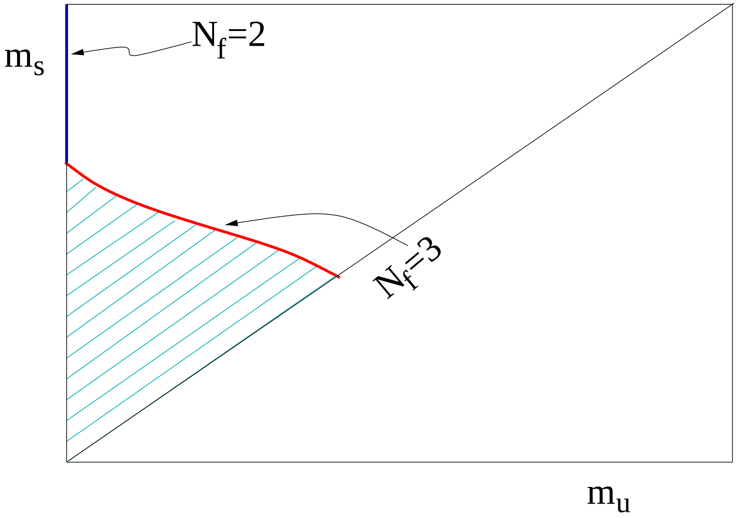

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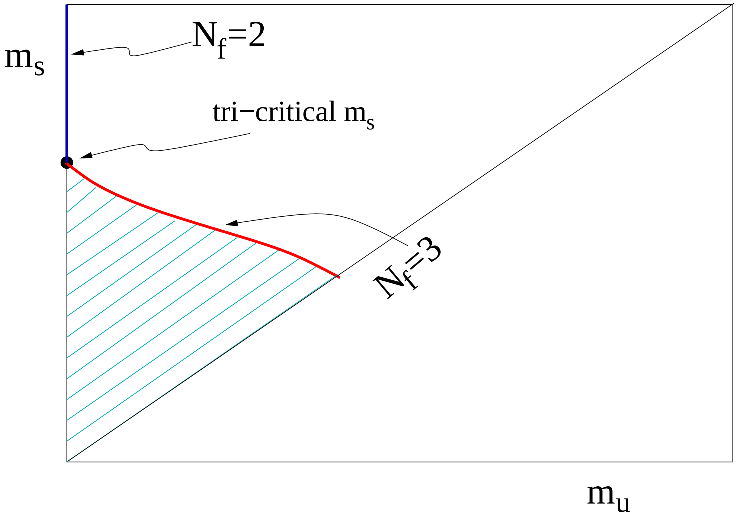

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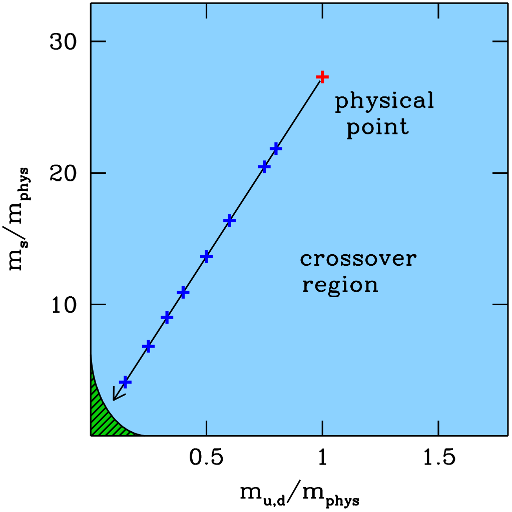
Lattice results for m_s^{3C}



In $N_f = 2 + 1$:

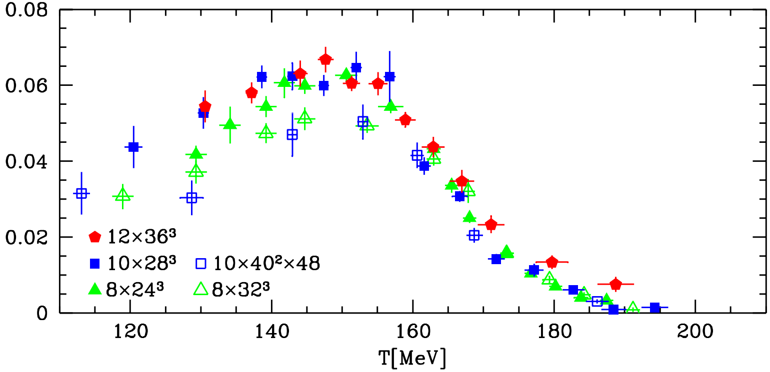
$$m_\pi^{crit} \begin{cases} = 0.07 m_\pi & (N_t = 4) \\ < 0.12 m_\pi & (N_t = 6) \end{cases}$$

Endrodi etal, 0710.0988 (2007)

Similarly for $N_f = 3$.

Karsch etal, hep-lat/0309121 (2004)

Crossover at realistic m_π



Broad crossover: figure shows chiral susceptibility; Even with one measure, T_c uncertain by 20 MeV.

Aoki et al, 0903.4155

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Crossover temperature in QCD

Chiral susceptibility	χ_m/T^4	$146 \pm 2 \pm 3$ MeV
	χ_m/T^2	$152 \pm 3 \pm 3$ MeV
	χ_m	$157 \pm 3 \pm 3$ MeV
Chiral condensate	Δ_{ls}	$155 \pm 2 \pm 3$ MeV
Polyakov loop		$170 \pm 4 \pm 3$ MeV
Strangeness suscepibility	χ_s	$169 \pm 3 \pm 3$ MeV

Aoki et al, 0903.4155

When to use an index

Because of the complexity of the underlying concept, an index might be particularly unhelpful or meaningless.
—Arvind Subramaniam, Chief Economic Advisor to the PM.

If a definition is arbitrary, then arbitrarily choose a definition, but insist that physics be independent of it.

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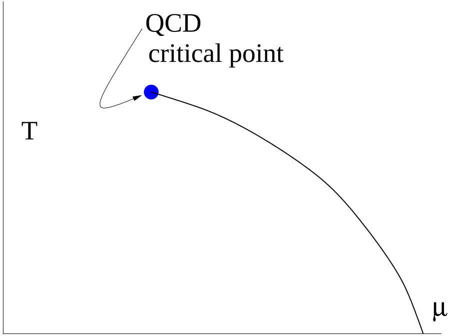

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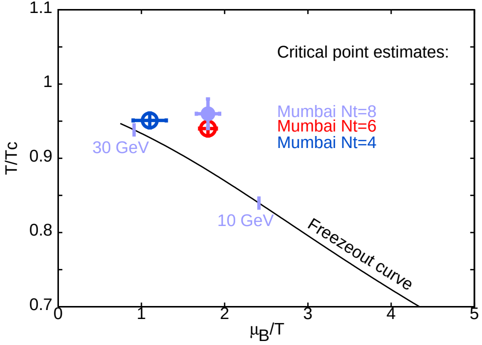
The critical point of QCD



Budapest: 2001; ILGTI: 2005, 2007, 2011, 2014

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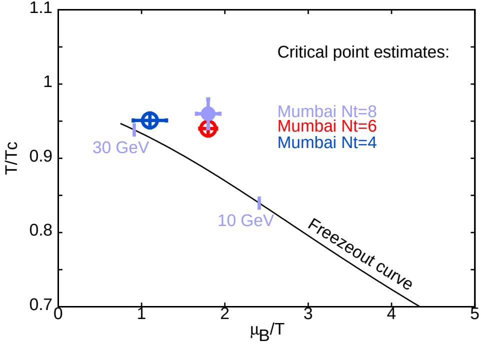
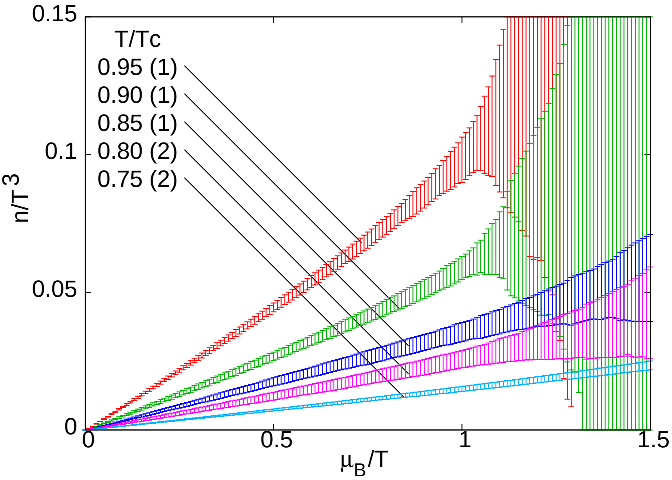


Budapest: 2001; ILGTI: 2005, 2007, 2011, 2014

$$\frac{\mu^E}{T^E} \simeq \begin{cases} 1.5 \pm 0.3 & N_f = 2, \textbf{ ILGTI, 1405.2206 (2014)} \\ 1.5 \pm 0.4 & N_f = 2+1, \textbf{ BNL-Bielefeld, unpublished, 2010} \end{cases}$$

comparable a, m_π; normalized to same estimator.

The Equation of State of QCD



ILGTI: 2015

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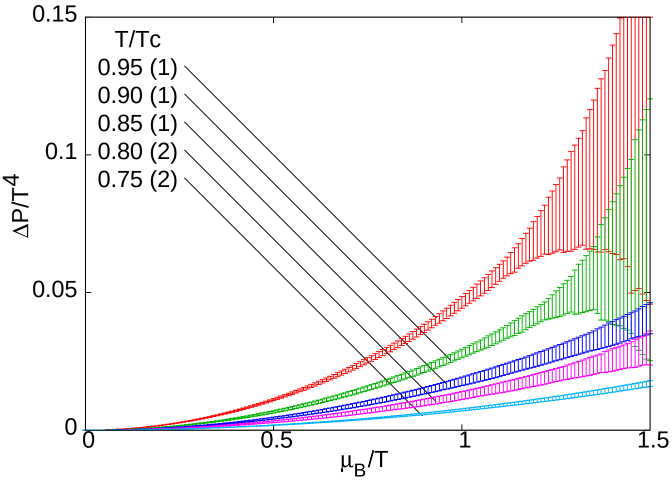

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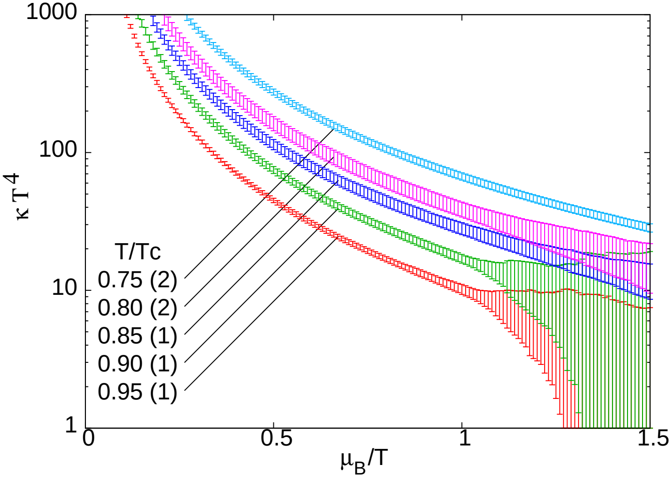

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1. Chiral QCD has a chirality restoring phase transition at finite T. Such a transition persists even in the presence of fairly light, but non-zero mass, strange quarks. Much heavier quarks have no effect on this phase transition.
2. At finite light quark masses, there is no phase transition at $\mu = 0$. The chiral transition becomes a cross over. The cross over temperature, T_c, depends on the definition and can vary between 170 MeV and 145 MeV. There is little effect of isospin symmetry breaking on the cross over. LHC operates in the regime where μ is extremely small, and therefore explores baryonless thermal QCD.
3. Even when the light quark masses are finite, there is a genuine phase transition at finite chemical potential. This occurs at

$$T_E \simeq 160 \text{ MeV}, \quad \text{and} \quad \mu_E \simeq 270 \text{ MeV}.$$

Explored in the RHIC BES, and at GSI, JINR.