

# Lattice QGP

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# Plan

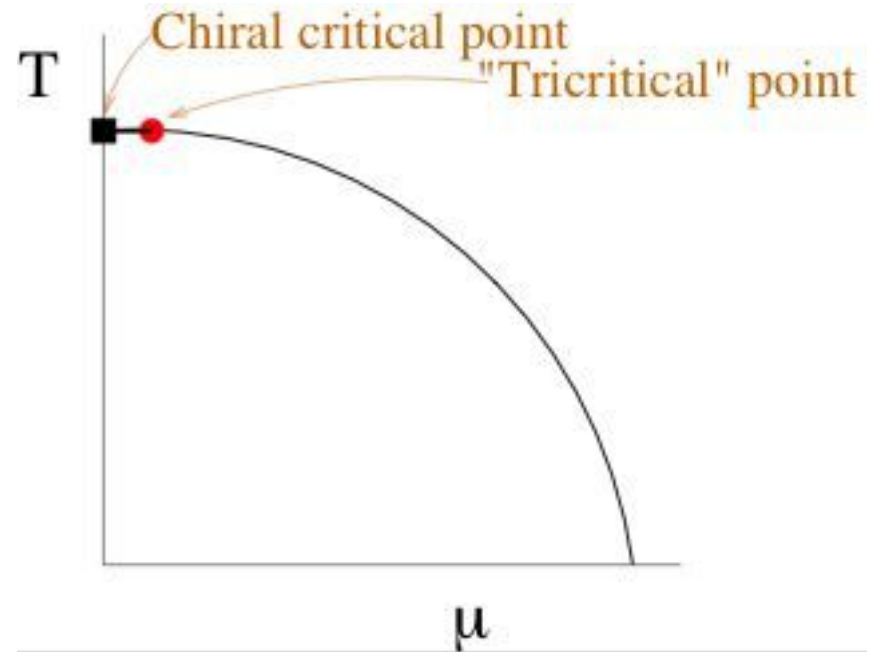
- The crossover temperature: BiBC ([hep-lat/0608013](#)), Wuppertal ([hep-lat/0609068](#))
- The QCD phase diagram: de Forcrand and Philipsen ([hep-lat/0607017](#)), Mumbai ([hep-lat/0412035](#))
- What the plasma is made of: Mumbai ([hep-lat/0510044](#))
- The equation of state: MILC ([hep-lat/0611031](#)), BIKR ([hep-ph/0611393](#))
- The melting of  $J/\Psi$
- Transport coefficients:  $\eta$  and  $\sigma$
- Screening masses and fluctuation measures

$$N_f=2$$

**Phase  
diagram**

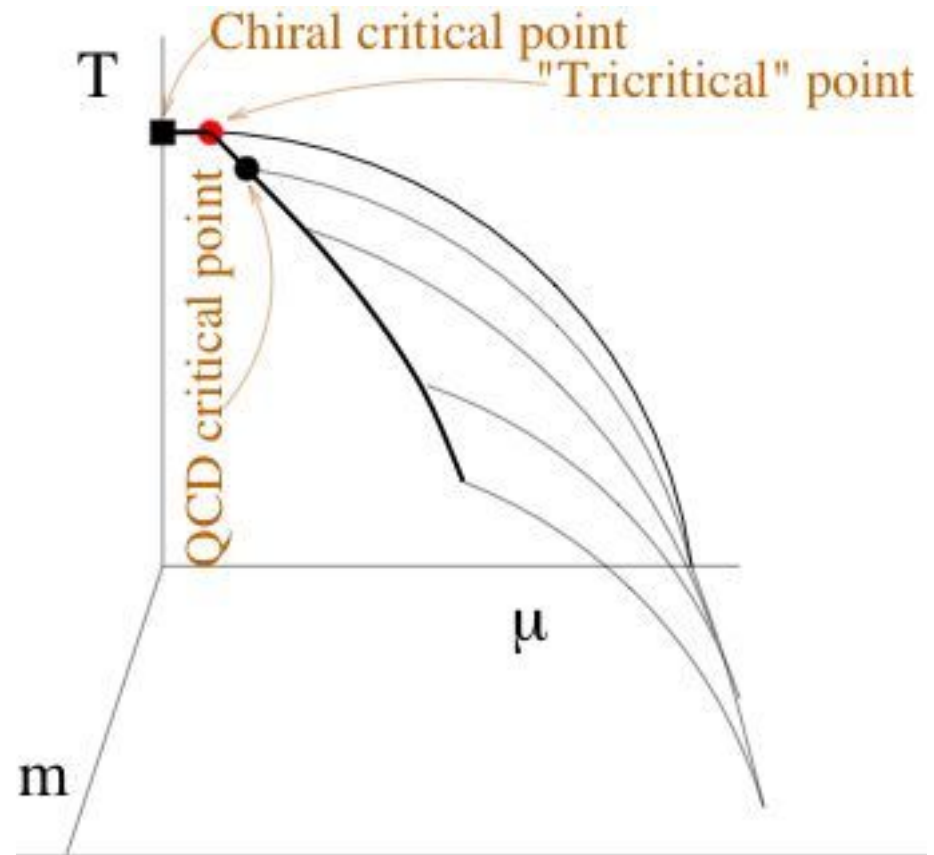
# Friendly neighbourhood phase diagram

- QCD phase diagram for two chiral flavours:  
 $\mu = m = 0$
- 2nd order transition at  $m = 0$  and TC develops into a critical line; turns into 1st order line at the tricritical point
- Each point of a phase diagram is an unique phase.



# Friendly neighbourhood phase diagram

- QCD phase diagram for two flavours has another direction: the light quark mass  $m$
- For any  $m \neq 0$ , there is no phase transition at  $m=0$ : only cross over (no divergences or discontinuities)
- Tricritical point develops into a critical point.



$$N_f = 3$$

**Phase  
diagram**

# Crossover in continuum

- Wuppertal ([hep-lat/0609068](https://arxiv.org/abs/hep-lat/0609068)) computation with fat link staggered quarks:  $N_t=4,6,8,10$
- Scale setting using  $f_K$  and Sommer scale  $r_0$  using realistic strange quark masses (ie,  $m_K/f_K$  and pion mass  $m_\pi/m_K$ ), pion 2 times heavier
- Spatial sizes range from 3 to  $6.7/m_\pi$ , which are reasonable. [Statistics on the smaller side.](#)
- Different indicators of phase transition do not agree. Typical of crossover.  $T_c=151\pm3\pm3$  MeV using  $\chi_m$  and [shifted by  \$28\pm5\pm1\$  MeV](#) using  $\chi_L$ .

# A different estimate

- RBBiC ([hep-lat/0608013](#)) compute with  $N_t=4$  and 6,  $m_{ss}/m_\pi=1.3$  instead of 3.6 (pion 3 times heavier than physical) using p4 action.
- Spatial size of lattice between  $3/m_\pi$  and  $6/m_\pi$  and [2/3 as large for  \$N\_t=6\$](#) .
- MD trajectories are short: autocorrelations could be more than twice as long as Wuppertal (if the action were the same). [Decreased effective statistics](#).
- Scale set by  $r_0$ , leads to  $T_c=192\pm 7\pm 4$  MeV.

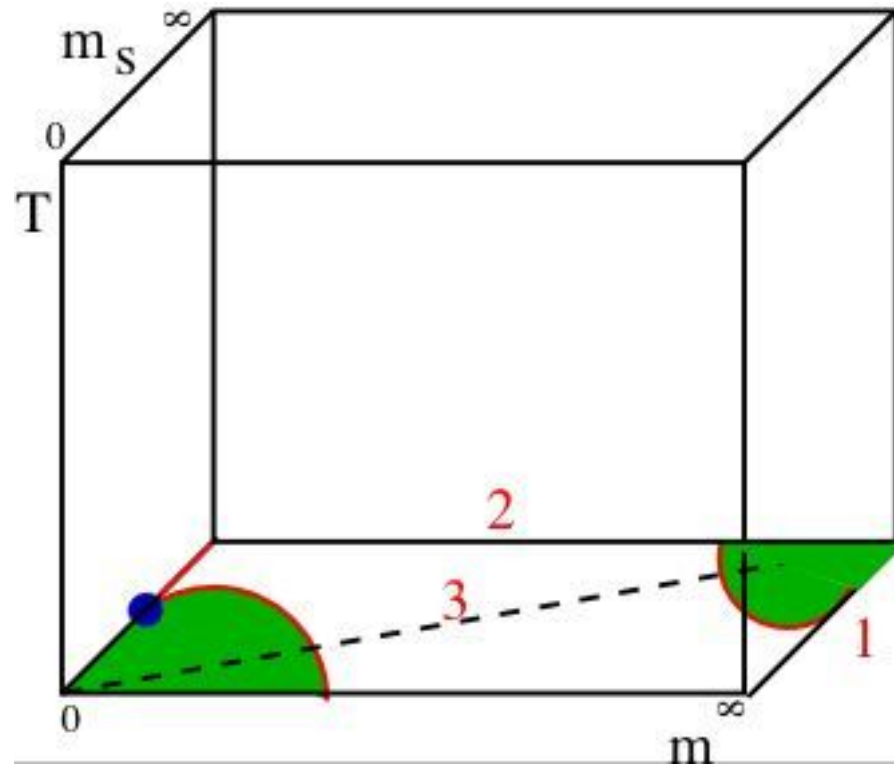


# Previous results

Old estimate by Bi ([hep-lat/0012023](#)):  $T_c = 173 \pm 8$  MeV using  $\sqrt{\sigma}$  to set the scale.

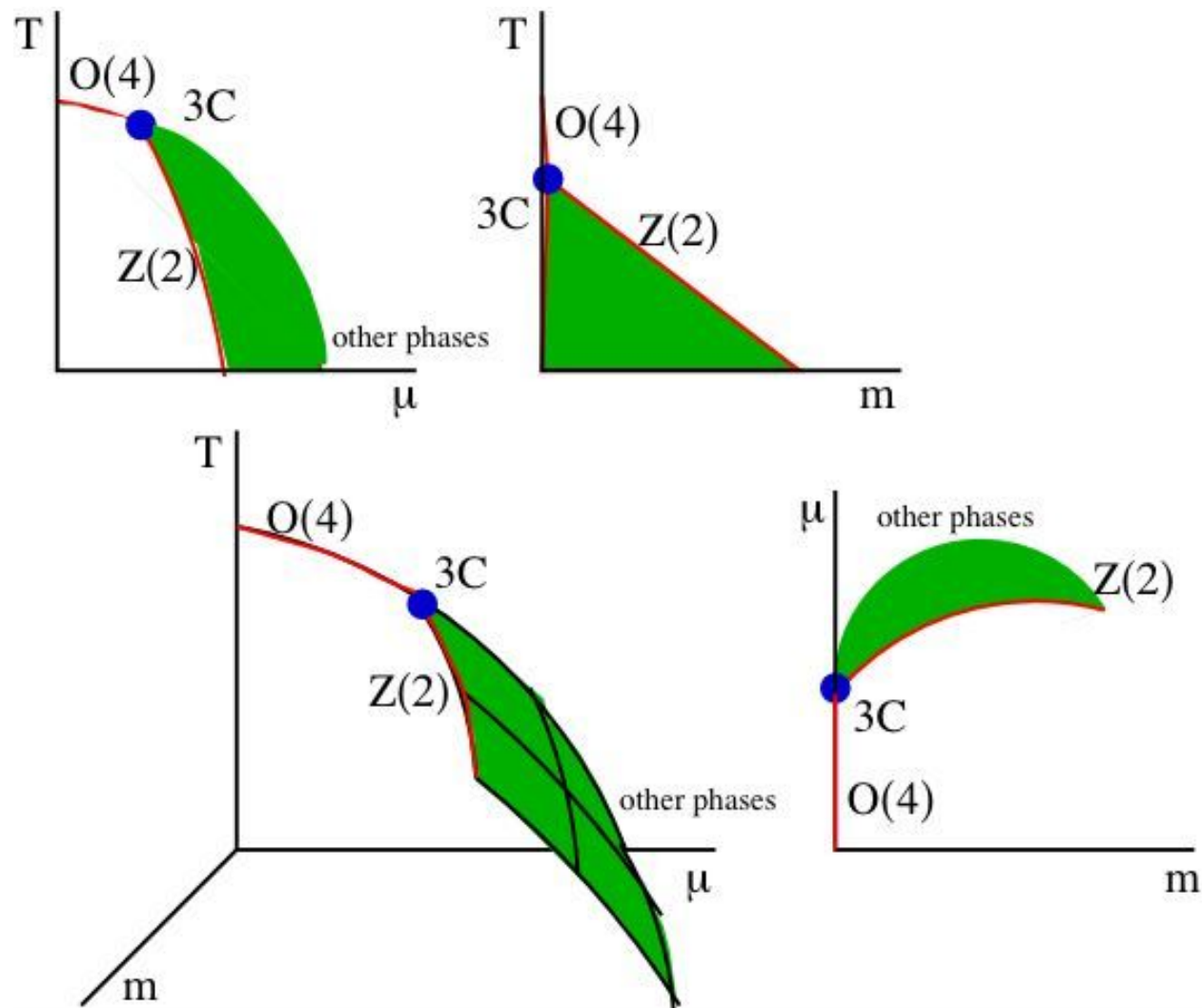
- Global analysis of  $N_f=2$  data from many collaborations (SG, [hep-lat/0010011](#)) gave  $T_c = 167^{+15}_{-14}$  MeV. Main uncertainty due to scale setting. Used  $\Lambda_{\text{QCD}}$  to set the scale.
- RBBiC quotes a 10% upward revision of  $\sqrt{\sigma}$  when  $r_0$  is used to set the scale.
- Wuppertal result is within 10% (below) old estimate. Claimed to be due to the continuum limit.
- At the moment, 40% uncertainty in  $\varepsilon$ .

# The phase diagram

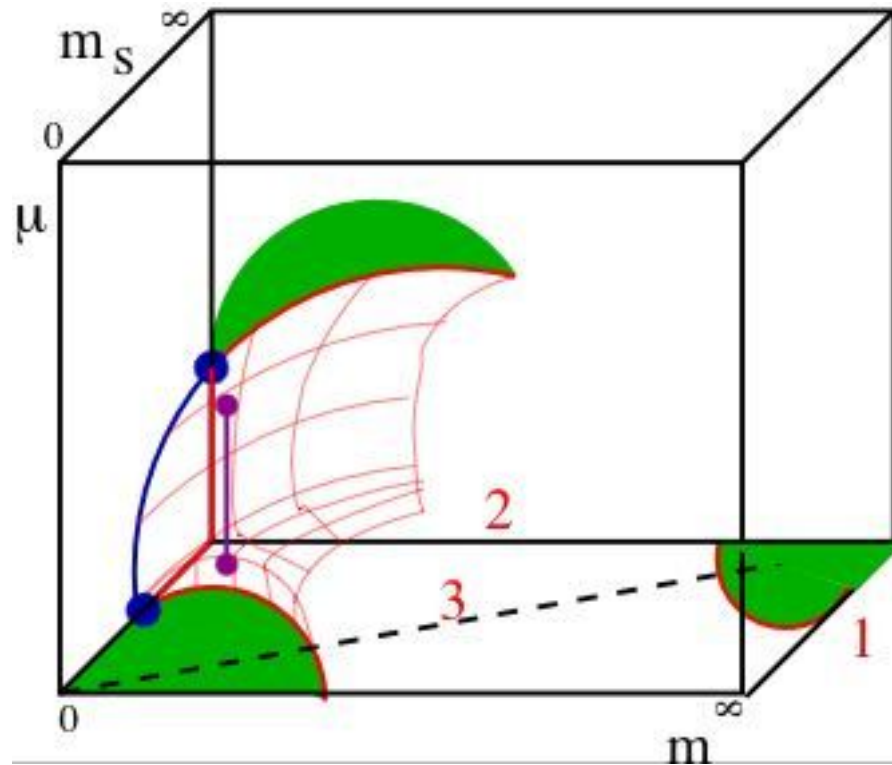


4 dimensional phase space:  $m$ ,  $m_s$ ,  $T$ ,  $\mu$ .  
Project to one less dimension: cut out  $T$

# Funny things happen

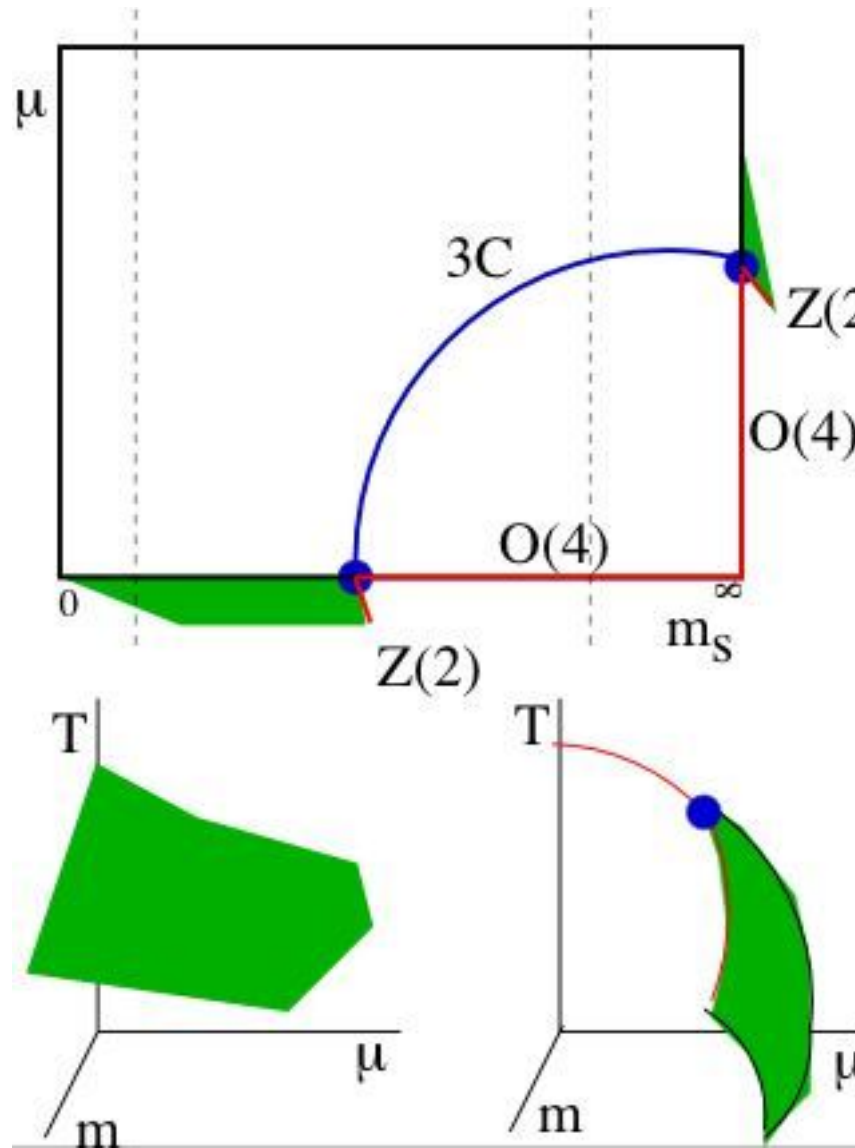


# Another dimension



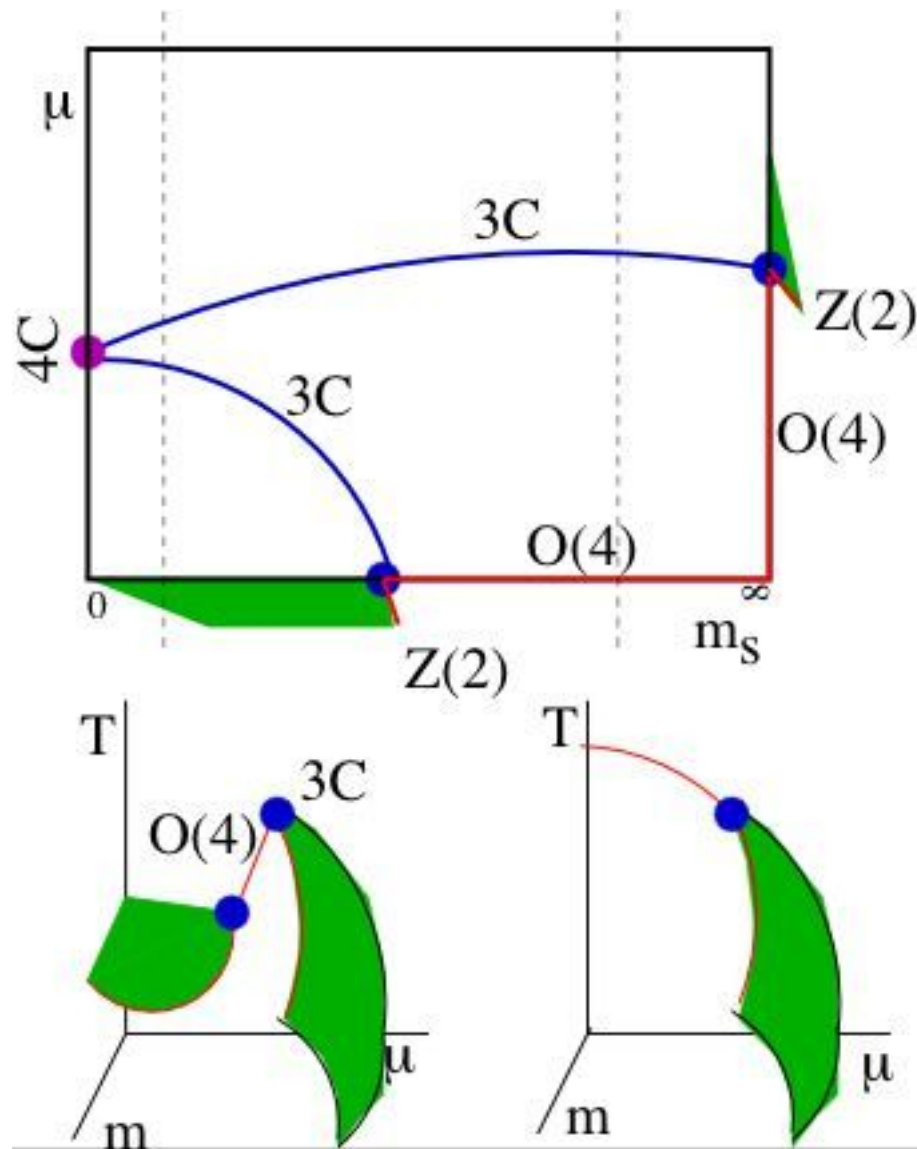
Two  $O(4)$  critical lines, two  $Z(2)$  critical lines, and two tricritical points organize the phase diagram.

# The tricritical line



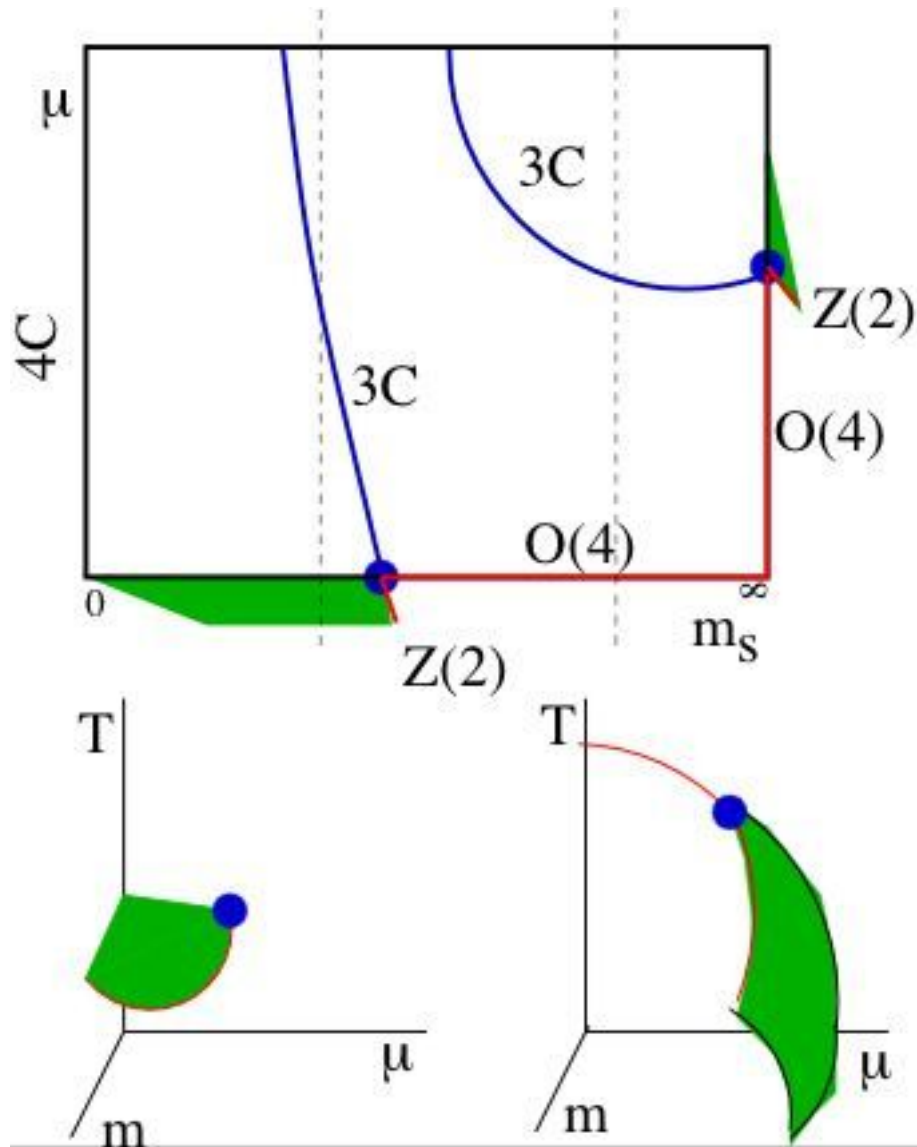
**Standard scenario.**  
 3C line meets  $\mu=0$  plane at a point which separates “large  $m_s$ ” from “small  $m_s$ ”. Above this, the  $N_f=2$  phase diagram is accurate. Dividing line must be  $m_s \approx \Lambda_{\text{QCD}}$ .  
 Accurate location of 3C point: future.

# A tetracritical point



Two 3C lines meet at a 4C point. But this is a point of higher symmetry. In this phase diagram it can only appear on the  $m_s = 0$  line. **4C point in  $N_f = 3$  is a new universality class.** Can be ruled out.

# Two tricritical lines



4C point can be avoided by taking the end points of the two 3C lines to infinity. Then phase diagram is **qualitatively similar to  $N_f=2$** , but **quantitatively very different**.  
Not ruled out.  
Effective models?

# On the method

- deF&P determine the critical line in the  $\mu=0$  plane using staggered quarks (2+1 flavours)
- Lattices are very small:  $Lm_\pi \approx 2$ , large finite volume corrections can be expected
- Lattices are coarse, lattice spacing artifacts?
- Repeated the simulations with  $\mu/T=2.4i$  and found that the critical line remains fixed.
- Negative slope inferred using non-perturbatively defined RG running. Careful!
- Made a linear fit to data!



# Where is the QCD critical point?

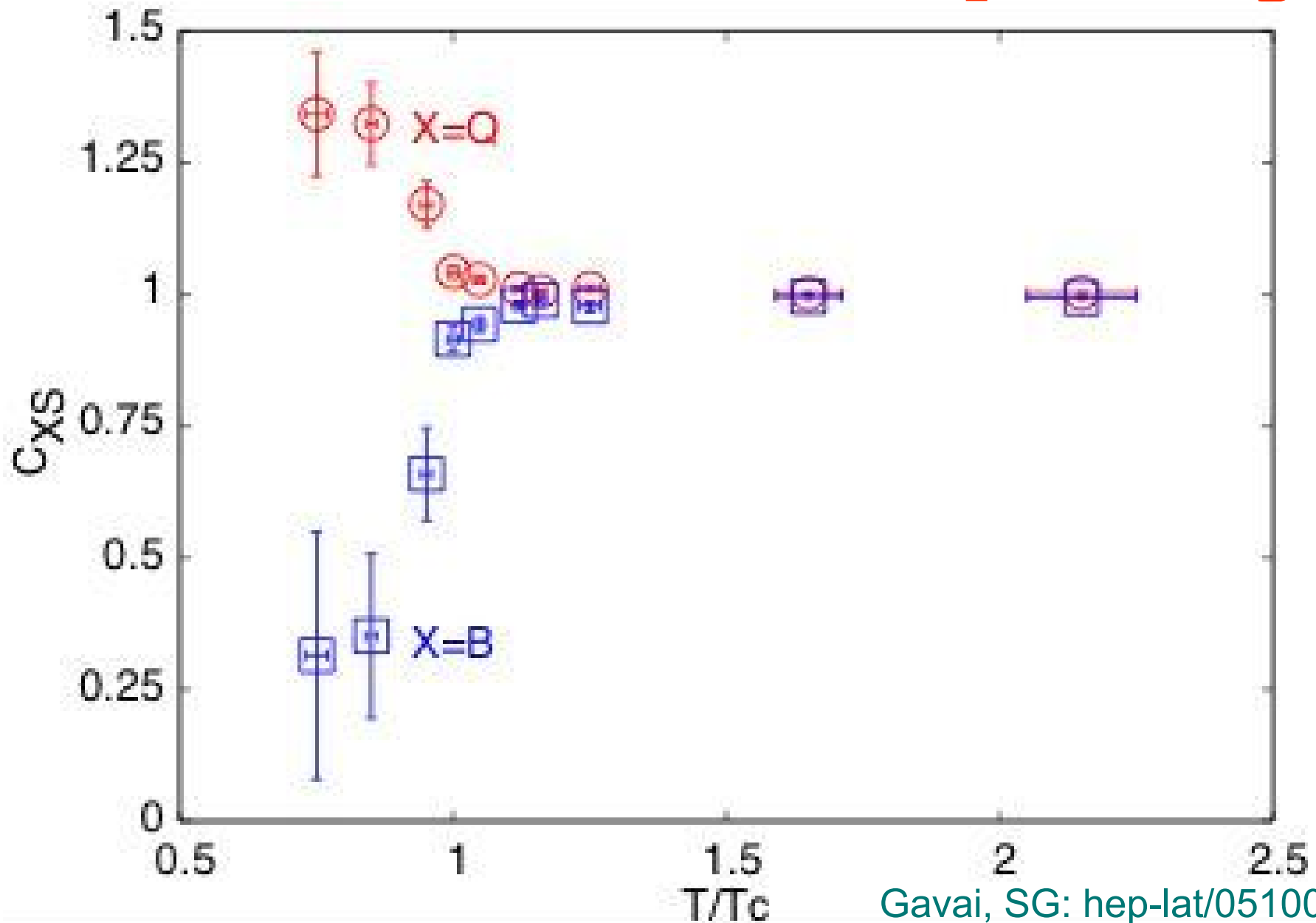
- In the phase diagram shown, there is always a QCD critical point (missed by deF&P). Forced by the fact that  $SU(3)_{\text{flav}}$  is always broken when  $m_s > m$ . (3C lines organize the phase diagram)
- If  $m_s$  is large enough ( $> \Lambda_{\text{QCD}}$ ), then  $N_f=2$  is a good quantitative guide to the real world. Is this true in our universe?
- Quantitative estimates for critical end point in  $N_f=2+1$  needed in future.
- For now, estimates for  $N_f=2$  from Mumbai are that  $\mu_e/T_e \sim 1$  and  $T_e/T_c \sim 0.95$

**What is the  
plasma made  
of?**

# Flavour carriers in the plasma

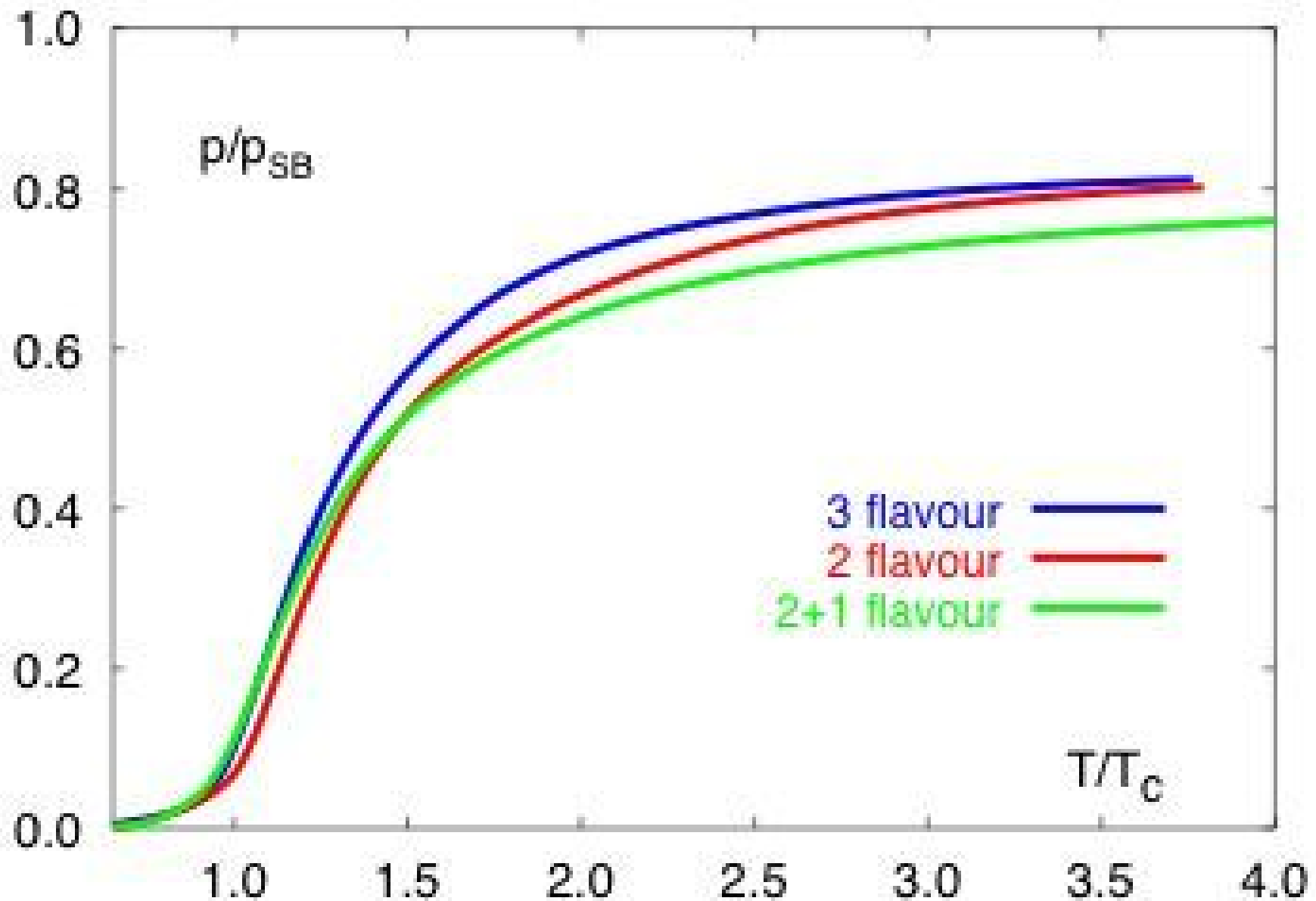
- Look for linkage of two flavour quantum numbers to find light degrees of freedom.
- If K meson then  $S=1$  comes with  $3B=0$ , and  $3Q=(3+0)/2=3/2$ . If s quark then  $S=1$  comes with  $3B=-1$  and  $3Q=1$ .
- Use fluctuation dissipation theorem to connect these linkages with quark number susceptibilities.
- Examine  $C_{BS}=-3L(B,S)$ ,  $C_{QS}=3L(Q,S)$  and  $C_{ud}=L(u,d)$  to look for free quarks.

# Freedom and equality



# The equation of state

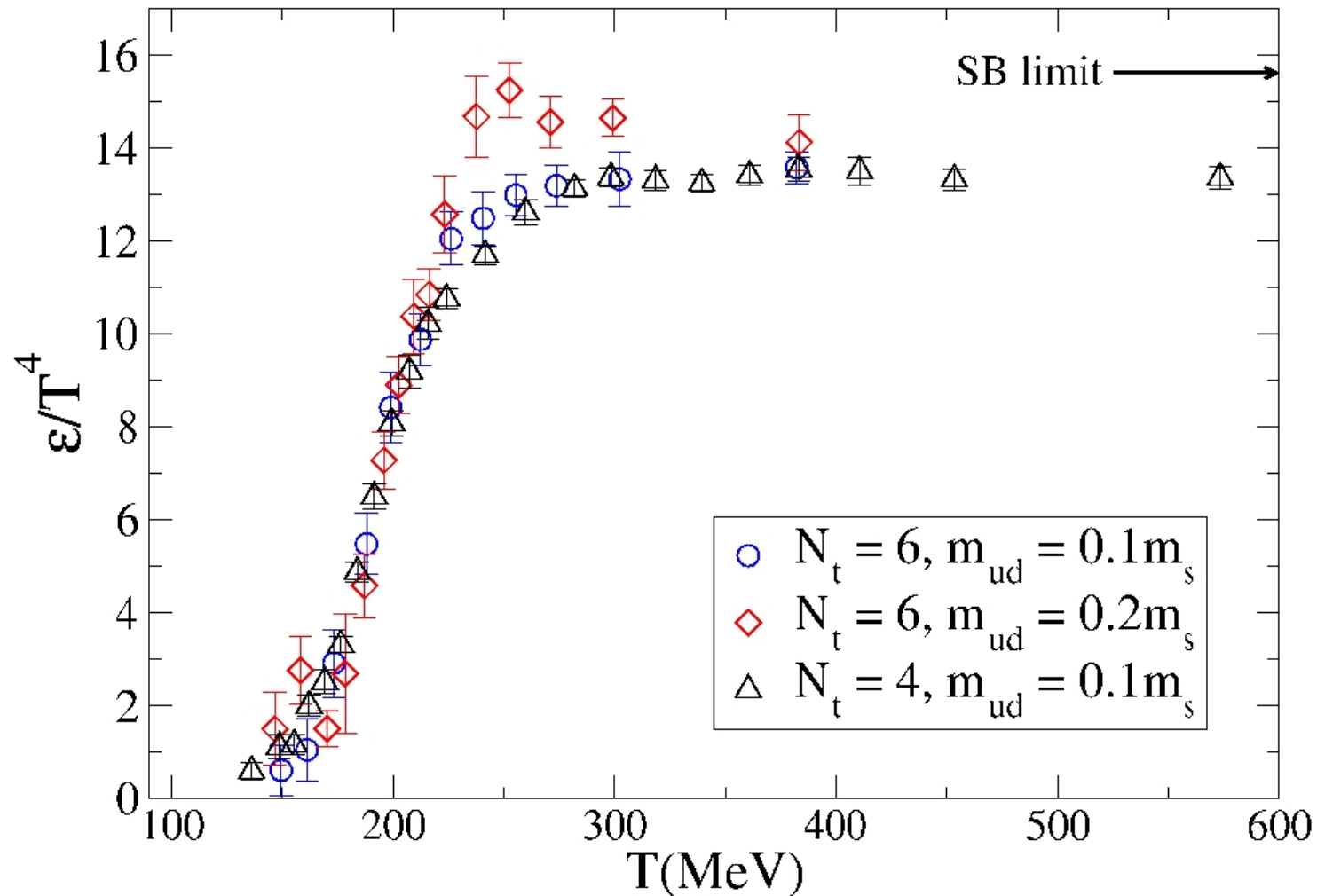
# Equation of state: circa 2001



# Equation of state: the history

- Lattice computations with quenched QCD showed large departure from ideal gas
- Interaction effects computed in weak coupling have trouble reproducing this data. Dimensional reduction, resummed perturbation theory do better.
- Strong coupling computation using AdS/CFT in toy model called N=4 supersymmetric QCD with  $N_c = \infty$ . (Note: quarks are adjoint in colour).
- New developments: fermions, weak coupling

# New lattice results on EOS



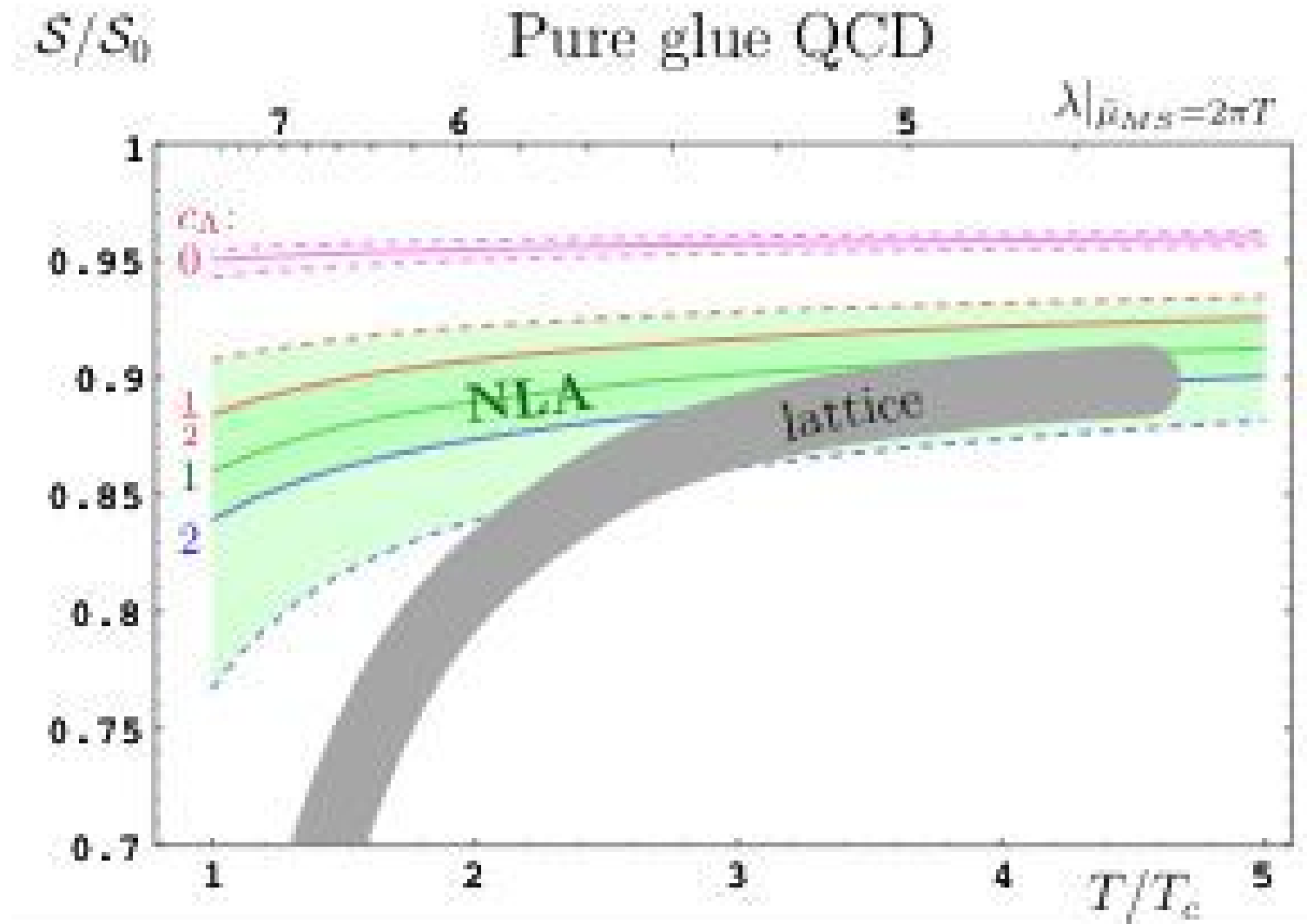
$N_f=2+1$  with improved staggered quarks. (MILC  
hep-lat/0611031)



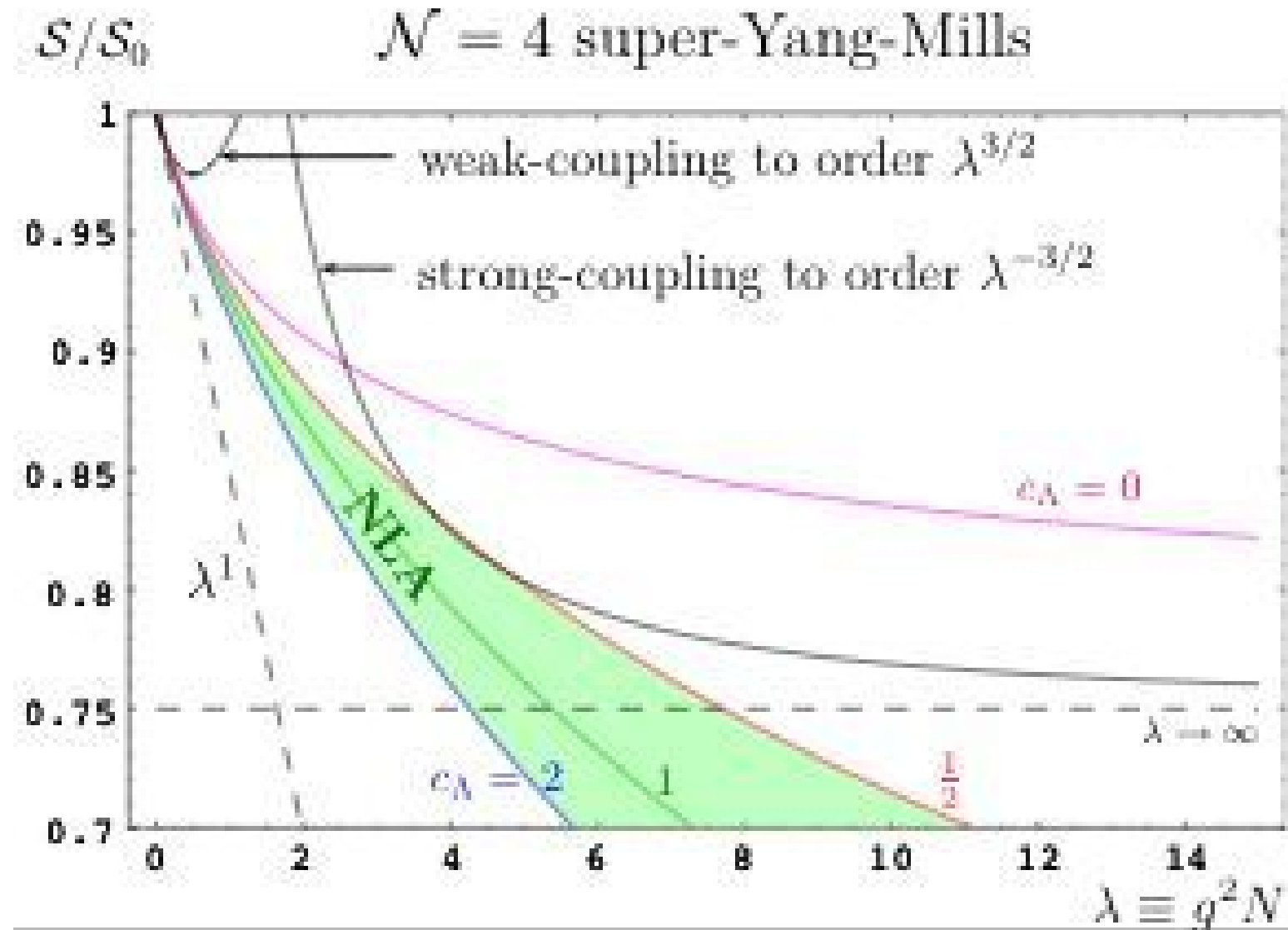
# N=4 SYM in weak coupling

- Blaizot-Iancu-Rebhan treat  $s$  (entropy density) in N=4 SYM in weak coupling, then adjust 't Hooft coupling to match lattice data
- Old Gubser-Klebanov strong coupling result is:  $s/s_{SB} = 3/4 + c/\lambda^{3/2}$ , where  $c$  is a known number.
- Compare strong and weak coupling. They agree at  $\lambda$  which is stronger than that required to explain the data for  $T > 3T_c$
- Small window in  $T$  left for agreement with N=4 SYM with  $N_c = \infty$  (too close to  $T_c$  the mass scale spoils conformal symmetry: lattice data)

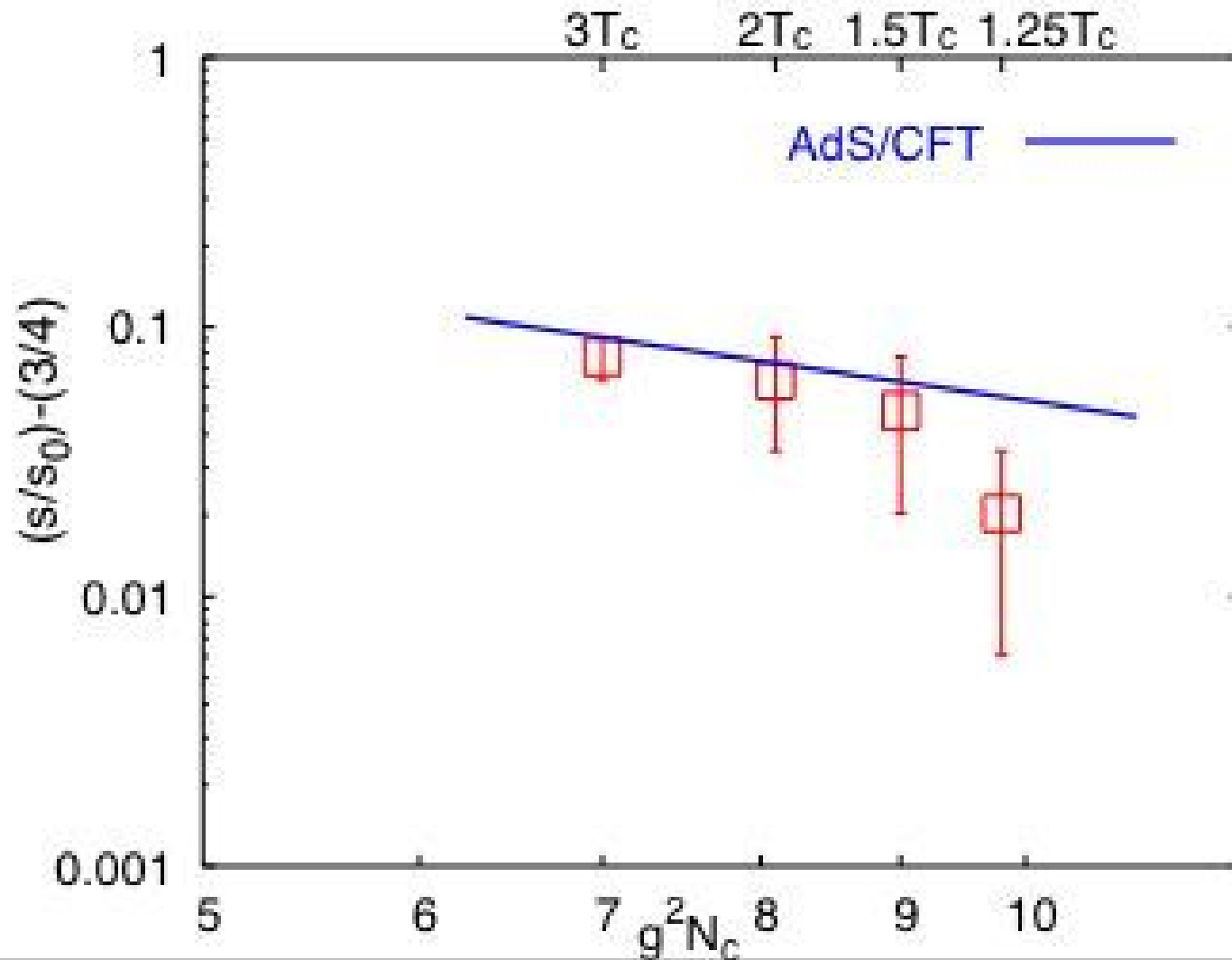
# N=4 SYM in weak coupling



# N=4 SYM in weak coupling



# N=4 SYM in strong coupling



# Conclusions

- $N_f=2$  QCD under reasonable control. **No phase transition for  $m>0$  and  $\mu=0$ .** Crossover temperature  $T_c$  fixed within 10%. **First estimates of QCD critical point available.**
- $N_f=2+1$  QCD under investigation.  **$T_c$  fixed within 10%.** Phase diagram constrained by deF&P as well as known results on  $N_f=2$  QCD because **order parameter of  $N_f=2$  QCD is also an order parameter of  $N_f=2+1$  QCD.** Global constraints on flag diagram forces critical point for  $\mu>0$ .
- **Quarks are liberated!** First direct proof.
- **EOS under control.** Window for AdS/CFT (strong coupling computation) shrinking.