Off the wall—
Taylor expansions in chemical potential

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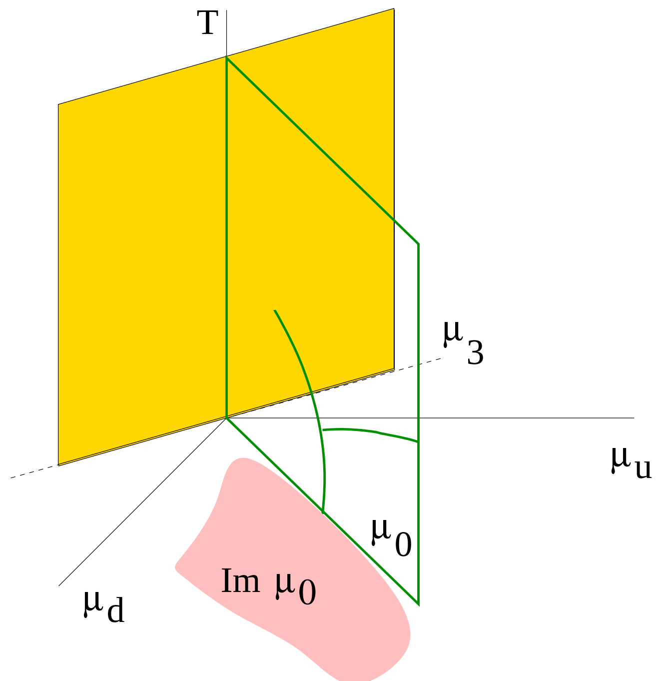
1. The wall, and moving out
2. Taylor expansion for pressure.
3. Taylor expansion for condensates
4. Taylor expansion for correlators
5. Main results

The wall

Lattice simulations are possible only when the measure is positive definite. For two quark flavours this happens on the plane of $\mu_0 = 0$, *i.e.*, for $\mu_u = -\mu_d = \mu_3$.



On wall: J. B. Kogut and D. K. Sinclair, *Phys. Rev.*, D66 (2002) 034505, *ibid.* D66 (2002) 014508, S. Gupta, hep-lat/0202005

Moving out

- Examine low-order Taylor expansions of pressure and other quantities.
  S. Gottlieb *et al.*, *Phys. Rev. Lett.*, 59 (1987) 2247
  R. V. Gavai *et al.*, *Phys. Rev.*, D 65 (2002) 054506, *ibid.* D 67 (2003) 034501
  O. Miyamura *et al.*, *Phys. Rev.*, D 66 (2002) 077502.

- Reweighting done for $N_t = 4$ and $N_f = 4$, 2 and 2+1.
  Z. Fodor and S. D. Katz, *J. H. E. P.*, 03 (2002) 014
  Z. Fodor, S. D. Katz and K. K. Szabo, hep-lat/0208078

- Partial Taylor expansion of Det M with respect to chemical potential.
  C. R. Allton *et al.*, *Phys. Rev.*, D 66 (2002) 074507, hep-lat/0305007

- Simulate imaginary chemical potential and do analytic continuation.
  P. De Forcrand and O. Philipsen, *Nucl. Phys.*, B642 (2002) 290
  M. D'Elia and M.-P. Lombardo, hep-lat/0209146

Taylor expansion for pressure

$$
\begin{aligned}
P(T,\mu) = -F/V &= P(T,0) + \chi_3(T)\mu^2 + \frac{1}{12}\chi_{uuuu}(T)\mu^4 + \mathcal{O}\left(\mu^6\right) \\
&= P(T,0) + \chi_3(T)\mu^2 \left[1 + \left(\frac{\mu}{\mu_*}\right)^2 + \mathcal{O}\left(\frac{\mu^4}{\mu_*^4}\right)\right].
\end{aligned}
$$

where $\mu_* = \sqrt{12\chi_3/\chi_{uuuu}}$ and other 2nd and 4th order terms have been neglected. Well-behaved for $\mu \ll \mu_*$ if all the higher order terms are small enough. All results can be obtained in the continuum. Term by term improvement of the series is possible. Series should fail to converge near a critical point. Series extrapolation methods should then be used to locate the critical point nearest to $\mu = 0$.

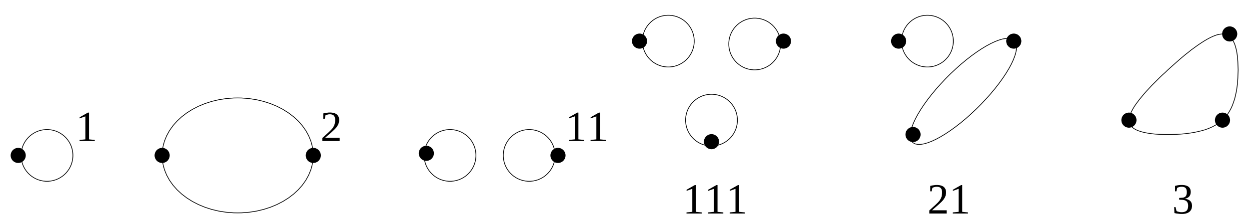
R. V. Gavai and S. Gupta, hep-lat/0303013

Derivatives

Derivatives of $\log Z$ can be expressed in terms of derivatives of Z. The latter can be constructed by the chain rule.

$$Z_f = \frac{\partial Z}{\partial \mu_f} = \int DU \mathrm{e}^{-\mathcal{S}} \mathrm{Tr}\ M_f^{-1} M_f'.$$

Odd derivatives vanish for $\mu_f = 0$ by CP symmetry.



S. Gupta, *Acta Phys. Pol.*, B 33 (2002) 4259

A lattice ambiguity and solution

In the continuum one adds chemical potential by adding a term $\gamma_0\mu$ to the Dirac operator. On the lattice one can add (almost) any term that reduces to this in the continuum. A popular (but not unique) choice is $\exp(\mu a)$ on each temporal finite difference in the Dirac operator. This gives rise to an ambiguity. For example—

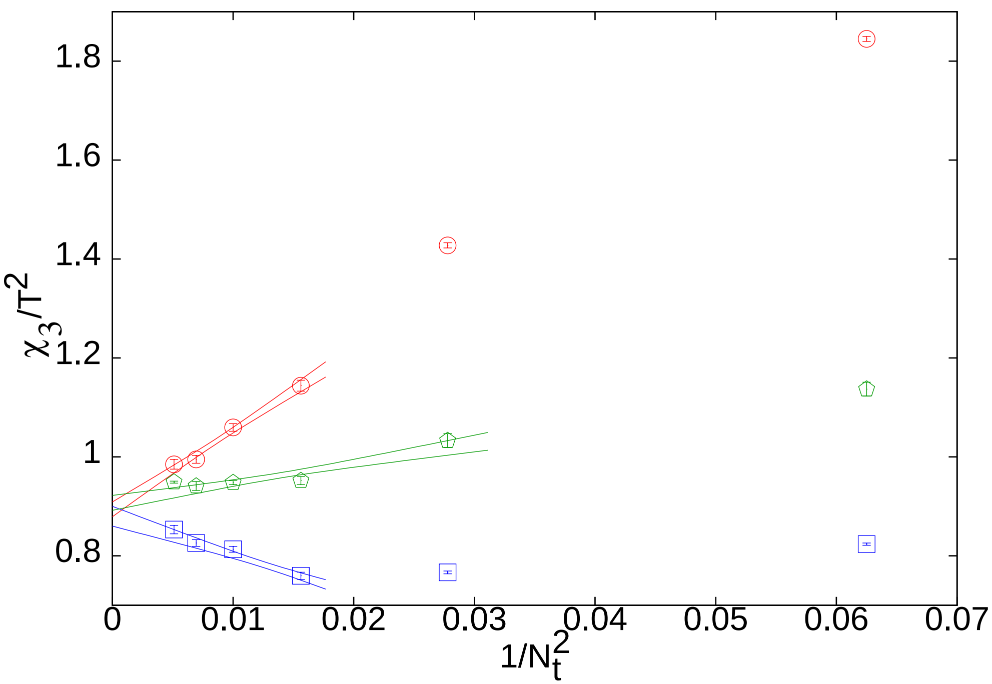
$$\mu_E = 725 \pm 35\,(\mathrm{stat}) \pm 35\,(\mathrm{this\ source}) \pm 20\%(?\ \mathrm{other\ sources})$$

Our solution is to make a Taylor expansion and take the continumm limit term by term in this expansion. Series breaks down at a phase boundary. As a result, there is a way to extract the location of the phase boundary and the critical end point.

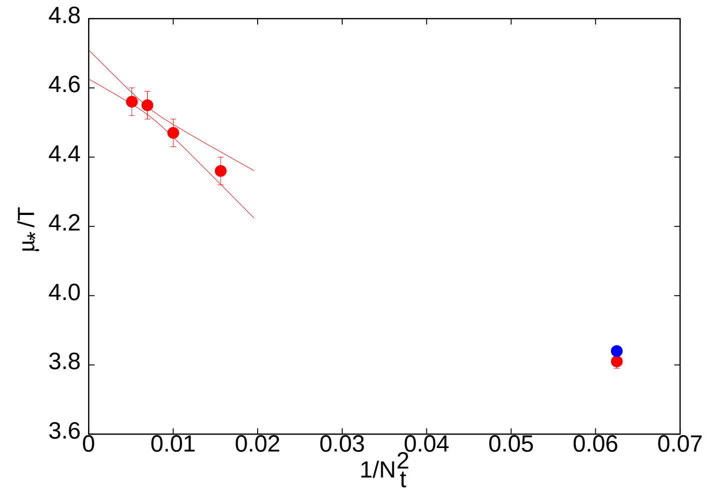
Continuum limit of χ_3

Main technical problem is to control the extrapolation to zero lattice spacing. For this we use two different kinds of Fermions (staggered and Naik) and perform simultaneous extrapolation with both: in the quenched theory.

R. V. Gavai and S. Gupta, *Phys. Rev.* D 67 (2003) 034501

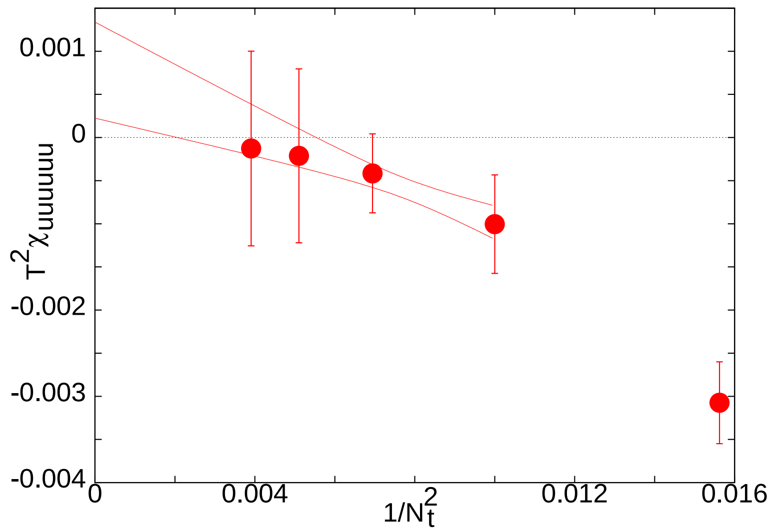


Continuum limit of μ^*, *i.e.*, χ_{uuuu}



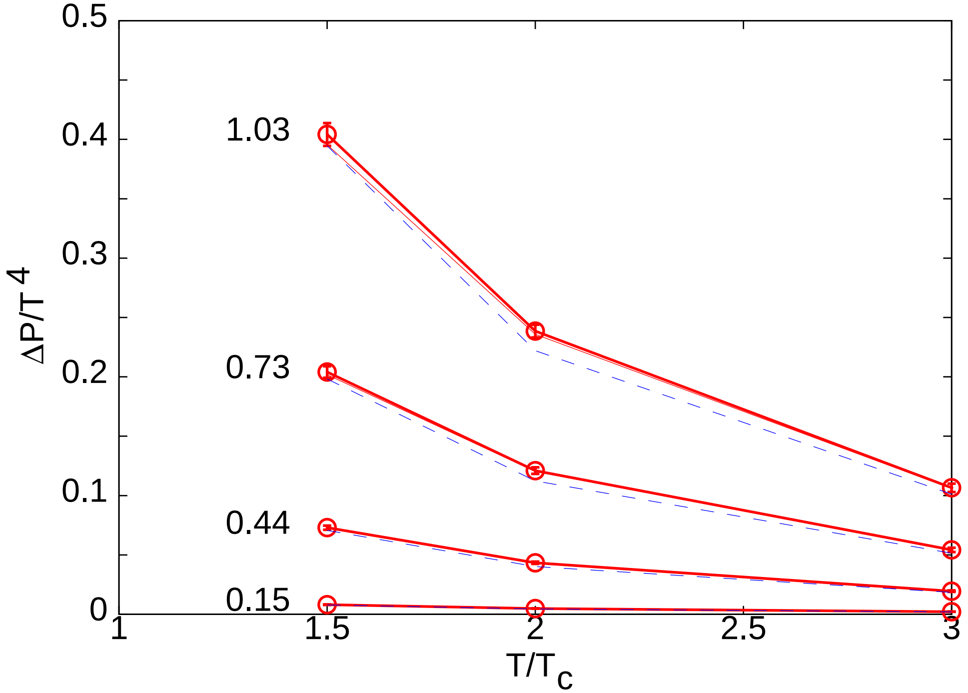
$\mu_* = \sqrt{12\chi_3/\chi_{uuuu}}$ at $T = 1.5T_c$. At finite N_t, the series is insensitive to prescription when $\mu \ll \mu_*$. In the continuum μ_* is the first estimate of the radius of convergence of the series.

Continuum limit of χ_{uuuuuu}



χ_{uuuuuu} at $2T_c$ for $m = 0.1T_c$. This is indistinguishable from zero in the continuum limit. However, this limit is approached much faster than free field theory would suggest.

The pressure



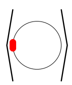
$$\Delta P(T) = P(T, \mu) - P(T, 0)$$

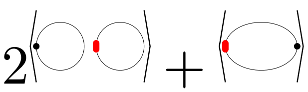
(Reweighting) Z. Fodor, S. D. Katz and K. K. Szabo, hep-lat/0208078,
(Taylor expn) R. V. Gavai and S. Gupta, hep-lat/0303013

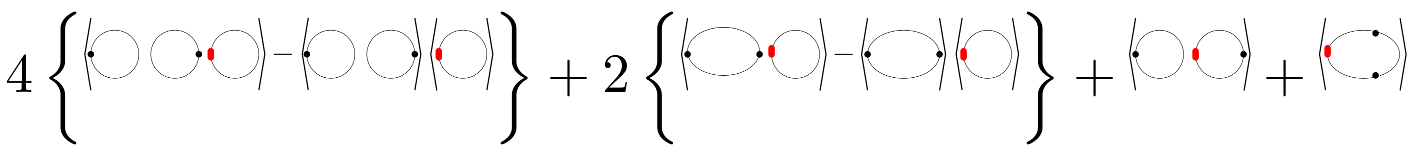
Condensates

$$C(T,\mu) \equiv \langle \overline{\psi}\psi \rangle_{T,\mu} = C(T,0) + c_1\mu + \frac{c_2}{2}\mu^2 + \cdots$$

where

$C(T,0) \;=$  $= \langle \mathcal{C} \rangle$

$c_1 \;=\; 2$  $= \langle 2\mathcal{O}_1\mathcal{C} + \mathcal{C}_1 \rangle = 0$

$c_2 \;=$ 

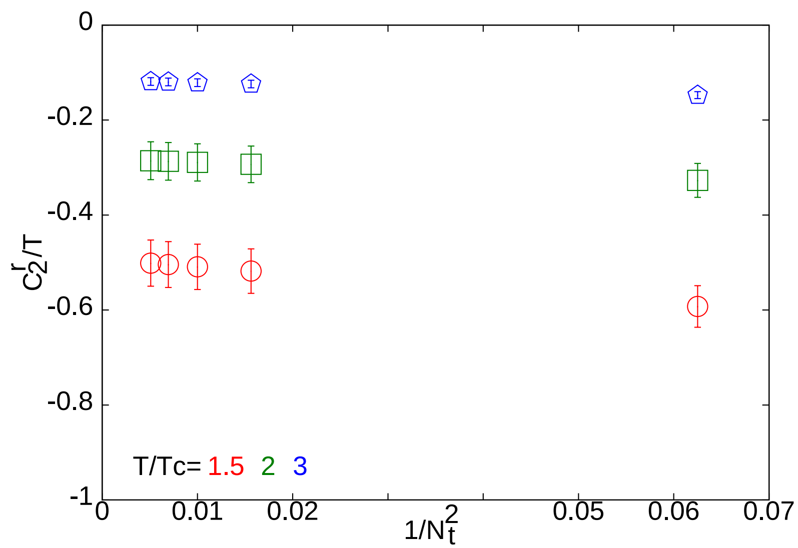
$$= 4\left\{\langle \mathcal{O}_{11}\mathcal{C}\rangle - \langle \mathcal{O}_{11}\rangle\langle \mathcal{C}\rangle\right\} + 2\left\{\langle \mathcal{O}_2\mathcal{C}\rangle - \langle \mathcal{O}_2\rangle\langle \mathcal{C}\rangle\right\} + \langle \mathcal{O}_1\mathcal{C}_1\rangle + \langle \mathcal{C}_2\rangle$$

c_2 dominated by last term, nearly equal for iso-vector and -scalar chemical potential, and related to λ_s in strangeness production through a Maxwell relation.

S. Gupta and Rajarshi Ray, in progress

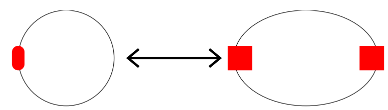
The second derivative of the condensate

c_2 has a logarithmic divergence in the zero lattice spacing limit and therefore has to be renormalised. After renormalization it is nearly independent of the lattice spacing. The dimensionless quantity c_2^r/T is strongly T dependent near T_c. The Maxwell relation indicates that at higher T it should begin to scale.



Correlators 1

For Taylor expansion of correlators in the iso-scalar chemical potential, everywhere in the expression for the condensate replace



This also gives the Taylor expansion of the meson susceptibilities, χ . Below T_c, since $\chi \propto 1/M^2$, this gives a Taylor expansion of the meson mass.

For the pseudo-scalar (PS) correlator, the chiral Ward identity

$$\langle\overline{\psi}\psi\rangle = m\chi_{PS}$$

(m is the quark mass) relates various Taylor coefficients.

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Correlators 2

In iso-vector chemical potential the explicit breaking of isospin symmetry breaks isomultiplets. It is useful to work in terms of the charge neutral meson and the indefinite charge combinations

$$S = (\mathcal{M}^+ + \mathcal{M}^-)/\sqrt{2} \qquad \text{and} \qquad V = (\mathcal{M}^+ - \mathcal{M}^-)/\sqrt{2}.$$

Then the Taylor expansions carried out to order μ^2 are—

$$\begin{aligned} S(T,\mu) &= S(T,0) + \left[2\left\{\langle\mathcal{O}_2 S\rangle - \langle\mathcal{O}_2\rangle\langle S\rangle\right\} + \langle\mathcal{O}_1 S_1 + S_2\rangle\right]\frac{\mu^2}{2} + \cdots \\ V(T,\mu) &= V(T,0) + \cdots \\ \mathcal{M}^0(T,\mu) &= \mathcal{M}^0(T,0) + 2\left[\langle\mathcal{O}_2\mathcal{M}^0\rangle - \langle\mathcal{O}_2\rangle\langle\mathcal{M}^0\rangle\right]\frac{\mu^2}{2} + \cdots \end{aligned}$$

Summary of Results

- Pressure can be extrapolated to finite chemical potential by Taylor expansion. The expansion coefficients are the (linear and non-linear) quark number susceptibilities.

- Taking the continuum limit term by term is computationally straightforward, and allows us to find the EOS in all regions of interest to experiments.

- Computation of several high order susceptibilities may allow estimation of the critical end point by series extrapolation methods.

- Taylor expansions of quark condensate allows us to explore the stability of the Wroblewski parameter as a function of the strange quark mass.

- Taylor expansions of various quantities relate simulations in iso-vector and imaginary iso-scalar chemical potential to those at real iso-scalar chemical potential.