Monday seminar series

Sourendu Gupta (TIFR, Mumbai)
University of Bielefeld

Winter semester 2005–6

Lecture 1

Linkage

Some remarks on Dirac eigenvalues

1. Glue sector seems harder to understand than the quark sector. Weak coupling theory at fairly high order needed to reproduce lattice data on P; quantitative agreement at $T \geq 4$–T_c. NLO corrections in the quark sector suffice for reasonable agreement with quark number susceptibilities (deriatives of P) at $T \geq 2T_c$.

2. First indication of the relative simplicity in the quark sector was the observation that $\langle\overline{\psi}\psi\rangle$ vanishes in the high temperature sector in the chiral limit.

3. Second indication of the relative simplicity in the quark sector was the observation by the MT_c collaboration (1990) that screening masses in some quark-bilinear sectors are "almost ideal". Following work on screening masses in glue sector showed lots of structure.

4. More recently it was found that the sign problem at finite chemical potential is under better numerical control at high temperature than at $T = 0$. This is the reason behind the surprising initial succcess of Fodor and Katz.

5. At $T \ll T_c$, infrared structure of Dirac eigenvalues is complex: quark mass is the infrared cutoff (when the quantization volume is large enough: outside the ϵ region of chiral perturbation theory). As a result, the chiral limit explores the full complications of the glue sector ("topology on the lattice").

6. For $T \geq T_c$, the infrared cutoff (in the chiral limit) is provided by T. For exploration of the long-distance physics this makes the weak-coupling theory somewhat easier to handle.

Definitions

Introduce chemical potentials into the partition function by adding a term $J = \sum_f \mu_f Q_f = \mu^T \cdot Q$ to the Hamiltonian. Now $PV = -F = T \log Z(T, \{\mu_f\})$. Define

$$
\begin{aligned}
n_f &= \left(\frac{T}{V}\right) \frac{\partial \log Z}{\partial \mu_f}\bigg|_{\{\mu_f=0\}} \\
\chi_{fg} &= \left(\frac{T}{V}\right) \frac{\partial^2 \log Z}{\partial \mu_f \partial \mu_g}\bigg|_{\{\mu_f=0\}}
\end{aligned}
$$

Change ensemble by $Q' = MQ$, giving $J = (\mu')^T Q'$, where $\mu' = (M^{-1})^T \mu$, and

$$
\frac{\partial}{\partial \mu'_i} = \frac{\partial \mu_j}{\partial \mu'_i}\frac{\partial}{\partial \mu_j} = (M^T)_{ji}\frac{\partial}{\partial \mu_j} = M_{ij}\frac{\partial}{\partial \mu_j}.
$$

Evaluation of off-diagonal susceptibilities

$$\mathrm{Tr}\, A = \mathrm{Tr}\, AI = \mathrm{Tr}\, A \left[\frac{1}{2} \sum_r |r\langle\rangle r| \right] = \overline{\langle r|A|r\rangle}.$$

The quantities $A(r)$ are (roughly) Gaussian distributed.

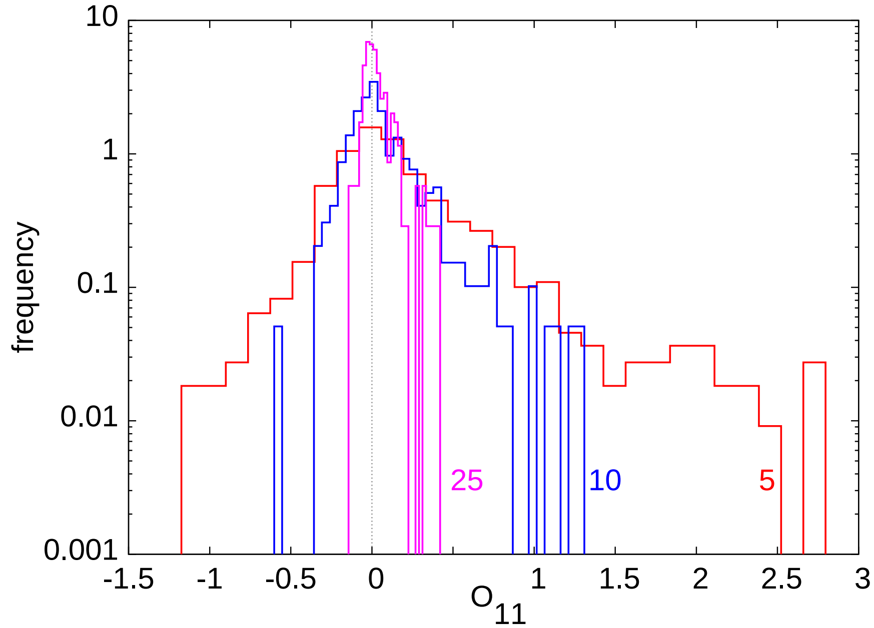
$$\langle \mathrm{Tr}\, A \mathrm{Tr}\, B\rangle = \left\langle \overline{\langle r|A|r\rangle\langle s|B|s\rangle} \right\rangle,$$

but product of Gaussian distributed numbers is not Gaussian distributed.

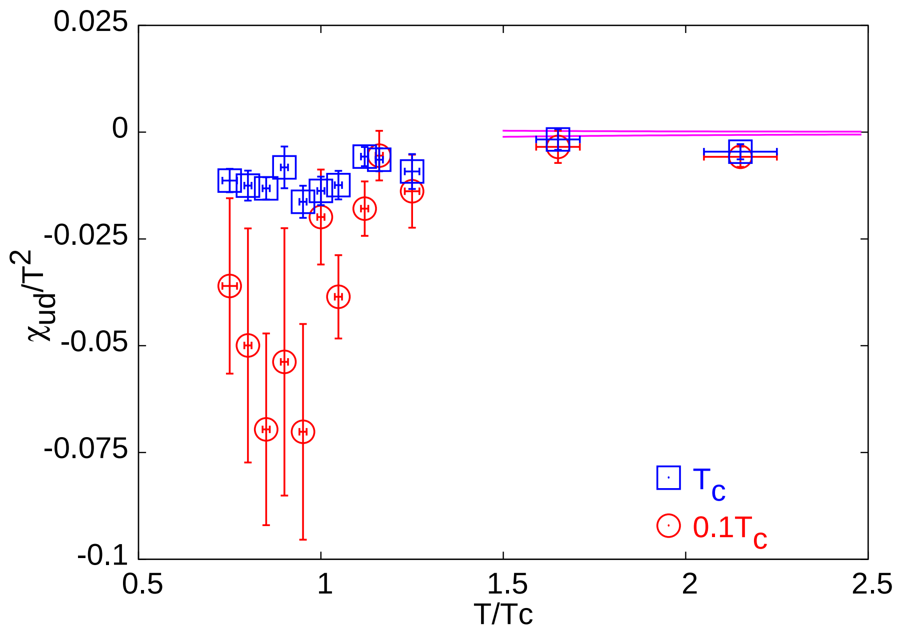
$$\int dz f(z) = \int dz dx dy \delta(xy - z) \exp(-x^2/2) \exp(-y^2/2) \propto dz \exp(-z).$$

More terms in the product gives stretched exponential. Long tailed distribution takes more statistics to evaluate. Similar to critical slowing down.

Off-diagonal distribution



Off-diagonal susceptibilities



Two flavour

Change ensemble from $\{u, d\}$ to $\{B, I_3\}$. Then

$$\chi_B = \frac{2}{9}\left(\chi_u + \chi_{ud}\right), \quad \chi_I = \frac{1}{2}\left(\chi_u - \chi_{ud}\right), \quad \chi_{BI} = \frac{1}{2}\left(\chi_u - \chi_d\right) = 0.$$

If $\Delta_{ud} = m_u - m_d$ then $\chi_{BI} = \chi_d' \Delta_{ud} + \chi_d'' \Delta_{ud}^2 + \cdots$. χ_d' is one of the QRC's studied in SG and R. Ray, PR D 70 (2004) 114015.

Change ensemble from $\{u, d\}$ to $\{B, Q\}$. Then

$$\chi_B = \frac{2}{9}\left(\chi_u + \chi_{ud}\right), \quad \chi_Q = \frac{1}{9}\left(5\chi_u - 4\chi_{ud}\right), \quad \chi_{BQ} = \frac{1}{9}\left(\chi_u + \chi_{ud}\right).$$

Three flavour

Change from $\{u, d, s\}$ to $\{B, I_3, Y\}$. Then

$$\chi_B = \frac{1}{9}\left(2\chi_u + \chi_s + 2\chi_{ud} + \chi_{us} + \chi_{ds}\right), \quad \chi_I = \frac{1}{2}\left(\chi_u - \chi_{ud}\right),$$
$$\chi_Y = \frac{2}{9}\left(\chi_u + 2\chi_s + \chi_{ud} - 4\chi_{us}\right), \quad \chi_{BY} = \frac{2}{9}\left(\chi_u - \chi_s + \chi_{ud} - \chi_{us}\right),$$
$$\chi_{BI} = \frac{1}{2}\left(\chi_u - \chi_d + \chi_{us} - \chi_{ds}\right), \quad \chi_{IY} = \frac{1}{2}\left(\chi_u - \chi_d - 2\chi_{us} + 2\chi_{ds}\right).$$

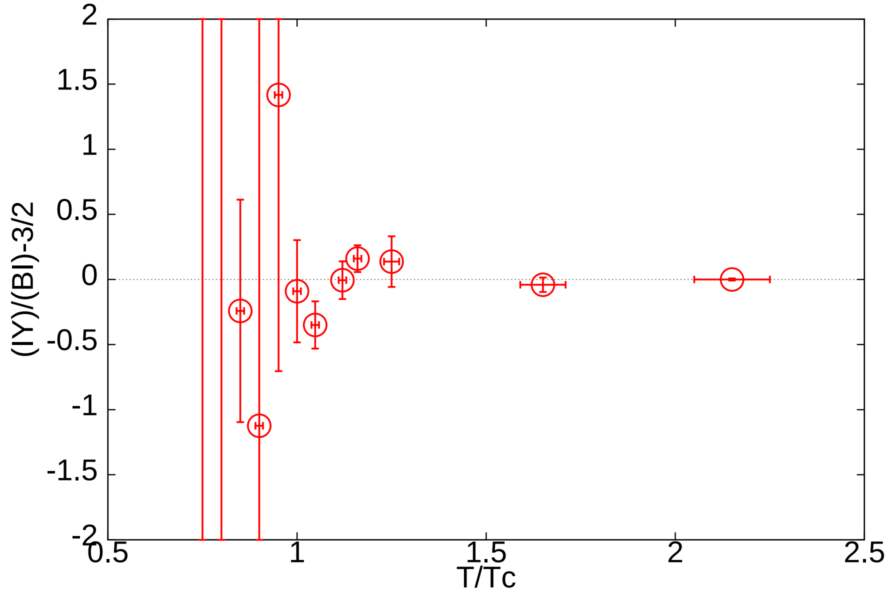
For small Δ_{ud}

$$\frac{\chi_{IY}}{\chi_{BI}} \approx \frac{3}{2}\left[1 - \frac{\chi'_{ds}}{\chi'_d}\right].$$

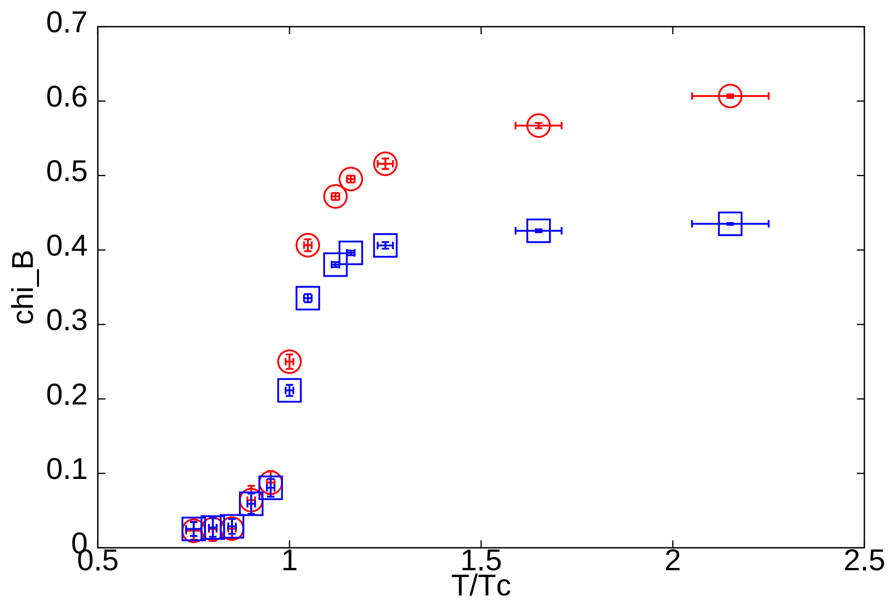
For small Δ_{us} (high temperature limit)

$$\chi_B = 2\chi_I/3 = \chi_Y/2.$$

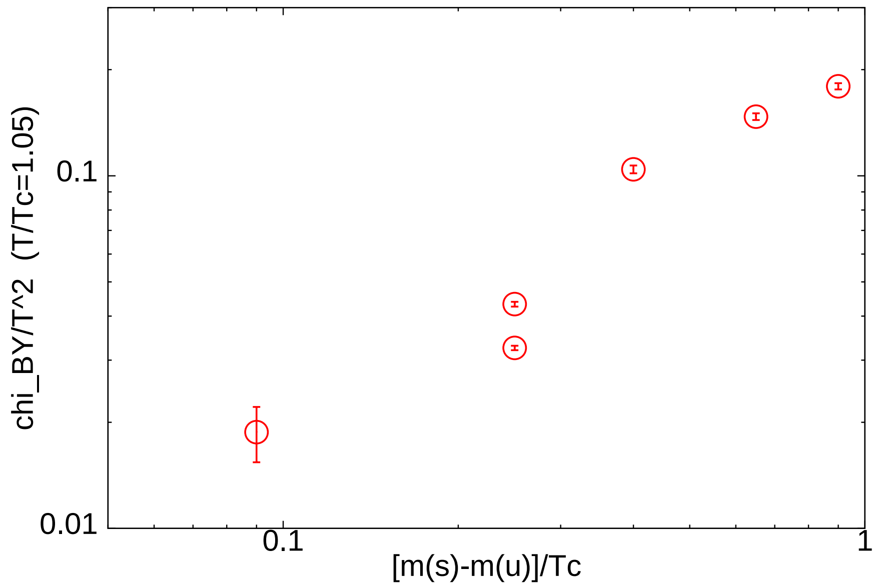
Isospin breaking



Importance of valence strange quark

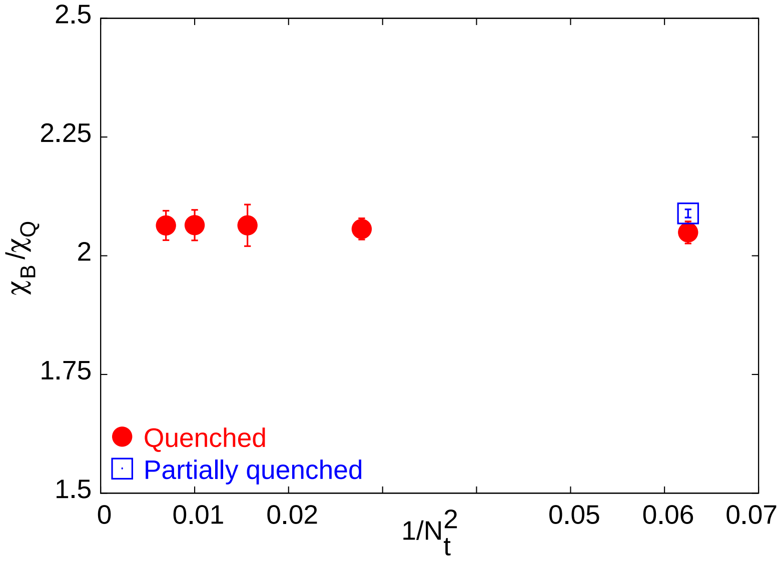


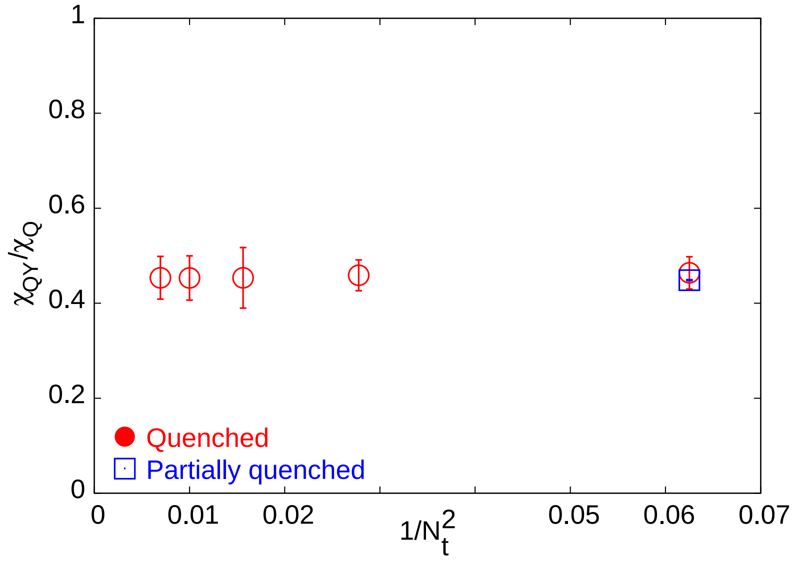
Strange quark mass is large



Easier to deal with this as a kinematic effect.

Robust variables





Sea quark mass effects negligible above T_c.

Linkage

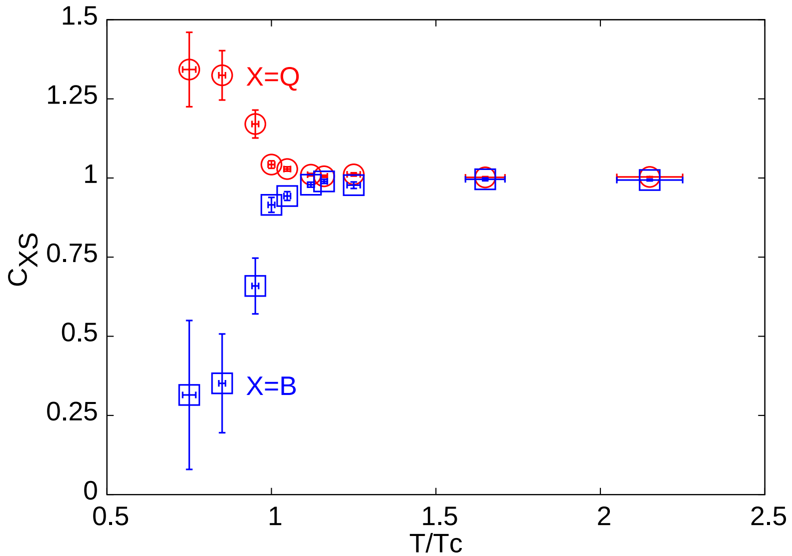
Observe the simultaneous occurrence of two quantum numbers in thermodynamic equilibrium—

$$Z = \sum_{\{Q_i\}} \langle Q_i | \, \mathrm{e}^{-\beta H} \, | Q_i \rangle \, .$$

What are the effective degrees of freedom in the high temperature phase? If some particle states dominate the sum, then these particle states carry linked quantum numbers Q_i.

Observe the linkage through robust variables: χ_{XY}/χ_X.

Strangeness is in quarks



$$C_{BS} = -3\frac{\chi_{BS}}{\chi_S} = -3\frac{\langle BS\rangle - \langle B\rangle\langle S\rangle}{\langle SS\rangle - \langle S\rangle^2}.$$

$$C_{QS} = 3\frac{\chi_{QS}}{\chi_S} = 3\frac{\langle QS\rangle - \langle Q\rangle\langle S\rangle}{\langle SS\rangle - \langle S\rangle^2}.$$

Flavours are decoupled