Temporal and spatial hadron correlators in finite temperature QCD

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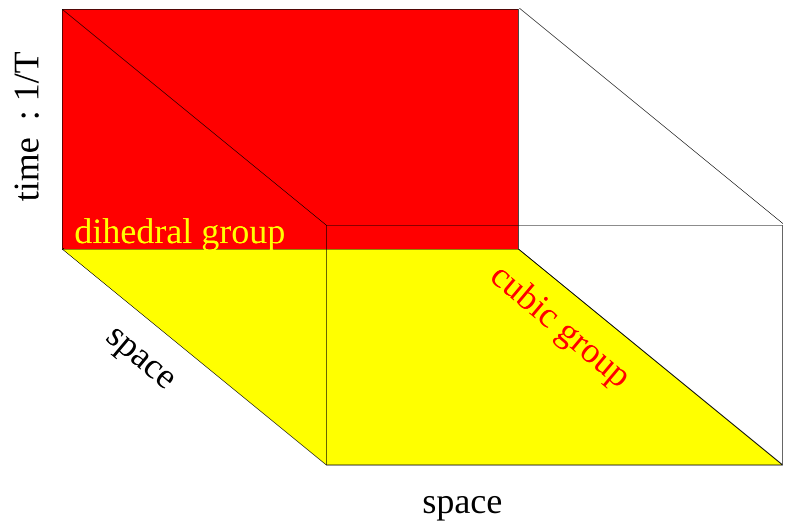
February 7, 2004

1. Screening correlators: departure from weak coupling theory?

2. Temporal correlators: transport coefficients and linear response theory.
   - Bayesian methods— functional and parametrised
   - Bayesian analysis of lattice data: yields small transport time scale

3. Some phenomenology

Hadronic Correlators for $T > 0$ or $\mu > 0$



Temporal correlators have the same classification (J^{PC}) as $T = 0$ correlators.
Screening (spatial) correlators are classified by the group $\mathcal{C} \otimes Z_2(T_E) \otimes Z_2(C_E)$.
The PS, S, V_0, V_{x+y}, AV_t and AV_{x+y} screening correlators are all in a single irrep of the latter and hence should be degenerate in (screening) mass.

Lattice simulations

1. Continuum extrapolation using lattice spacings $1/4T \leq a \leq 1/14T$.

2. High temperature phase investigated for $1.5 \leq T/T_c \leq 3.0$.

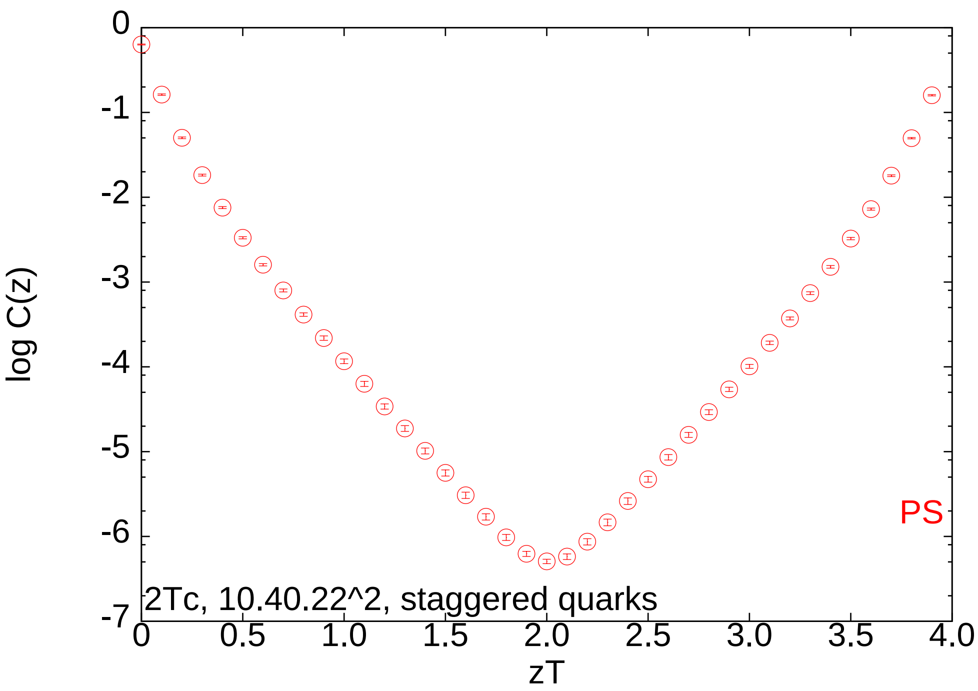
3. Quenched QCD, Wilson action, $N_f = 2$ staggered valence quarks. At most 5% influence of unquenching in quantities under study. Further unquenched computations under way.

4. Volume dependence controlled by comparing some of the results obtained on $T \times (2T)^3$, $T \times (3T)^3$ and $T \times (2T)^2 \times (4T)$ lattices.

5. Renormalisation of operators extracted by $T = 0$ simulations at the same lattice spacings.

Screening correlators

For $T > 0$ the pattern of degeneracies expected for lightest states is observed at small a. (Fat-link fermions have this behaviour even on coarse lattices.)



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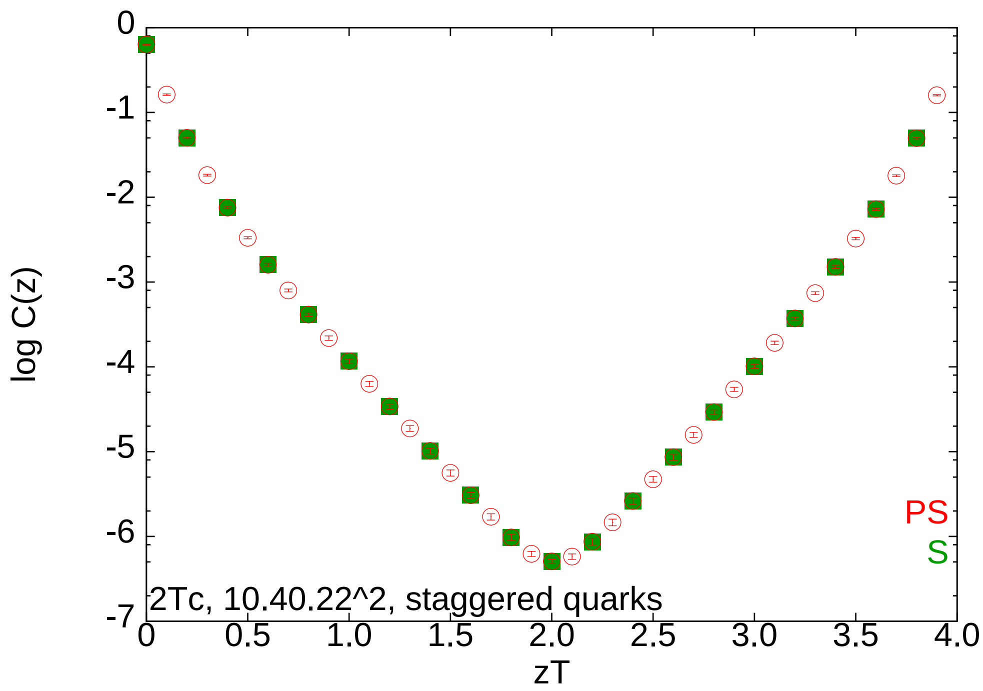

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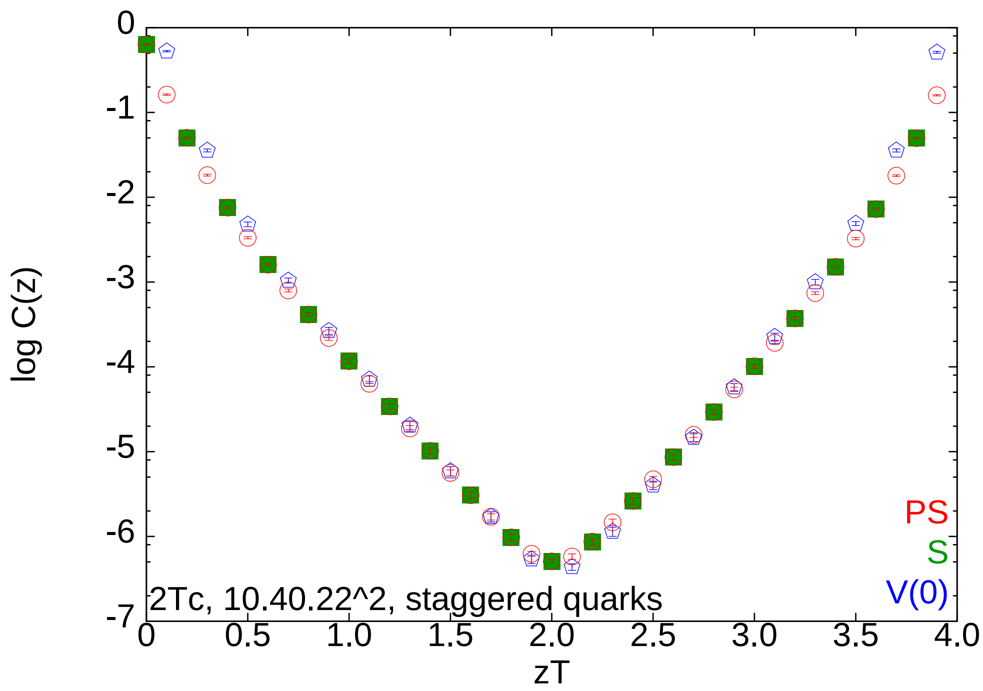
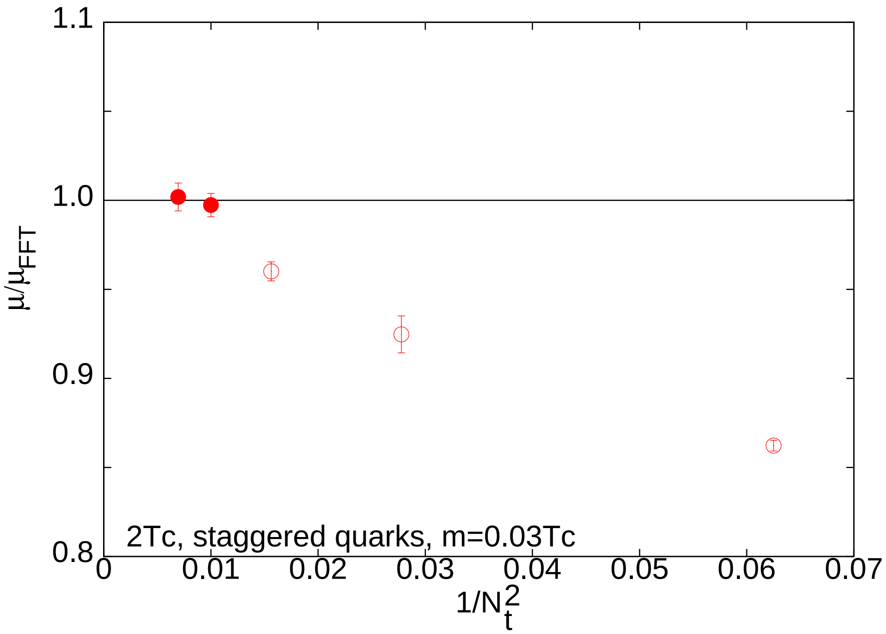


Screening masses

Staggered quark screening masses agree with free field theory. Insignificant quark mass dependence for $m \leq T$. Overlap quarks at coarse lattice spacing give similiar results.

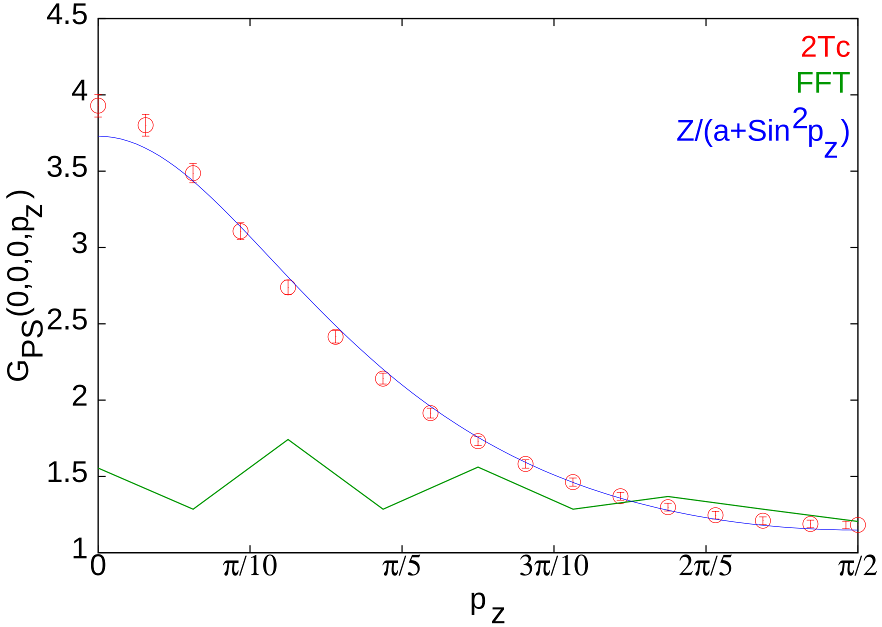


Gavai and SG, *Phys. Rev.*, D 67 (2003) 034501

Laine and Vepsalainen, hep-ph/0311268 and Hansson and Zahed, *Nucl. Phys.*, B 374 (1992) 277

Long distance physics

Screening mass does not capture the full physics. Momentum space correlator shows that long distance physics is different from free field theory of quarks. There is also too much power in the long distance part of the correlator when compared to a single boson exchange.



Temporal correlators: electrical conductivity and photon emissivity

The differential photon emissivity is given by—

$$\omega\frac{d\Omega}{d^3p} = \frac{C_{EM}}{8\pi^3} n_B(\omega;T)\rho^\mu_\mu(\omega,\mathbf{p};T) \qquad \text{where} \qquad C_{EM} = 4\pi\alpha\sum_f e_f^2 \approx \frac{1}{21}.$$

In terms of the DC electrical conductivity ($\mathbf{j} = \sigma\mathbf{E}$)

$$\sigma(T) = \frac{C_{EM}}{6}\;\frac{\partial}{\partial\omega}\rho^i_i(\omega,\mathbf{0};T)\Big|_{\omega=0}, \qquad \frac{8\pi^3\omega}{C_{EM}T^2}\frac{d\Omega}{d^3p} = 6\,\frac{\sigma}{T}.$$

Since $k^\mu\rho_{\mu\nu} = 0$, we have $\rho_{00} = 0$ along the line $\mathbf{p} = 0$. Formally,

$$\rho_{00}(\omega,\mathbf{0};T) = 2\pi\chi_Q\omega\delta(\omega),$$

where χ_Q is the charge susceptibility.

Linear Response Theory

The response, $\mathbf{A}(t)$, of a system to a force $\mathbf{F}(t)$ if non-linear terms are neglected—

$$\mathbf{A}(t) = \int_{-\infty}^{\infty} dt' \chi(t-t')\mathbf{F}(t') \qquad \text{hence} \qquad \mathbf{A}(\omega) = \chi(\omega)\mathbf{F}(\omega).$$

Causality implies $\chi(t) = 0$ for $t < 0$. As a result $\chi(\omega)$ is regular in the upper half plane and dispersion relations follow. The spectral density is the imaginary part of $\chi(\omega)$ as ω approaches the real axis from above. A microscopic computation explicitly relates $\chi(\omega)$ to the retarded propagator. From this follow the Kubo formulæ relating the transport coefficient and the zero energy limit of the spectral density—

$$\chi \propto \lim_{\epsilon \to 0} \int d^3x' \int_{-\infty}^{t} dt'' \mathrm{e}^{\epsilon(t''-t)} \int_{-\infty}^{t''} dt' \langle \mathbf{A}(\mathbf{x},t)\mathbf{A}(\mathbf{x}',t')\rangle.$$

J. Hilgevoord, *Dispersion Relations and Causal Description*, North-Holland, 1960

Euclidean Correlators

In the Euclidean theory one constructs equilibrium correlation functions which are related to the spectral function by the relation—

$$G(\tau, \mathbf{p}; T) = \int_0^\infty \frac{\omega}{2\pi} K(\omega, \tau; T) \rho(\omega, \mathbf{p}; T).$$

In a lattice theory there are N_t points in the τ direction, but there is a continuous infinity of ω.

Replace integral by sum, the linear relation above becomes a set of linear equations: more variables than equations. Inverse of K is ill defined. Convert to a minimisation/Bayesian problem.

(Opposite of least-squares fit: more equations than unknowns)

Regularisation

Maximize the Bayesian probability—

$$P(\rho|G) \propto P(G|\rho)P(\rho) = \exp[-F(\rho)],$$

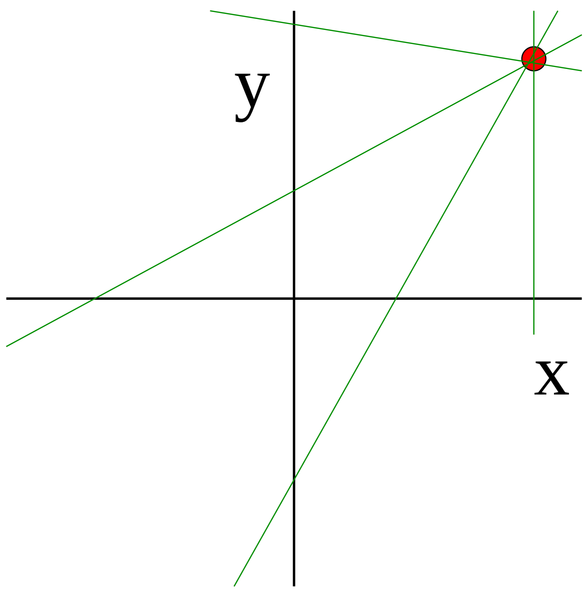
$$F(\rho) = (G - K\rho)^T \Sigma^{-1} (G - K\rho) + \beta U(\rho)$$

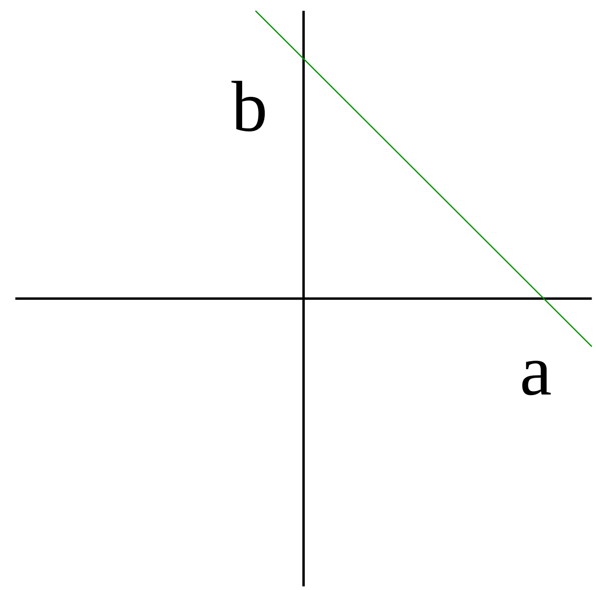
β is a regularisation parameter, and $U(\rho)$ is a function which we are free to choose. This encodes our prior knowledge of the system.
A. N. Tikhonov and V. Y. Arsenin, *Solutions of Ill-posed Problems*, Wiley, New York (1977)

1. Maximum Entropy Method: $U = \sum \rho \log(\rho/\rho_0) - \rho$ Y. Nakahara, M. Asakawa and T. Hatsuda, *Phys. Rev.*, D 60 (1999) 091503, also QCD TARO, *Nucl. Phys.* B (Proc. Suppl.) 63 (1998) 460
2. Linear: $U = \rho^T L^T L \rho$, $L = 1$, D, D^2, *etc.*SG, hep-lat/0301006
3. Include known information into the Bayesian probability G. P. Lepage *et al.*, *Nucl. Phys.*, B (Proc. Suppl.) 106 (2002) 12.

An example

Determine the parameters of the line $y = a + bx$ passing through (1,1)

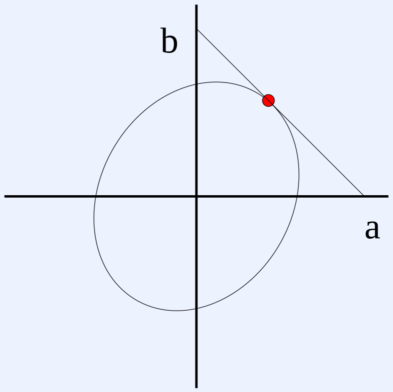




Simplified version of the actual problem to be solved: $2 \times L^3$ lattice.

Solution: method 1

Method 1: General linear regulator $U = \rho^T L^T L \rho$



For all L the best fit passes through the data.

$$F(a,b) = (1-a-b)^2 + \beta(l_{11}a^2 + l_{22}b^2 + 2l_{12}ab)$$

U is positive definite. The minimum occurs at

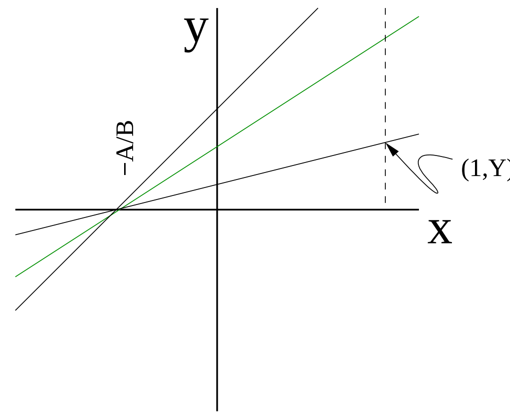
$$M\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \quad \text{where} \quad M = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} + \beta\begin{pmatrix} l_{11} & l_{12} \\ l_{12} & l_{22} \end{pmatrix}$$

$$\text{Most probable } \beta = 0: \quad \begin{pmatrix} a \\ b \end{pmatrix} = \overbrace{\frac{1}{1+x}\begin{pmatrix} x \\ 1 \end{pmatrix}}^{l_{11}\neq 0\ (x=l_{22}/l_{11})} \quad \text{or} \quad \overbrace{\frac{1}{1+x}\begin{pmatrix} 1 \\ x \end{pmatrix}}^{l_{22}\neq 0\ (x=l_{11}/l_{22})} \qquad (l_{12}=0).$$

Solution: method 2

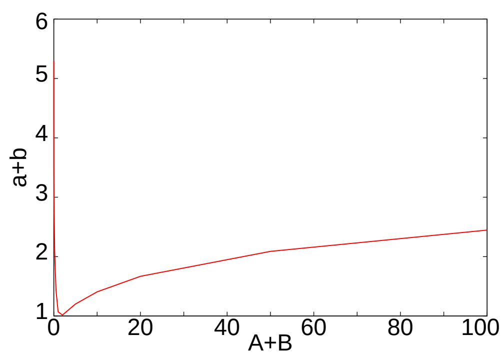
Method 2: MEM

$$F(a,b) = (1-a-b)^2 + \beta\left(a\log\frac{a}{A} + b\log\frac{b}{B} - a - b\right)$$



The minimum is at

$$\frac{a}{A} = \frac{b}{B} = u \quad \text{where} \quad 1 - Yu = \frac{\beta}{2}\log u,$$



and $Y = A + B$. Solutions exist only for $Y > 0$.
If $Y < 1$ then $u > 1$ and vice versa.

The best fit does not pass through the data except when $A + B = 1$

Solution: method 3

Method 3: Partial knowledge

Prior: the line passes through the origin. We can choose the Bayesian probability distribution to be a Gaussian of width σ around the origin—

$$F = (1 - a - b)^2 + \frac{a^2}{2\sigma} \quad \text{giving} \quad \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}.$$

If only information on b is needed then a can be integrated out of the Bayesian probability to give the marginal distribution

$$P(b)db \propto db \exp\left[-(1 - b)^2\right].$$

In this case marginalisation gives b=1 which is the same result as minimisation. This is true if the distribution is unimodal.

Lattice gauge theory with functional Bayesian methods

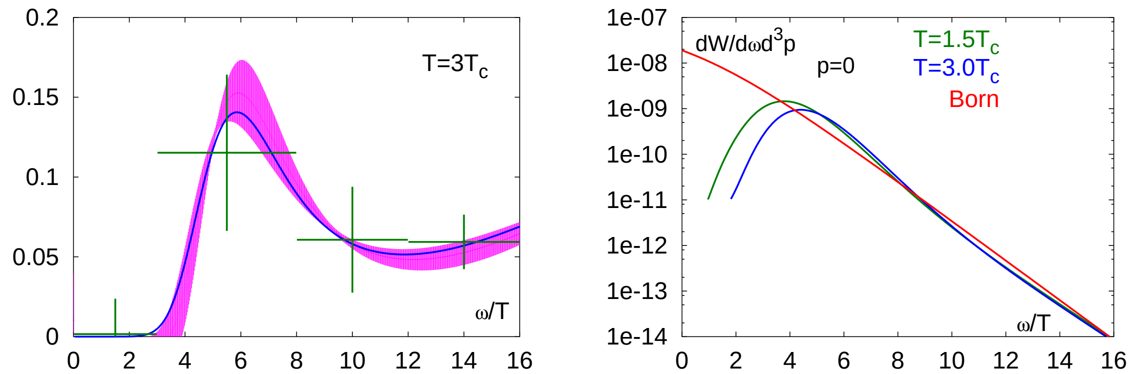
The lattice problem is to use—

$$G(\tau, \mathbf{p}; T) = \int_0^\infty \frac{\omega}{2\pi} K(\omega, \tau; T) \rho(\omega, \mathbf{p}; T),$$

and the data on G to extract ρ.

One has only limited data from the lattice. This can be treated in different ways to extract different kinds of physics. One method of analysis will not give us all the information needed. We must tune the method to the problem.

Large ω using MEM



F. Karsch et al, Phys.Lett.B530:147,2002— Wilson quarks

Full agreement with Born for $\omega/T \geq 4$.

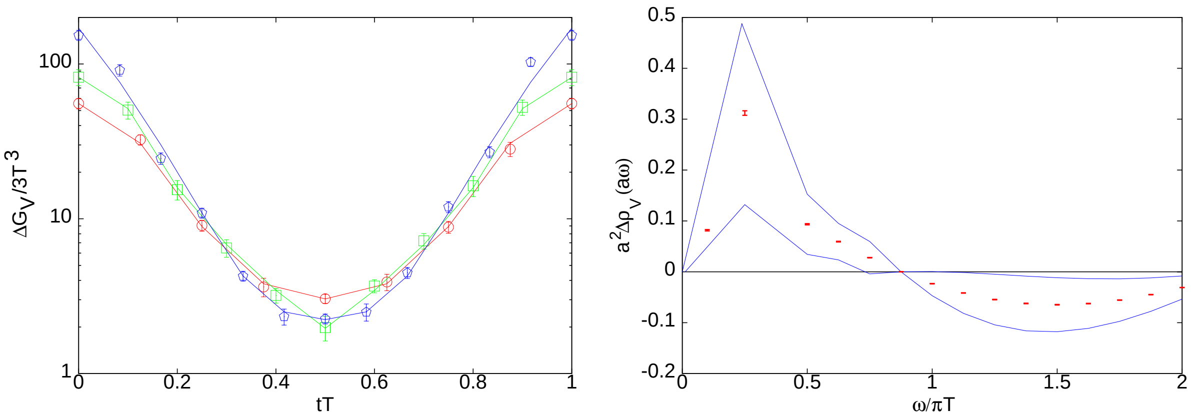
Default model: ideal gas behaviour. Output: $\rho(\omega)$ grows as ω^2 at large ω. Extracted value vanishes as $\omega \to 0$. Need to examine low ω region by another method in more detail.

Small ω using linear regulator

Since the problem is linear, work with

$$\Delta G(\omega, \mathbf{p}; T) = G_{full}(\omega, \mathbf{p}; T) - G_{ideal}(\omega, \mathbf{p}; T).$$

This gets rid of the ω^2 divergence at infinity, at the cost of the positivity of $\Delta\rho$. Use a linear regulator. This shows a bump at small ω.



Second and higher bump at $\omega/T \approx$8–9.

SG, hep-lat/0301006

Lattice gauge theory with parametrised Bayesian methods

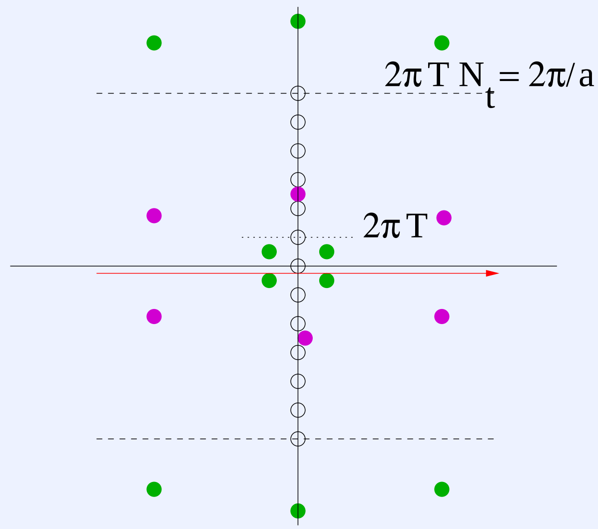
Use a sequence of parametrisations for the spectral density

$$\frac{\Delta\rho}{T^2} = \frac{z\sum_{n=0}^{N}\gamma_n z^{2n}}{1+\sum_{m=1}^{M}\delta_m z^{2m}}.$$

Use with Fourier space correlators—

$$\Delta G(\omega_n, \mathbf{p}; T) = \oint \frac{d\omega}{2i\pi}\,\frac{\Delta\rho(\omega, \mathbf{p}; T)}{\omega - \omega_n}$$

where $\omega_n = 2i\pi nT$.



Two types of poles: S_2 is "relaxation time" and S_4 is "Landau damping".

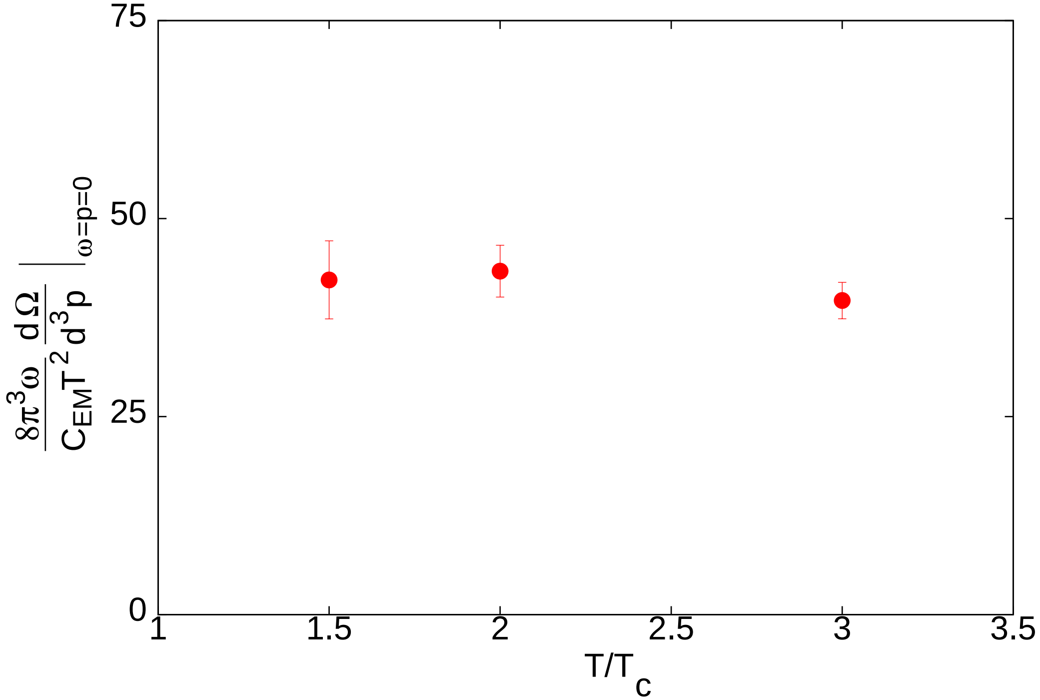
Use χ^2 parameter fitting if $N + M + 1 \leq N_t$, Bayesian otherwise.

F. Karsch and H. W. Wyld, *Phys. Rev.*, D 35 (1987) 2518; S. Sakai *et al.*, hep-lat/9810031

Electrical conductivity: continuum limit

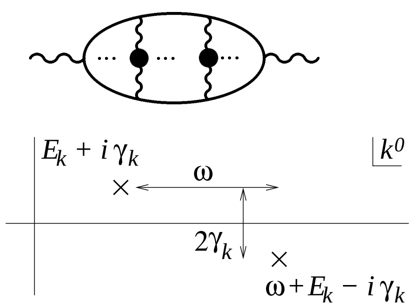
Electrical conductivity depends only on the parameter γ. Obtain this by marginalising over the remaining parameters. SG, hep-lat/0301006

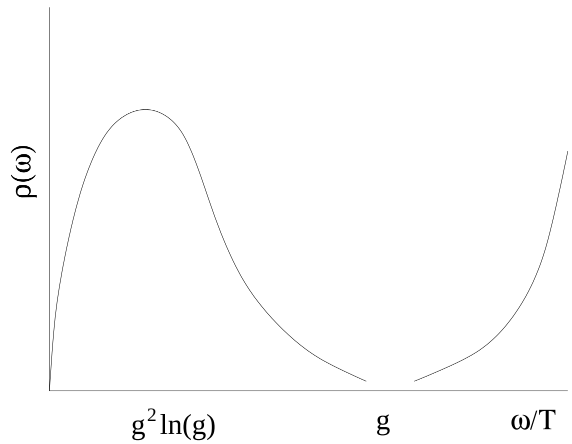
$$\frac{\sigma}{T} \approx 7C_{EM} \text{ for } 1.5 \leq T/T_c \leq 3$$



Pinch singularities and transport

There are pinch singularities at small external energy, ω, from ladder diagrams. These ladder diagrams correspond to multiple scatterings off particles in the plasma.





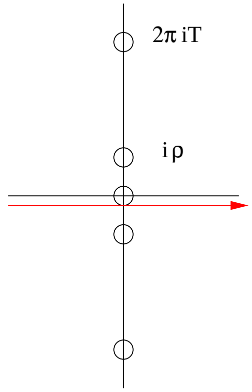
Transport: G. Aarts and J.M.M. Resco JHEP 0204:053,2002

Parametric behaviour

$$\Delta G(\omega_n, \mathbf{p}; T) = \oint \frac{d\omega}{2i\pi} \frac{\Delta\rho(\omega, \mathbf{p}; T)}{\omega - \omega_n}$$

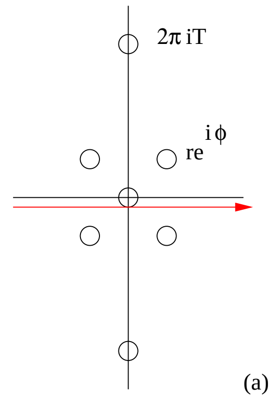
dominated by poles closest to the origin. If these are poles of $\Delta\rho$ then

Relaxation time



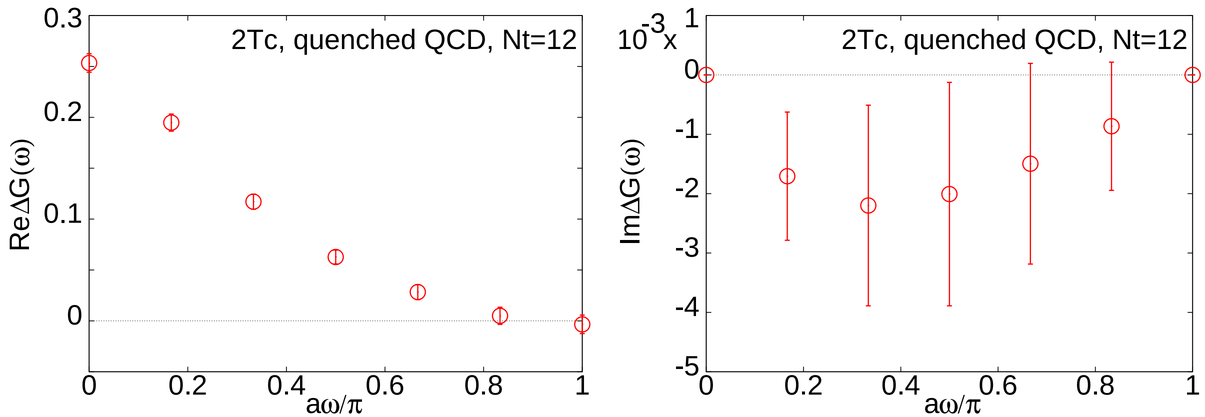
- $|\Re G(\omega_n)| \gg |\Im G(\omega_n)|$ $\quad(\star)$
- $\text{sign}\, \Re G(\omega_n) = \text{sign}\, \Im G(\omega_n)$ $\quad(\star)$
- $\Re G(\omega_n) < 0$.

Landau damping



- $|\Re G(\omega_n)| \gg |\Im G(\omega_n)|$ $\quad(\star)$
- $\text{sign}\, \Re G(\omega_n) \neq \text{sign}\, \Im G(\omega_n)$ $\quad(\star)$
- $\Re G(\omega_n) > 0$.

Lattice results



- Relaxation time approach ruled out. Landau damping possible.
- For $N_t = 12$ fits possible with $|n| \leq 3$.
- Smaller lattices give qualitative results but a fit is not possible.
- S_4 poles at $\omega/2\pi T \approx 0.15$ and $\phi \approx 0.2$.

Dynamical scales and phenomenology

1. A soft photon ($\omega \approx T$) emitted in the plasma is reabsorbed if its path length is

$$\ell = \frac{1}{\sigma} \approx \frac{1}{7C_{EM}T} \approx 3 \text{ fm}.$$

   Typical fireball dimensions at RHIC are a few fm, so the fireball is marginally transparent to soft photons. This may change at LHC.

2. Typical hadronic length/time scales in the plasma are

$$\tau \approx \frac{1}{7T} \approx 0.15 \text{ fm}.$$

   Hydrodynamic description of the final state in the plasma works if its thermalisation time is less than 1 fm. Therefore hydrodynamics may work at both RHIC and LHC.

3. Spontaneous thermal fluctuations of flavour even out by diffusion. The diffusion constants, D, are related to the electrical conductivity as

$$\sigma = \sum_f e_f^2 D_f \chi_f,$$

where χ_f is the thermal susceptibility for particle number. We find $D \approx \frac{1}{7T} \approx 0.15 \text{ fm}$, and hence the only visible chemical signals are those at freeze out. Strong implications for strangeness production.
R.V. Gavai and SG, Phys.Rev.D65:094515,2002

4. Jets traversing the plasma are quenched. This calls for high shear viscosity, η. Expect $\eta/S \approx 1/7(1 + c_s^2)$, *i. e.*, $\Gamma_s T \approx 1/7$. Try a computation of η in near future.

Summary

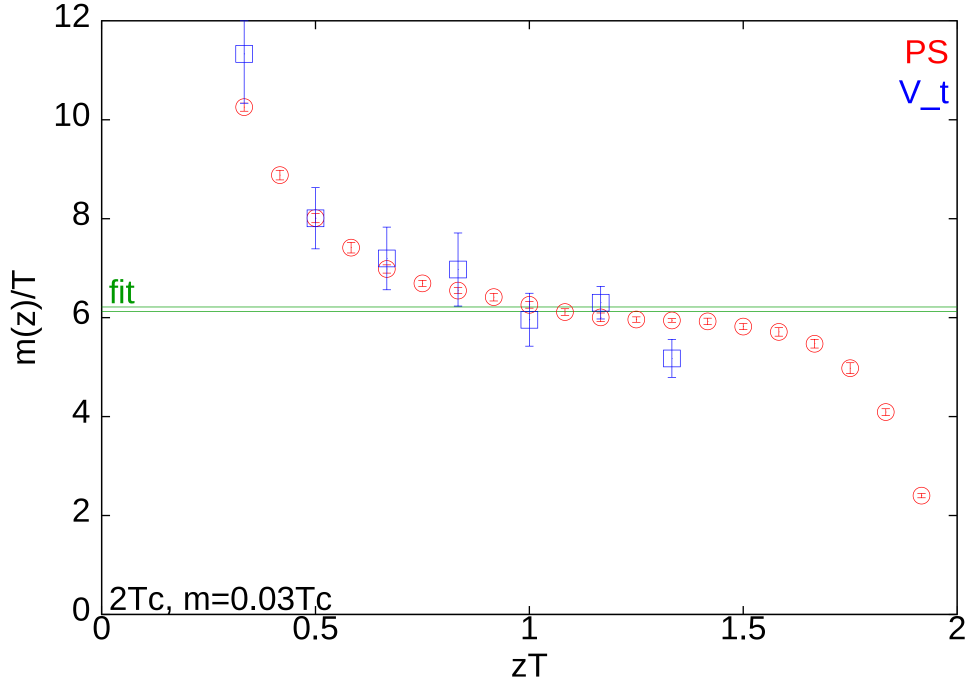
1. Screening masses close to free field theory, but shape of correlators strongly differs from this.

2. Lattice computations of temporal correlators contain too little information to parametrize near-equilibrium physics completely. Prior assumptions necessary.

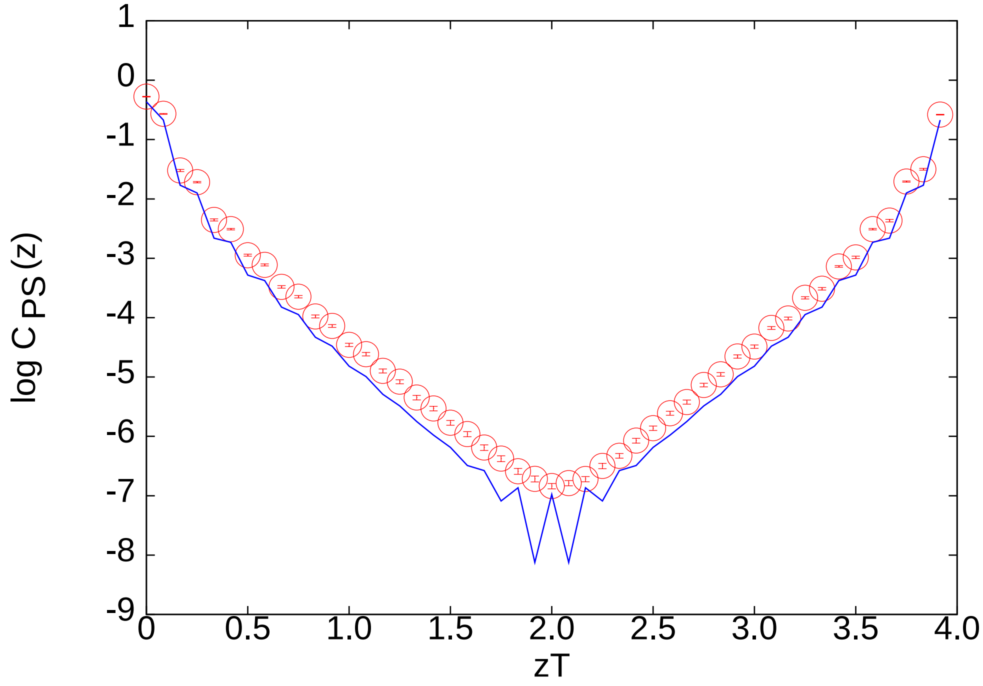
3. Analysis methods must try to identify or isolate important physical behaviour. MEM has done well in the large energy region. After subtracting the effect of the large energy regime, linear Bayesian methods show an additional bump in the small energy region.

4. A parametrised Bayesian method gives the electrical conductivity. Small time scale for transport seen. Parametrised forms check specific models of interactions and supports physics of Landau damping.

Screening masses



Screening correlators



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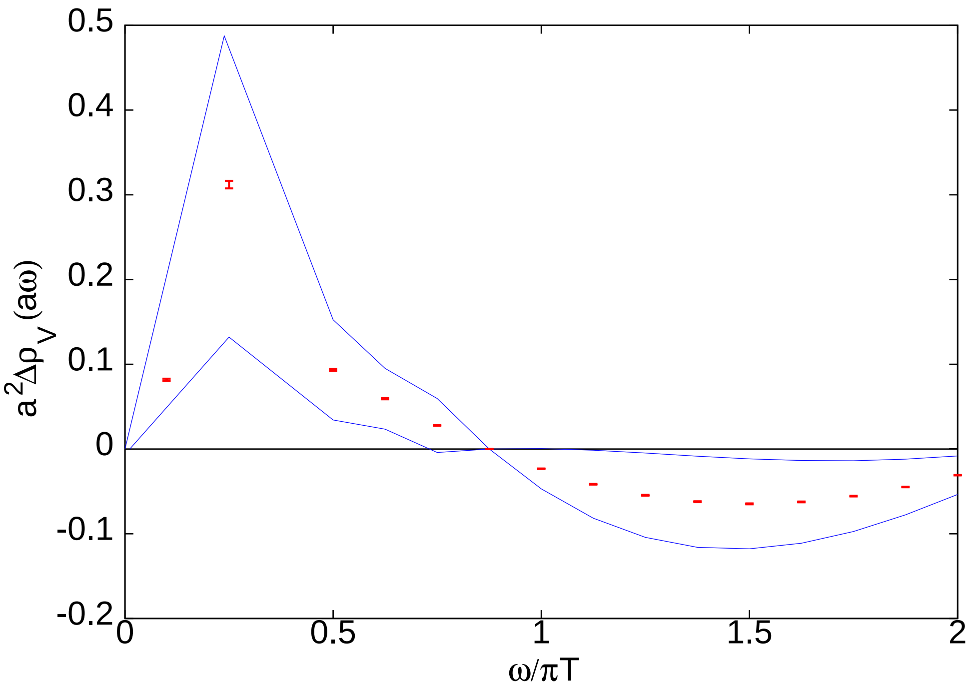

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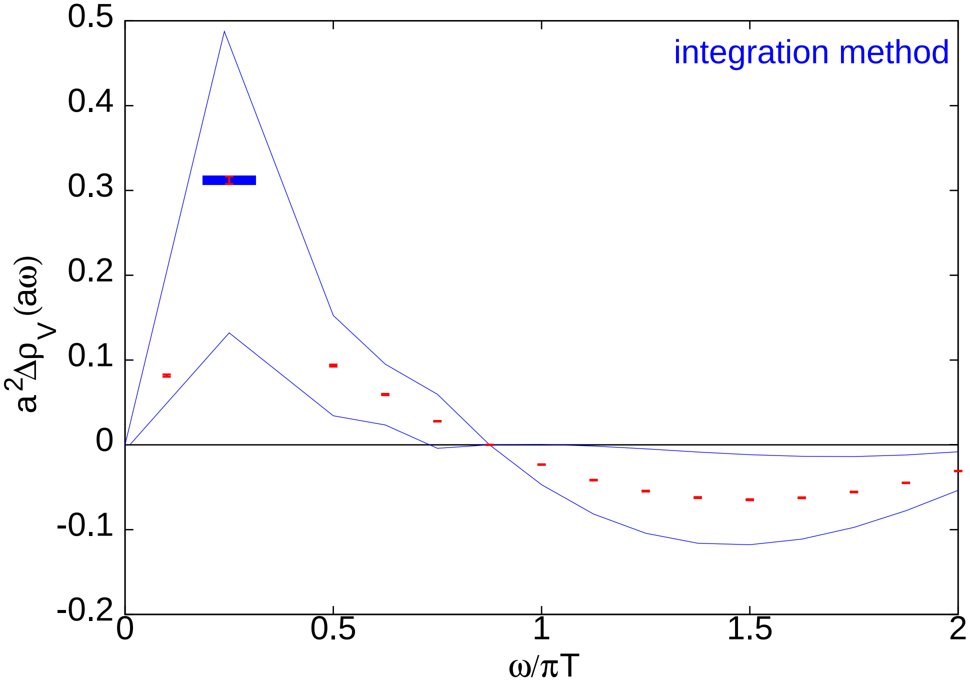

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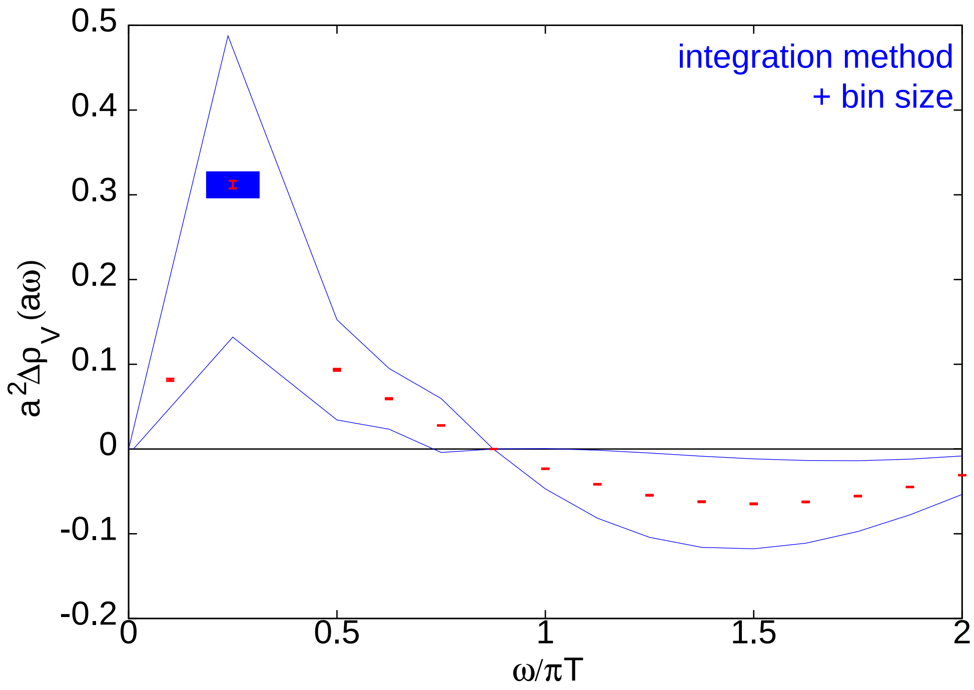

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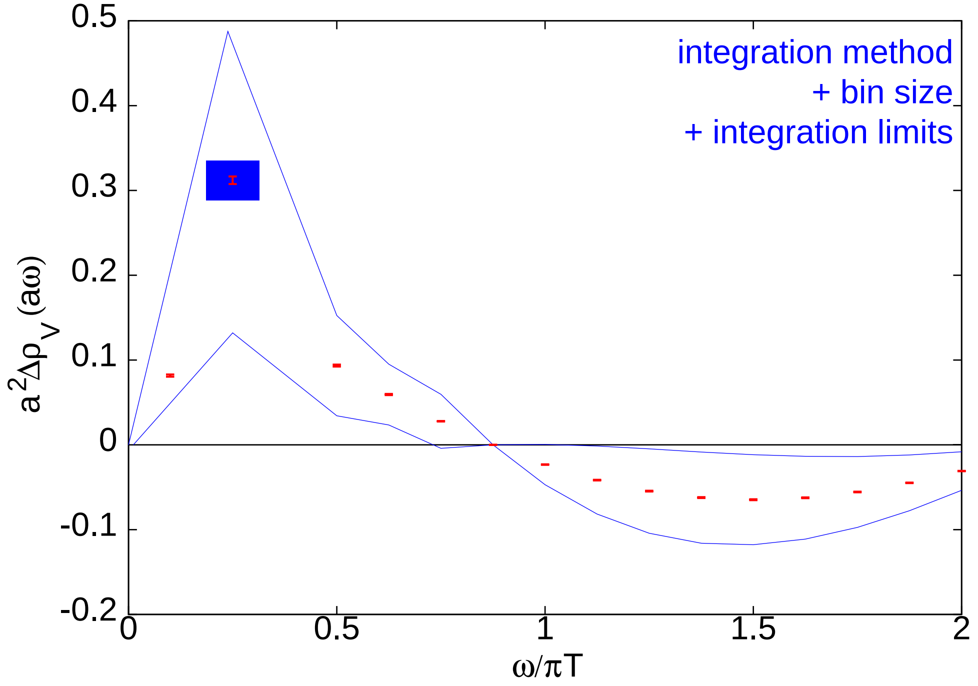

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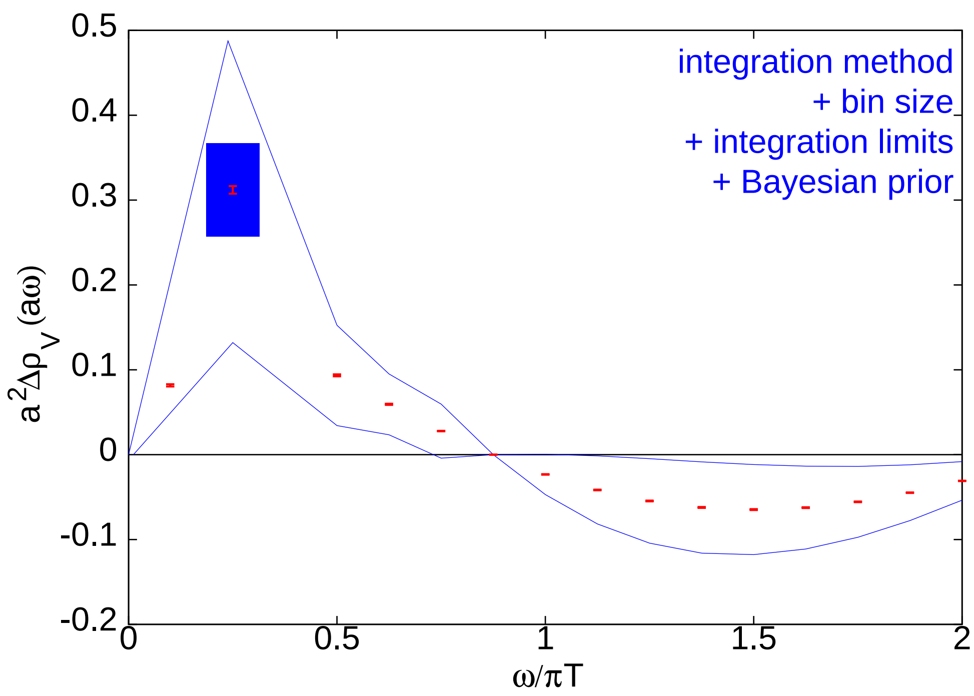

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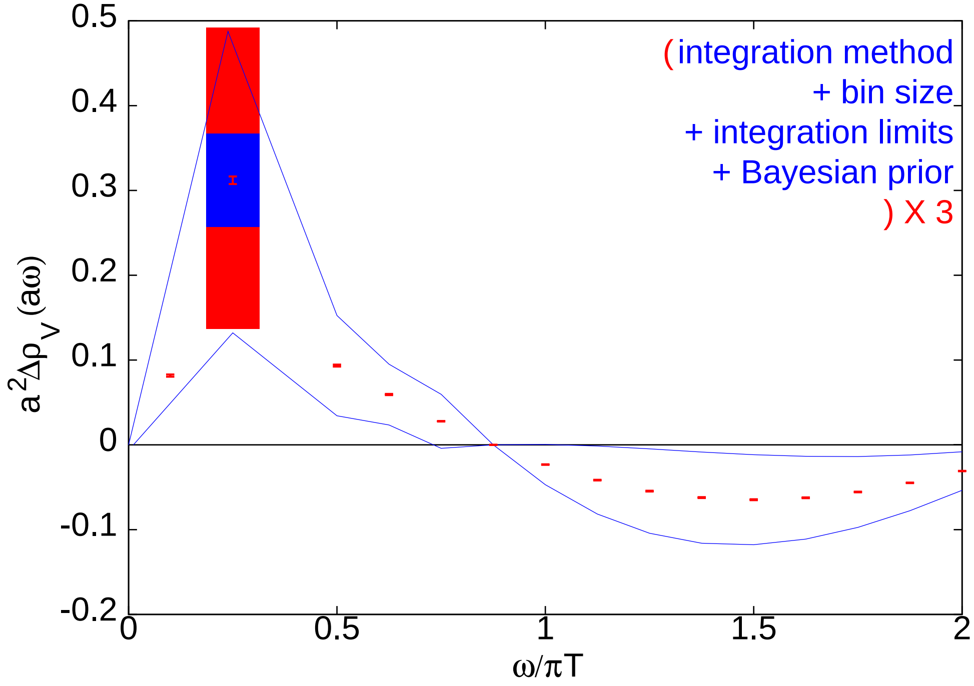
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