Predictions for QCD matter

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October 6, 2004

QGP Meet
Institute of Physics
Bhubaneshwar

Plan of the talk

1. Aspects of high T QCD
2. The speed of sound and specific heat
3. Electrical conductivity and viscosity
4. Fluctuations and chemistry
5. Jet quenching crabwise; no J/ψ suppression

Collaborators: Saumen Datta (Bielefeld), Rajiv Gavai (TIFR), Robert Lacaze (Saclay), Manu Mathur (SNBNCBS), Swagato Mukherjee (TIFR), Rajarshi Ray (TIFR).

Machines: 486, Pentiums, SGI Origin, DEC α, NEC SV5, CRAY X1

The **I**ndian **L**attice **G**auge **T**heory **I**nitiative

Aspects of high T QCD matter

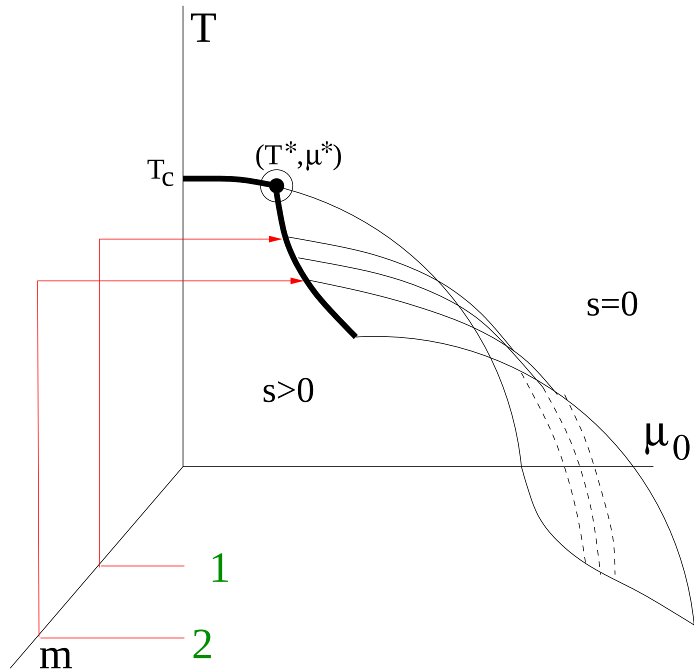
It is a plasma

Overall charge/colour neutrality— Debye screening, Landau damping.
Probes: all quantitative— need to measure correlations between fluctuations and viscosities or other dissipative response functions.

It is a strongly dissipative fluid

Large viscosity, small correlation lengths.
Consequences: many qualitative— probes lose momentum quickly (jet quenching?), early thermalization, independent fluctuations.
Caution: May mimic ideal fluids— independent fluctuations, speed of sound, etc.

The phase diagram of QCD (1)



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2. **Allton et al (2003)** $m_\pi/m_\rho = 0.70(1)$, $m_\rho/T_c = 5.5(1)$, $Lm_\pi = 15.4(5)$

Compensation between m_π dependence and L dependence: accidental?

Berges and Rajagopal, Halasz, Jackson, Shrock, Stephanov and Verbaarschot: (1998)

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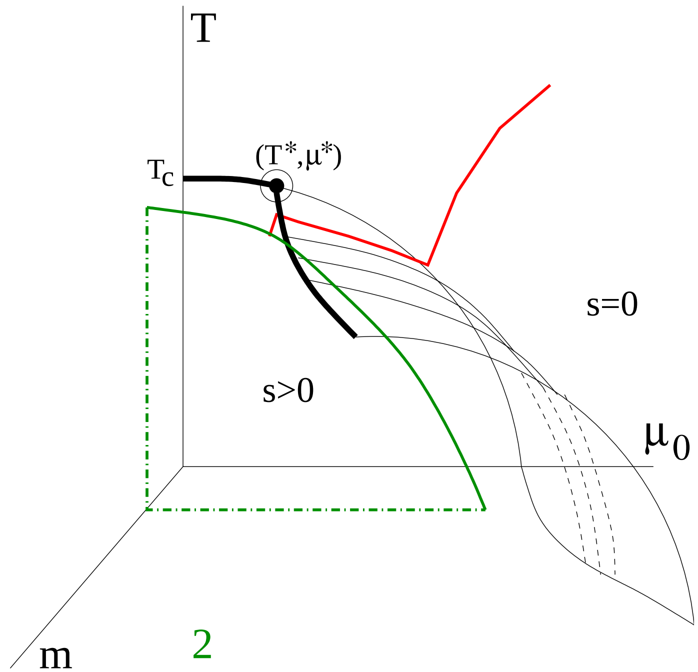

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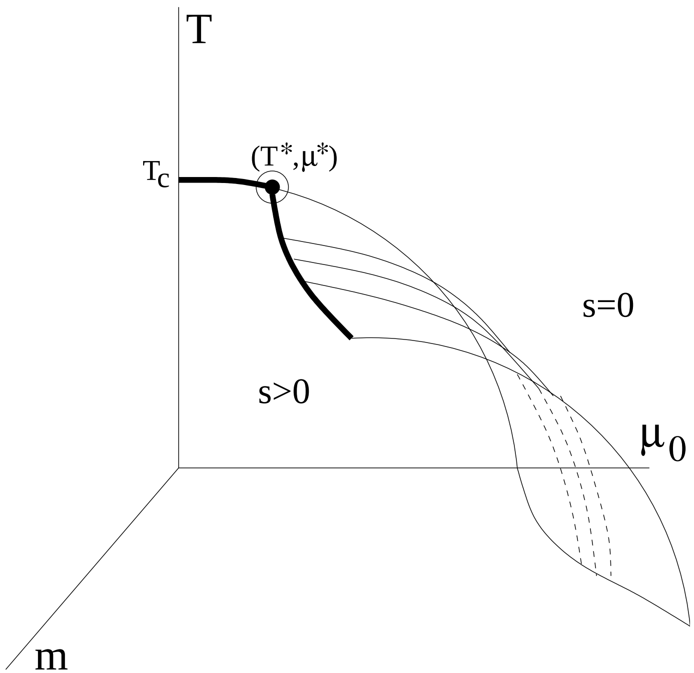
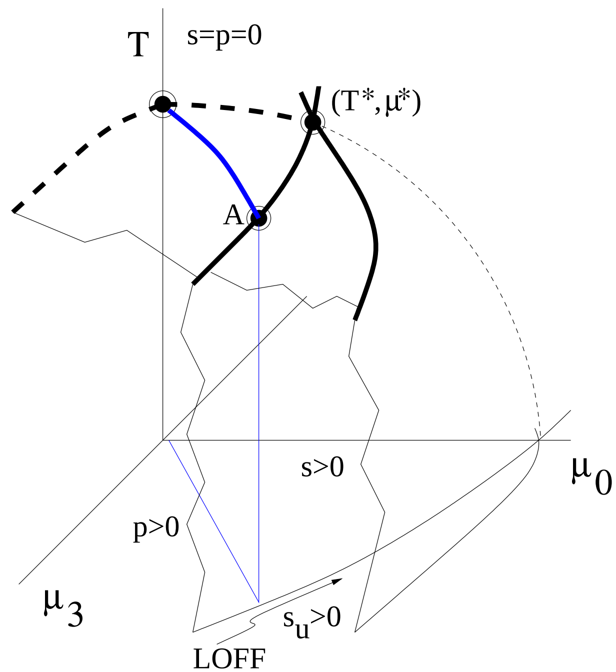
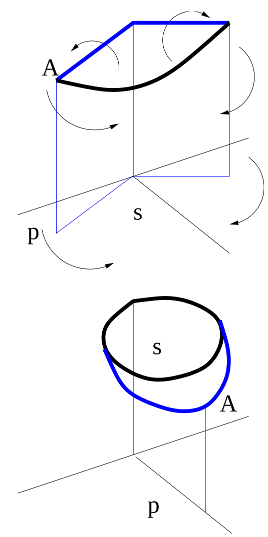


Parameters in $N_f = 2$: T, $\mu_0 = (\mu_u + \mu_d)/2 = \mu_B/3$, $\mu_3 = (\mu_u - \mu_d)/2 = \mu_I$, $m = (m_u + m_d)/2$, $\Delta m = m_u - m_d$.

Parameters in $N_f = 2 + 1$: T, μ_B, μ_I, μ_Y (μ_s), m, Δm, m_s.

The phase diagram of QCD (3)



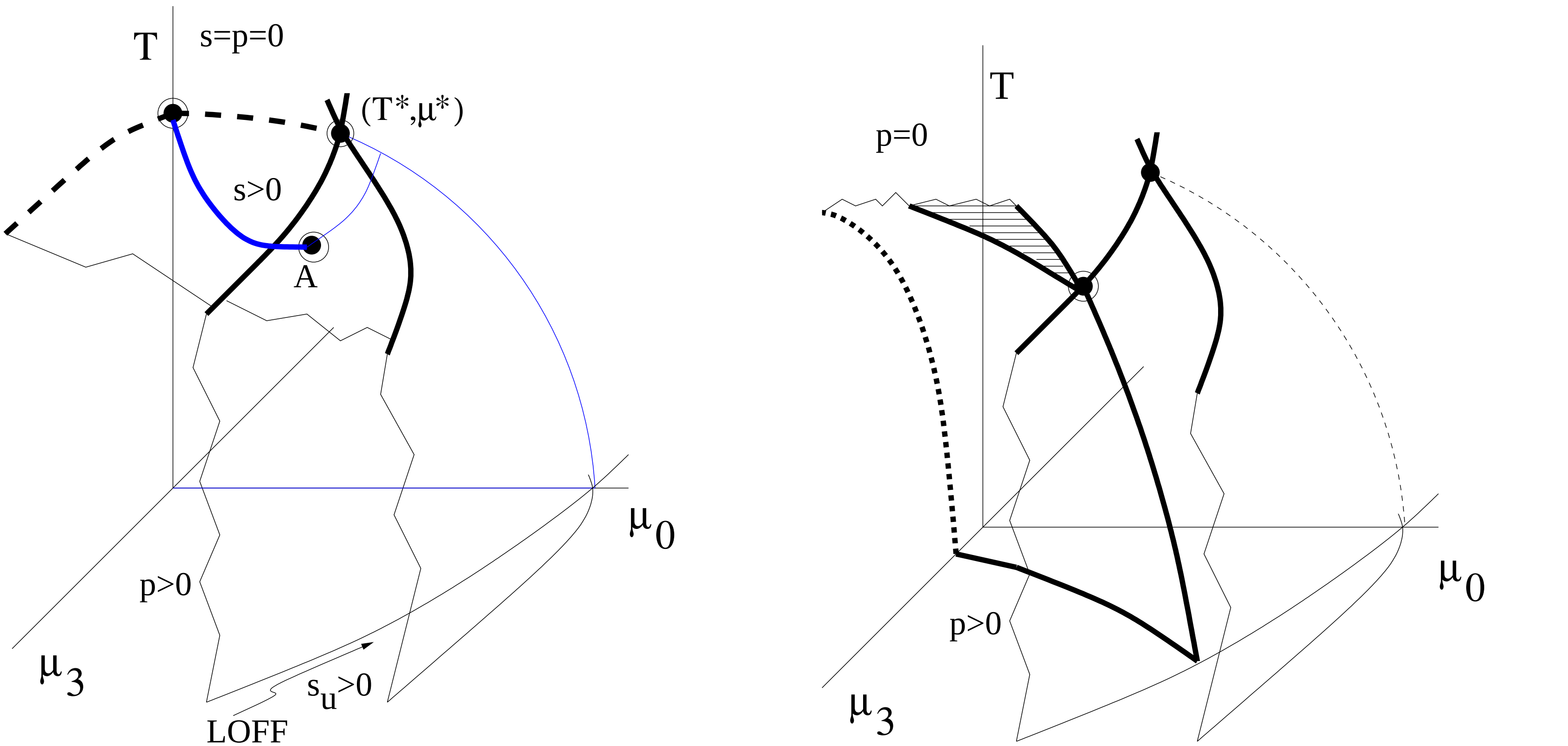


Son and Stephanov (2000), Klein Toublan and Verbaarschot (2003), Nishida (2003), Barducci, Casalbuoni, Pettini and Ravagli (2004)

(T^*, μ^*)— penta-critical point (earlier called tri-critical: only for $m_u = m_d =$)
A— tri-critical point (remains for arbitrary m_u and m_d).

SG and R. Ray, in progress

The phase diagram of QCD (4)



All is not lost— there are critical points and phase transitions to be discovered even in the real world of strong interactions.

Beyond EOS: speed of sound and specific heat

In a relativistic theory: $P(T, \mu)$, *i.e.*, pressure adjusts itself if T is changed. Hence compressibility is not well defined for $\mu = 0$. As a result there are only two second derivatives of the free energy—

$$c_s^2 = \frac{1}{V} \left.\frac{\partial E}{\partial P}\right|_S, \qquad c_V = \frac{1}{V} \left.\frac{\partial E}{\partial T}\right|_V.$$

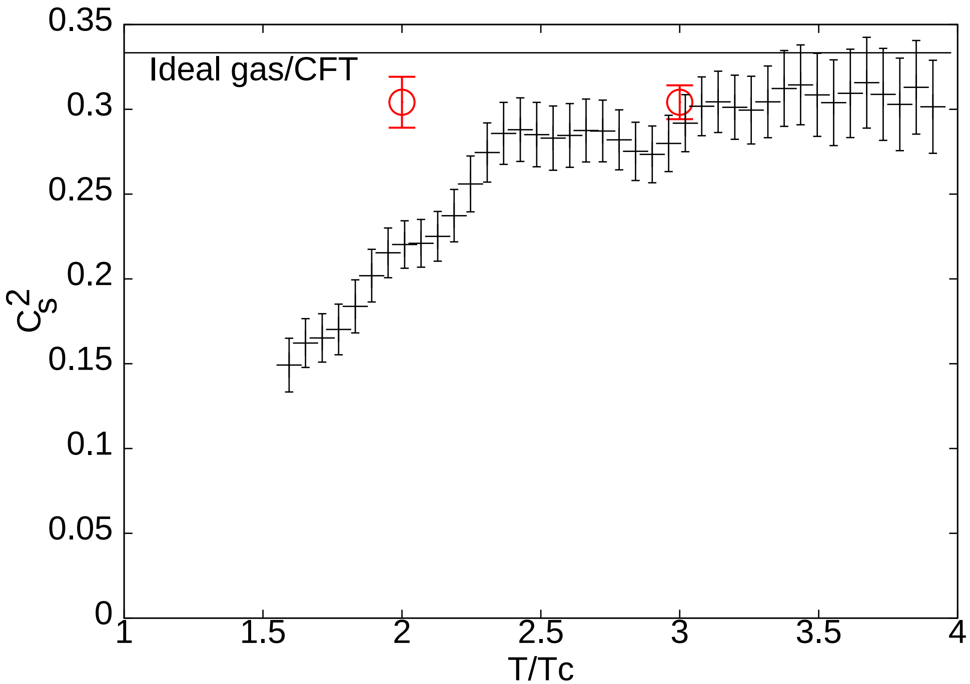
Recall the definition of the trace of the stress-energy tensor—

$$\frac{\Delta}{T^4} = \frac{E/V - 3P}{T^4}.$$

$\Delta = 0$ is a conformal field theory (*i.e.*, scale symmetric). In QCD $\Delta \neq 0$.

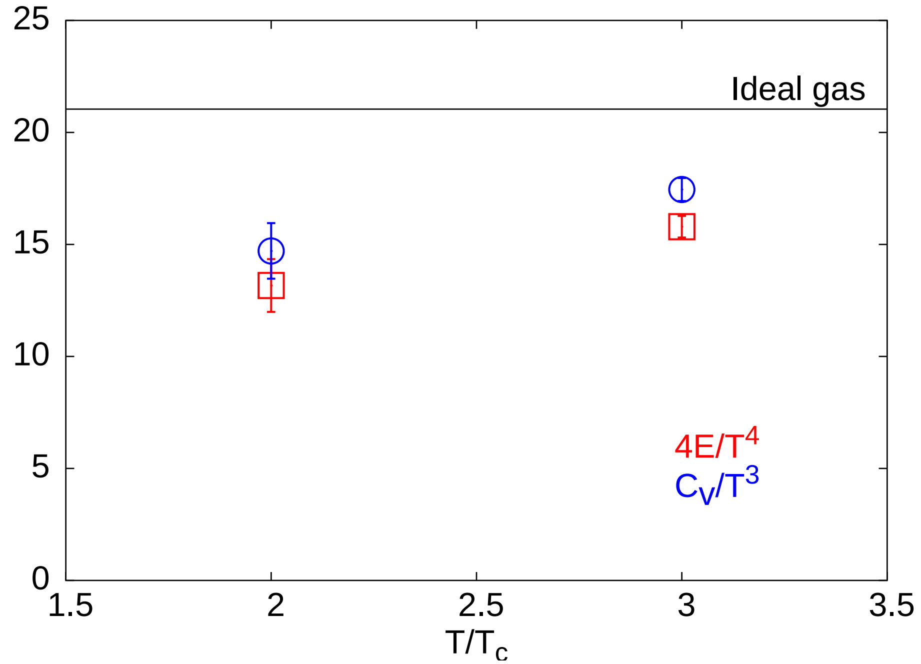
Breaking of scale symmetry implies $c_s^2 \neq 1/3$ and $c_V/T^3 \neq 4E/VT^4$.

Speed of sound



Black: re-analysis of Bielefeld's $N_t = 4$ data (SG, QCD 2002: IIT-K)
Red: new results for continuum limit (Mukherjee, Gavai, SG, in progress)
Coming soon: closer to T_c and below T_c, continuum limit.

Specific heat



Results are far from ideal gas (6–8σ), but close to CFT (1–2σ).

Gavai, SG. Mukherjee, in progress

Experimental question: is it feasible to measure event-to-event $\Delta T/T$ in the fireball?

Transport coefficients: Shear Viscosity

1. AdS/CFT gives $\eta/S \geq 1/4\pi$. (Son, Starinets et al.)

2. Perturbation theory contains a $\log(1/g)$ factor which is negative for $T/T_c \leq 7$. (Arnold, Yaffe and Moore)

3. $2 \rightarrow 2$ collisions with constant cross section, $\sigma_0 = 10$ mb gives $\eta = 1.264T/\sigma_0$. (Gyulassy and Molnar)

4. RHIC data can be fitted in blast wave model with $\eta/S \approx 0.14$. (Teaney)

5. Lattice? Direct computation difficult but another method possible. Compute electrical conductivity and extract thermalization time scale from it using $\sigma = \langle e^2 \rangle n_q \tau_q / m$, where $n_q \simeq S/V$, m is thermal mass. $\tau_q = 0.3$ fm. Assume $\tau_g \simeq \tau_q/2$. Then $\eta/S \approx 0.2$. **(SG, [PL B597 57, 2004])**

Electrical conductivity and photon production

The differential photon emissivity is given by—

$$\omega \frac{d\Omega}{d^3p} = \frac{\langle e^2 \rangle}{8\pi^3} n_B(\omega; T) \rho^\mu_\mu(\omega, \mathbf{p}; T) \qquad \text{where} \qquad \langle e^2 \rangle = 4\pi\alpha \sum_f e_f^2 \approx \frac{1}{21}.$$

In terms of the DC electrical conductivity ($\mathbf{j} = \sigma \mathbf{E}$)

$$\sigma(T) = \frac{\langle e^2 \rangle}{6} \left. \frac{\partial}{\partial \omega} \rho^i_i(\omega, \mathbf{0}; T) \right|_{\omega=0}, \qquad \frac{8\pi^3 \omega}{\langle e^2 \rangle T^2} \frac{d\Omega}{d^3p} = 6 \frac{\sigma}{T}.$$

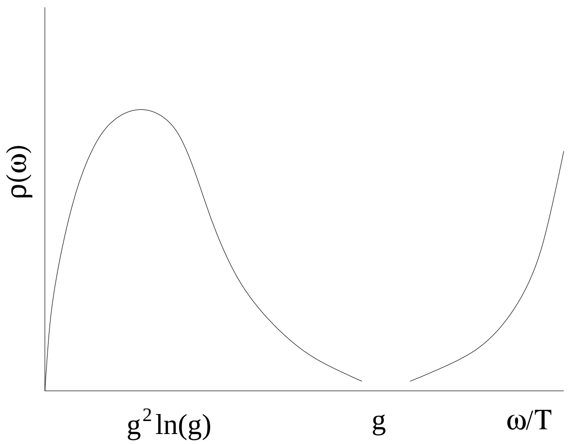
Since $k^\mu \rho_{\mu\nu} = 0$, we have $\rho_{00} = 0$ along the line $\mathbf{p} = 0$. Formally,

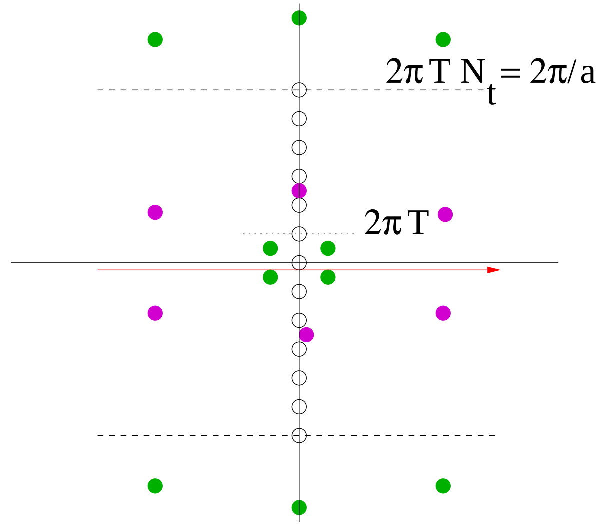
$$\rho_{00}(\omega, \mathbf{0}; T) = 2\pi \chi_Q \omega \delta(\omega),$$

where χ_Q is the charge susceptibility.

Perturbation theory: pinch singularities

There are pinch singularities at small external energy, ω, from ladder diagrams—corresponding to multiple scatterings off particles in the plasma.





Arnold, Moore and Yaffe,

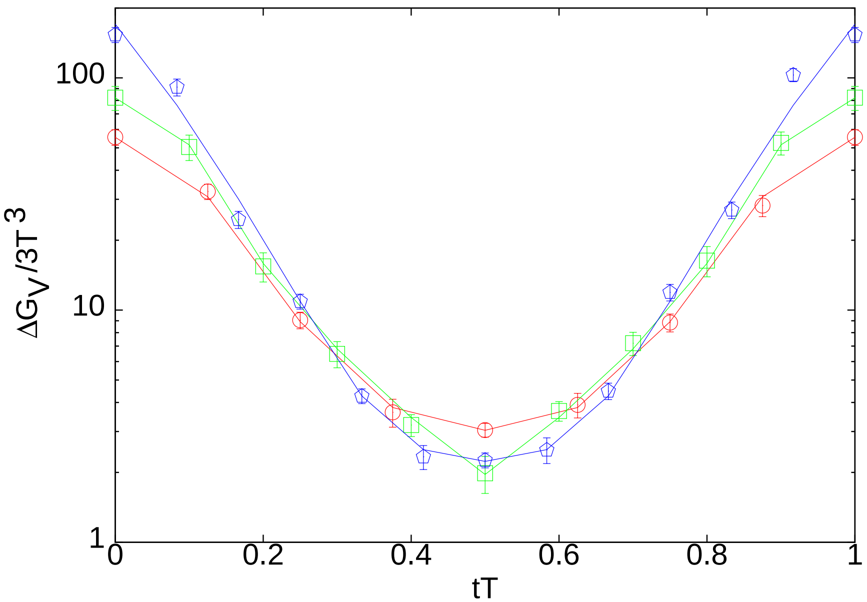
G. Aarts and J.M.M. Resco JHEP 0204:053,2002

The spectral function

Since the problem is linear, work with

$$\Delta G(\omega, \mathbf{p}; T) = G_{full}(\omega, \mathbf{p}; T) - G_{ideal}(\omega, \mathbf{p}; T) = \int_0^\infty d\omega K(\omega, \tau; T) \Delta\rho(\omega, \mathbf{p}; T).$$

This gets rid of a ω^2 divergence at infinity and shows a bump at small ω corresponding to Landau damping.



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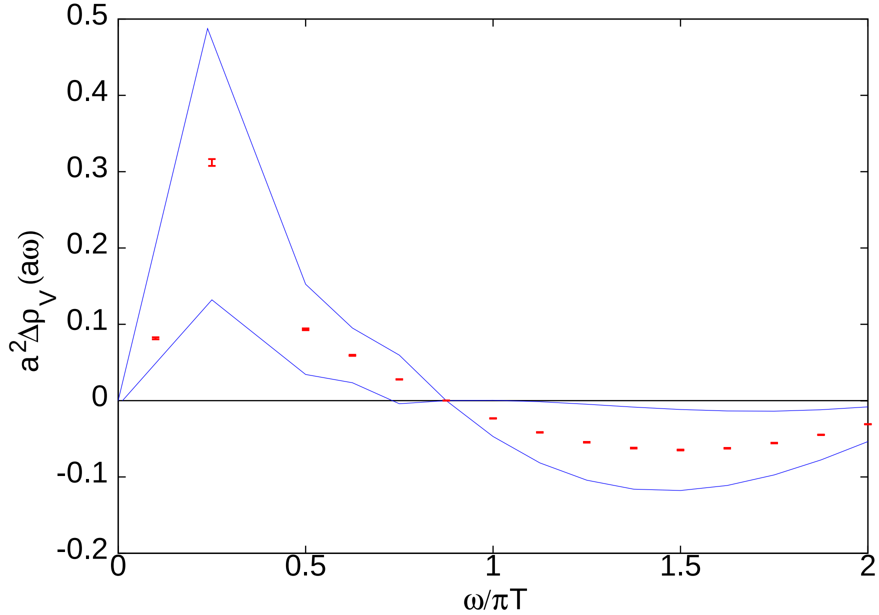

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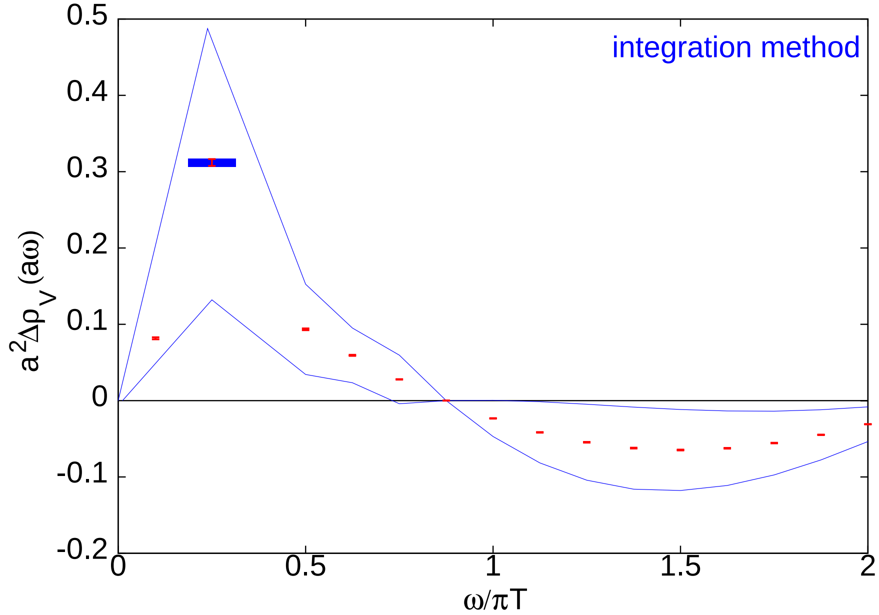

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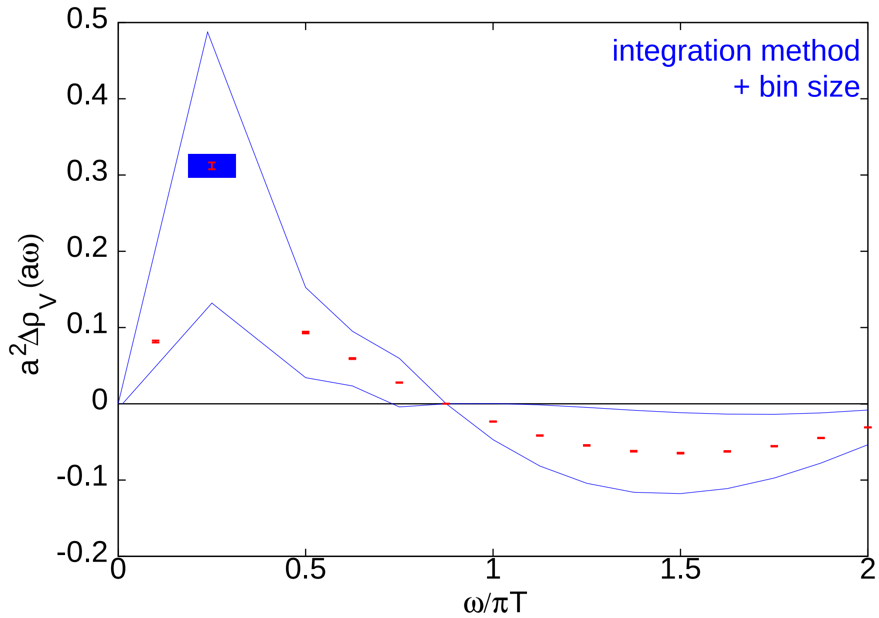

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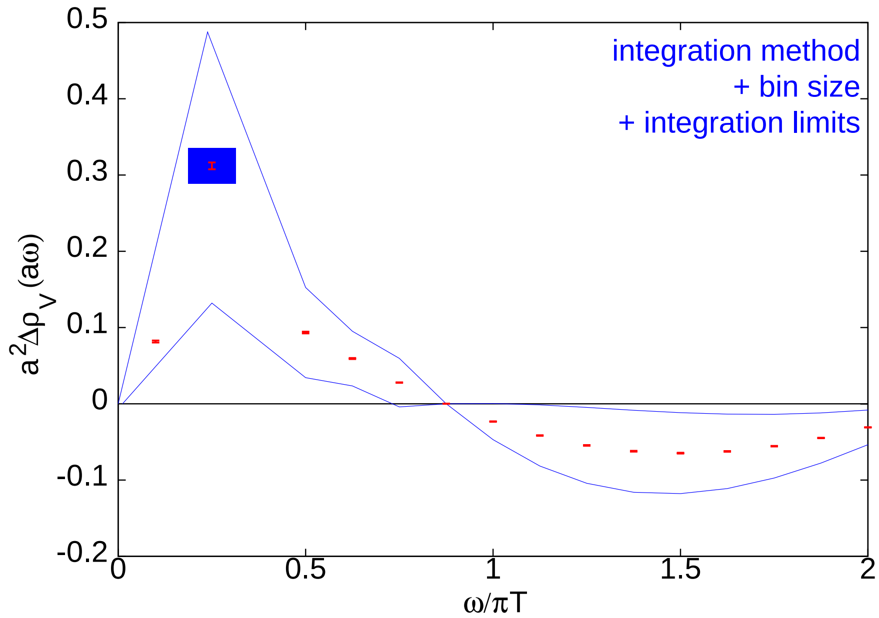

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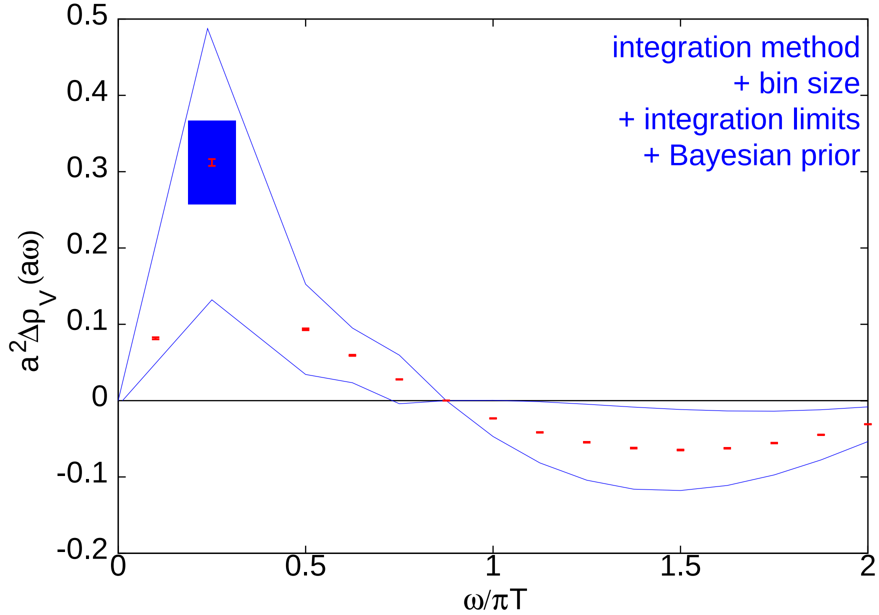

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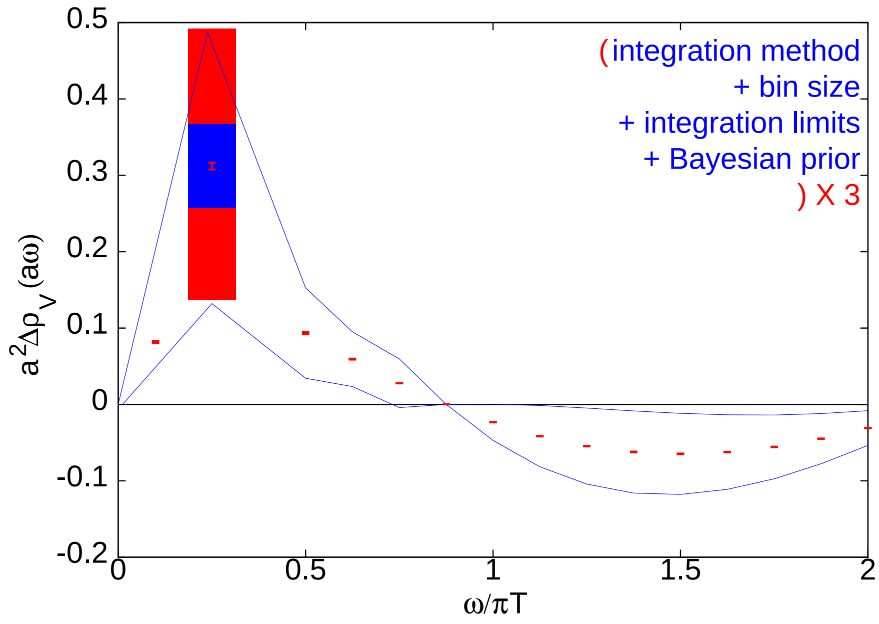
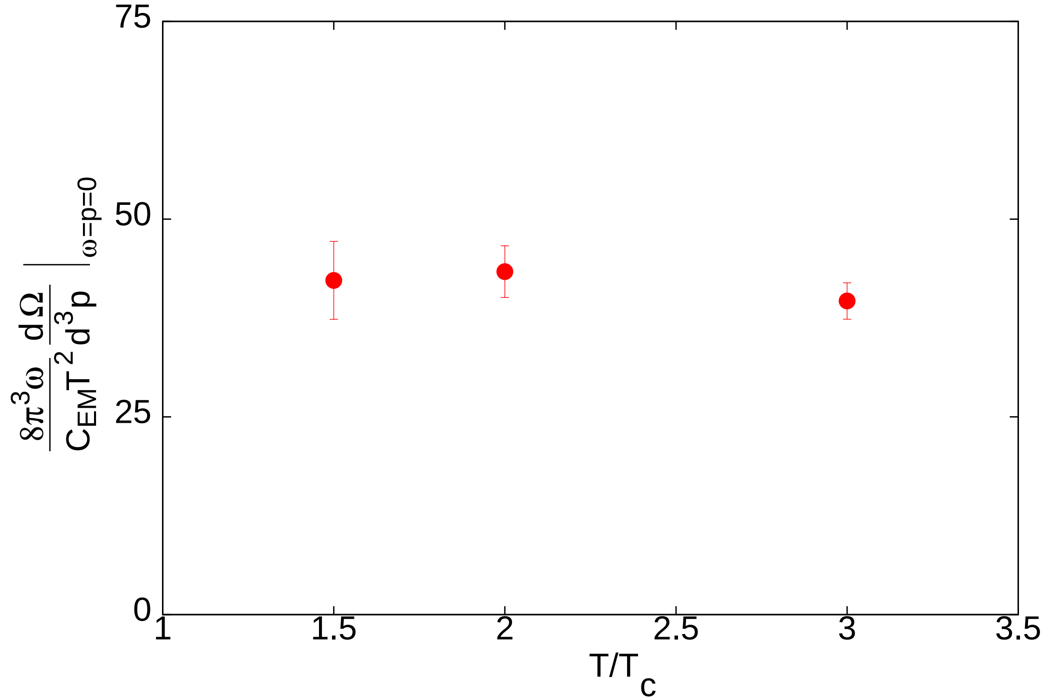


Electrical Conductivity and damping of low frequency photons



Estimate Landau damping at $\omega_L \approx 200$ MeV, and $\tau_q \approx 0.3$ fm. Mean free path of photons with $E \leq \omega_L$ is $\tau_q/\langle e^2\rangle \approx 20 \times 0.3 = 6$ fm. (Another approach: plug $\sigma \approx 1/7T$ into Maxwell's equations and find skin depth)

Question to experimentalists: can you see this independent check of the shear viscosity? Connect hydrodynamics to photons— single source of dissipation.

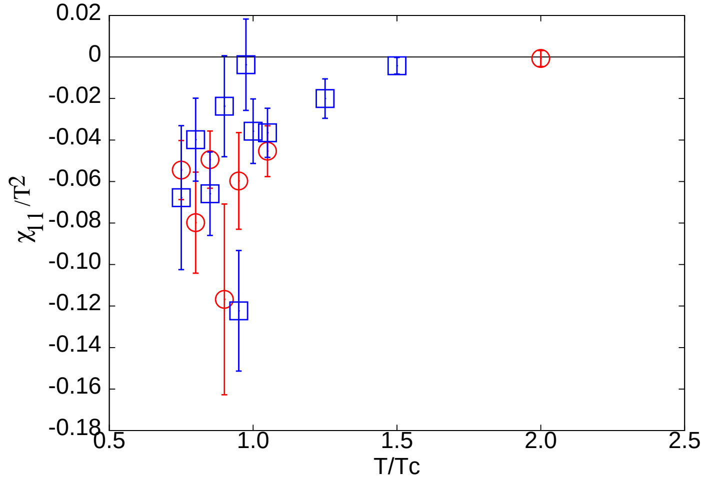
Fluctuations and chemistry

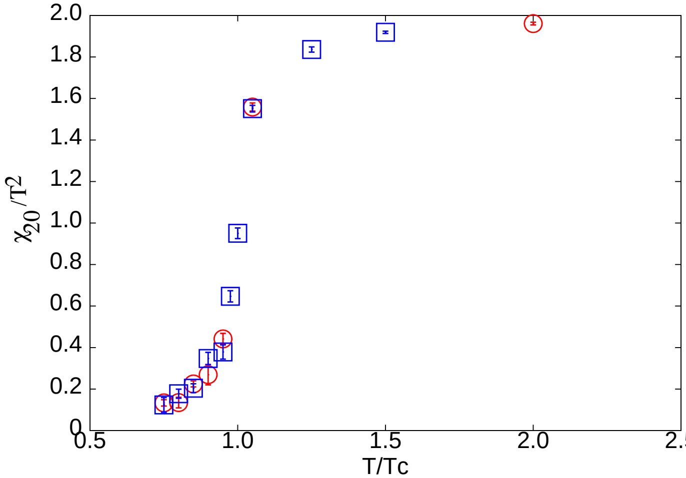
Taylor coefficients of various quantities in μ—

1. of free energy gives charge and baryon number fluctuations

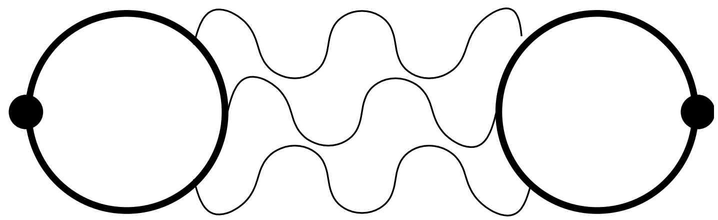
2. of chiral condensate gives correlation volume of charge fluctuations (R. Ray's talk)

3. ratio of QNS gives ratio of production rates of quarks of different mass (Wroblewski parameter)





Correlation volume of baryon number fluctuations (1)



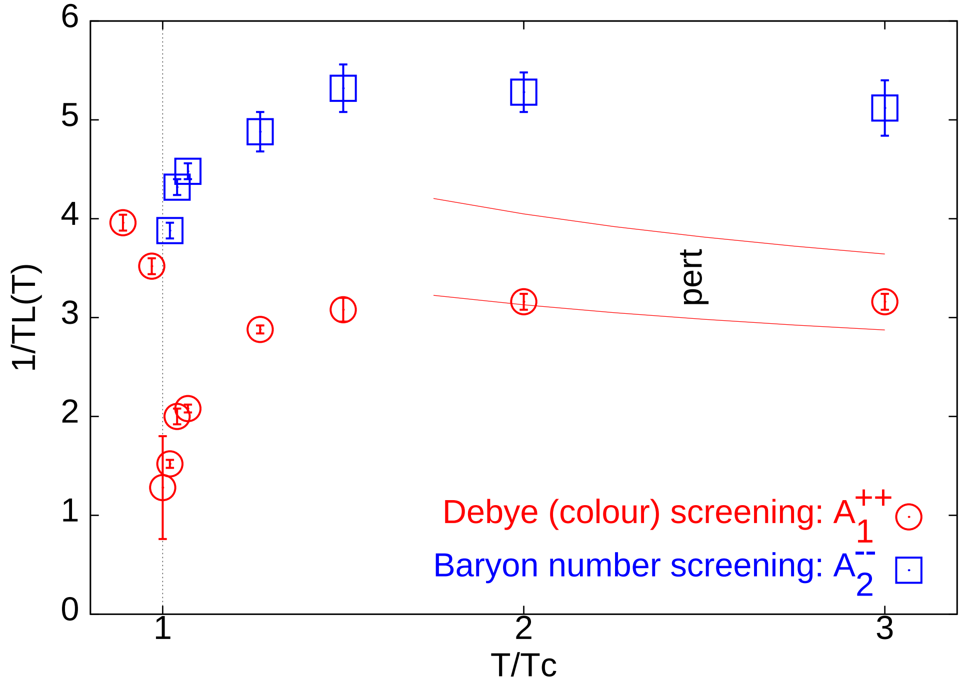
Correlations between baryon number fluctuations (flavour singlet, $C = -1$) are carried by pure glue operators. Correlation lengths are given by the screening mass of the glue operator A_0^3.

Bödeker and Laine [JHEP 0109 029, 2001]

Leading order pertubative evaluation performed recently, agrees with lattice measurements of χ_{11} for $T \geq 2T_c$.

Gavai and SG [PR D64 074506, 2001], Blaizot, Iancu and Rebhan [PL B523 143, 2001]

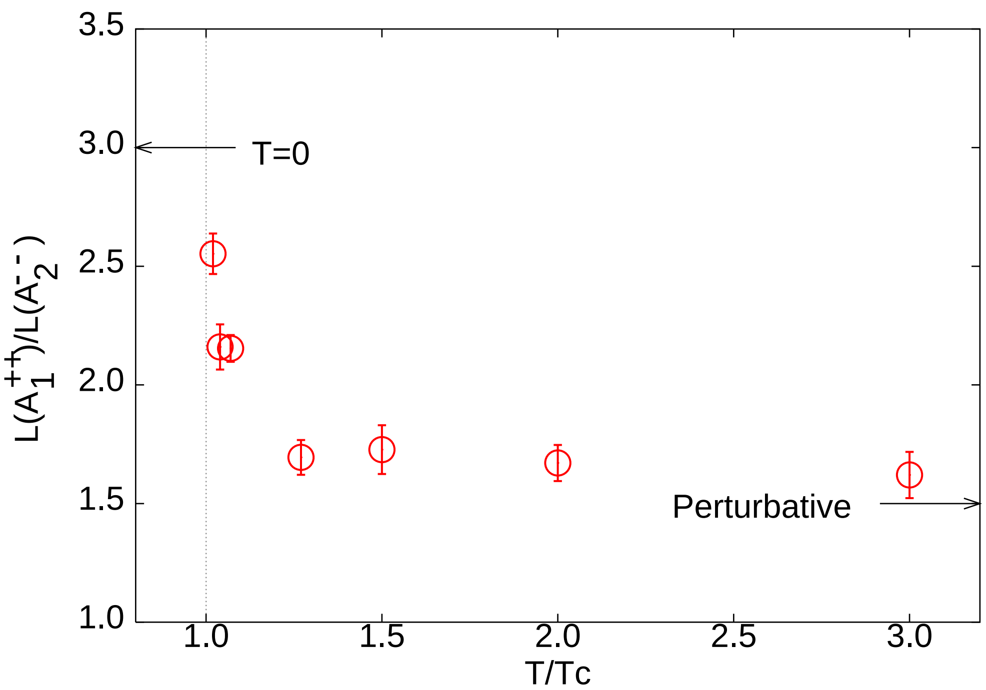
Correlation volume of baryon number fluctuations (2)



Glueball-like screening masses

Baryon number screening length increases near T_c.
Probably drops again below T_c.
Diffusion rate expected to decrease near T_c.
Datta and SG [PR D67 054503, 2003]

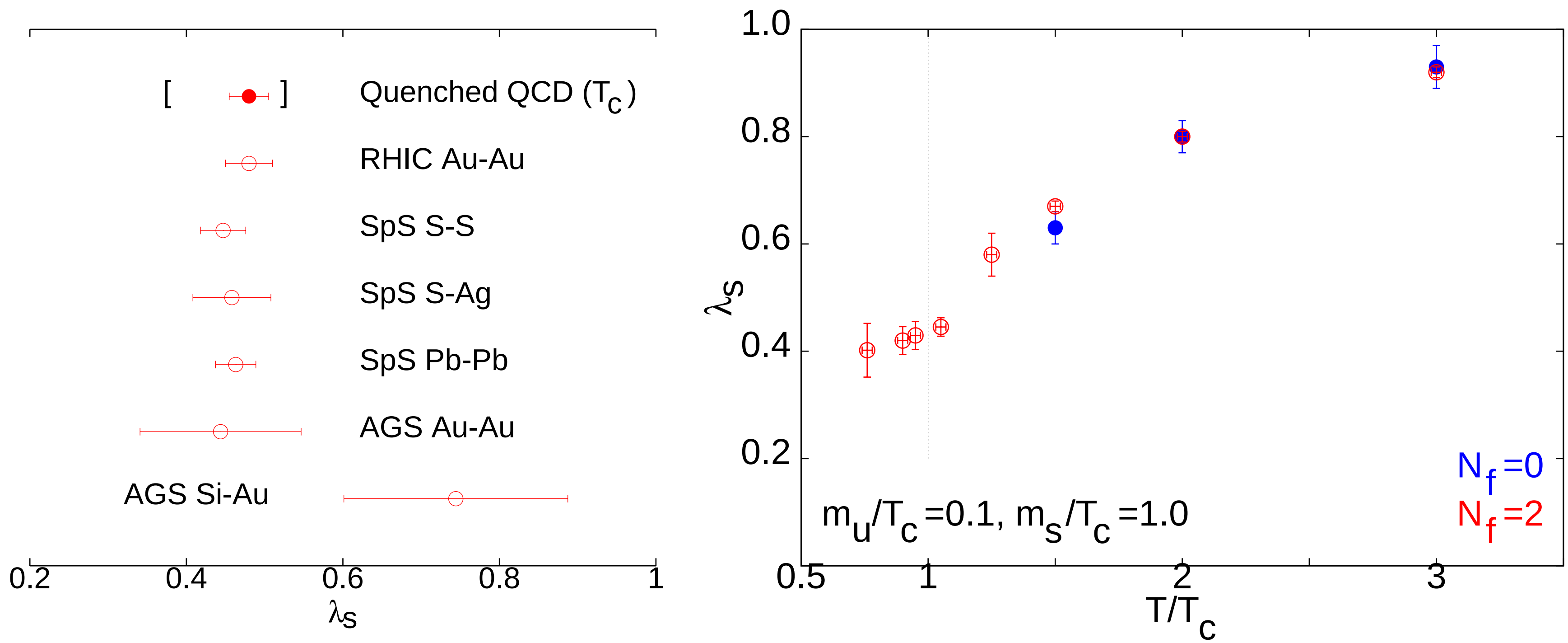
Correlation volume of baryon number fluctuations (3)



Ratio of Debye screening length to baryon number correlation length increases near T_c. Evidence for non-perturbative physics.

Datta and SG [PR D67 054503, 2003]

Strange quark production rate



Ratio of production rates for heavy and light quarks equals the ratio of the QNS for the two quarks. Agrees with data on Wroblewski parameter extracted from experiments.

Gavai and SG (2002), Cleymans (2002), Gavai and SG in preparation.

More in talk by R. Ray

Summary

1. No phase transition at $\mu = 0$ when $m_\pi > 0$, but many phases reachable in heavy ion collisions by tuning T, μ_B and μ_q. Many different critical points may be within reach— need intense new lattice computations.

2. Speed of sound may be compatible with ideal gas or CFT. Specific heat lifts the degeneracy, and shows QCD is close to CFT and far from ideal gas.

3. Viscosity inferred from lattice data is not far from the CFT limit, and agrees with value obtained by fitting to experimental data. Consequence is that soft photons feel a skin-depth in the plasma. Like jet quenching can experiments see also soft photon dimming?

4. Theory of fluctuations now very firm. Fluctuation length scales and time constants are computable on the lattice. How much fluctuation survives the quark-hadron cross over? Need a firmer estimate of the critical end point to pin this down.