Thermal effects on localized eigenvectors in QCD

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New directions in non-perturbative QCD, ECT* Trento

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Quarks in the QCD plasma, screening phenomena, Dirac eigenvalues, localization of Dirac eigenvectors.

Default collaborators: Gavai, Lacaze (Thanks: IFCPAR)

Plan

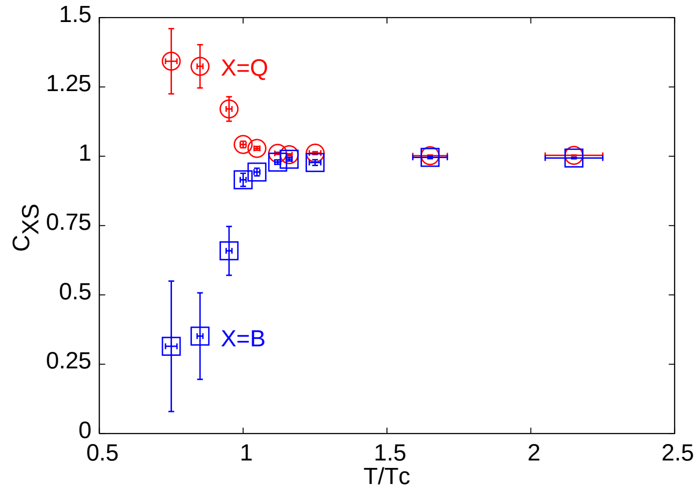
1. Evidence for unbound quarks in the plasma: QNS and (overlap) screening correlators.

2. The spectrum of Dirac eigenvalues: temperature dependence and effective theories.

3. Localization of Dirac eigenvectors: measures of localization, and stability of localized eigenvectors.

Quarks in the plasma

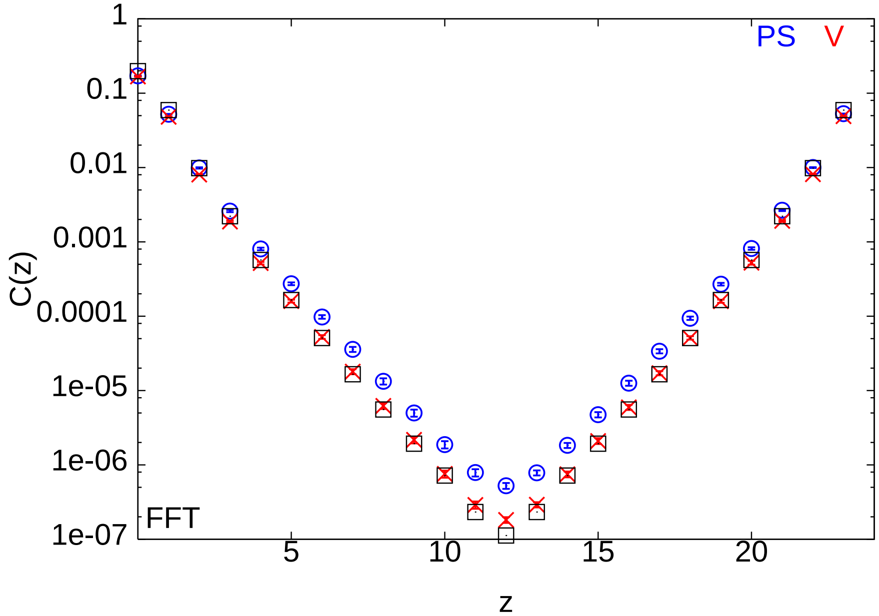
Look for **linkage** between flavour quantum numbers of excitations in the plasma. Linkage of two quantum numbers P and Q is the thermal expectation value of P when unit value of Q is excited: $C_{P|Q} = \chi_{PQ}/\chi_Q$.

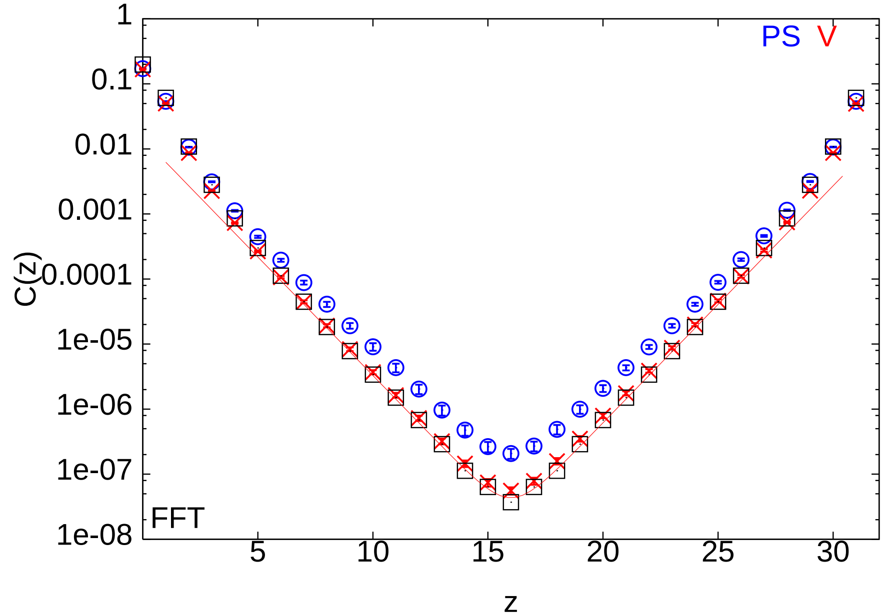


$C_{BS} = -3C_{B|S}$ and $C_{QS} = 3C_{Q|S}$. Note very rapid crossover to value expected of quarks above T_c. Weak coupling fails very close to T_c.

Gavai and SG, Phys. Rev. D

Screening correlators

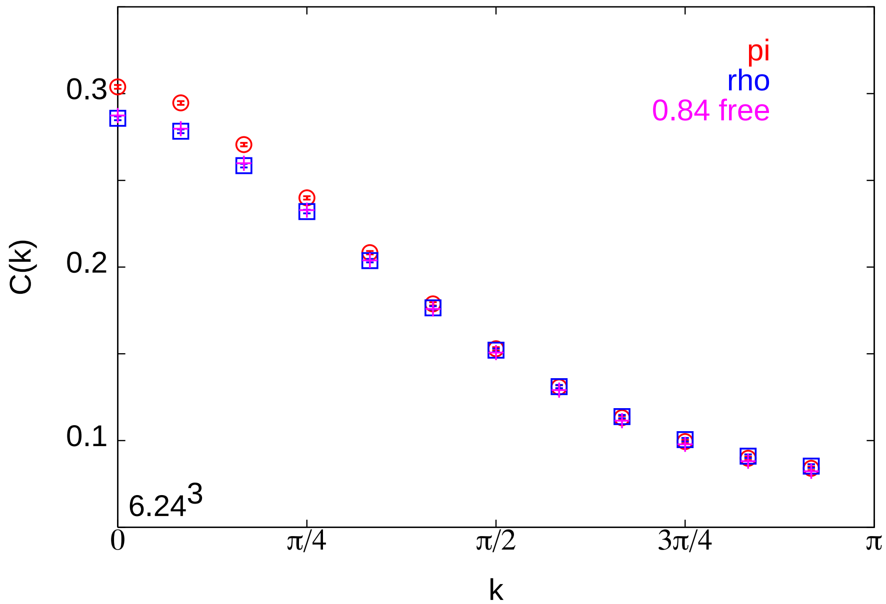


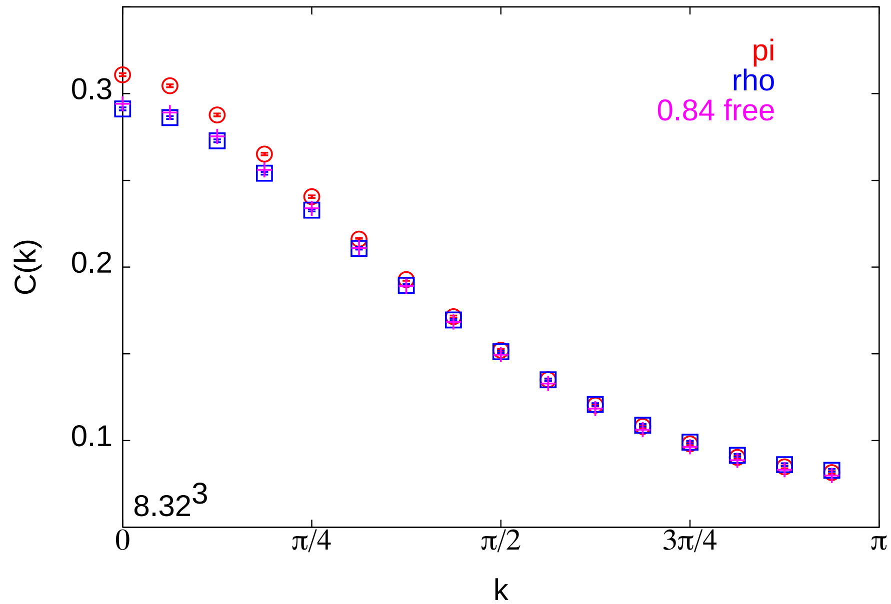


$N_s = 4/T$; $a = 1/6T$ and $a = 1/8T$; overlap. Fit to Bose gas good only over limited range of z, so do not quote screening masses. Vector correlator close to ideal gas of quarks. Is this a good starting point for weak coupling theory?

Pseudoscalar correlator badly described by ideal quark gas at long distances and by Bose gas at short distances. Does not seem to be a finite lattice spacing artifact.

Screening correlators in momentum space

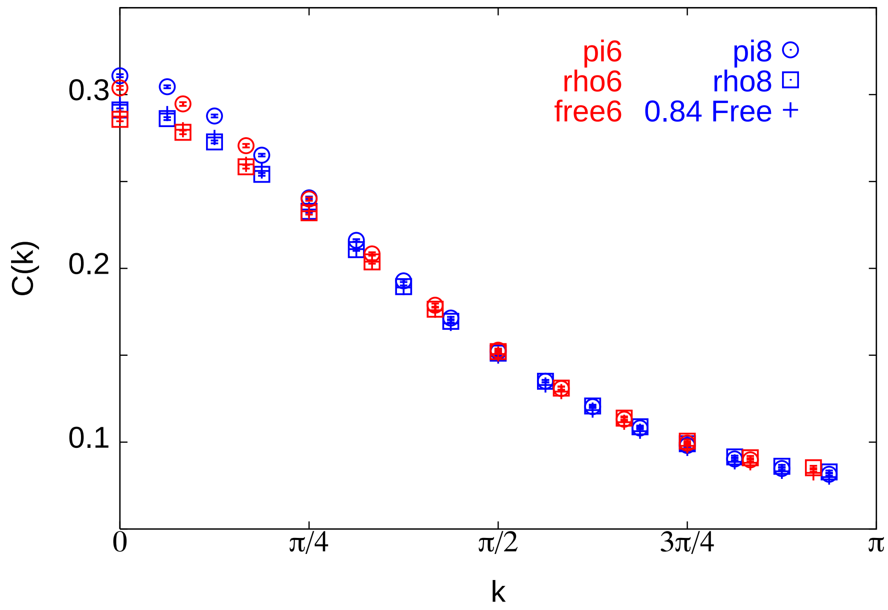




Vector screening masses roughly compatible with ideal quark gas upto overall normalization. $\chi^2/\mathrm{DOF} \approx 4$, implying shape change is a small effect but necessary. Try weak coupling expansion?

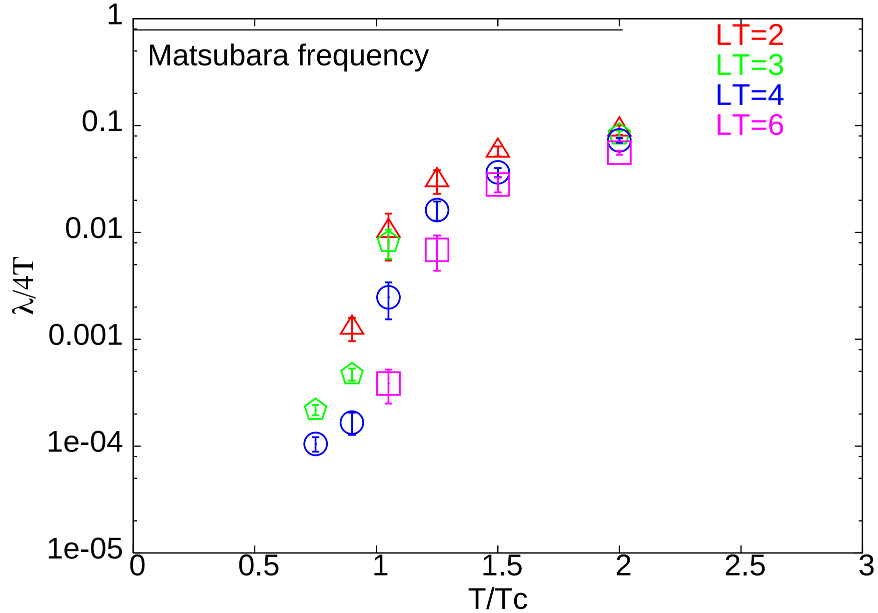
Pseudoscalar is definitely different at small momentum, but agrees at large momentum.

Lattice spacing effects in momentum space correlators



Some remnant lattice spacing dependence— going in the direction of the pion less compatible with weak coupling theory!

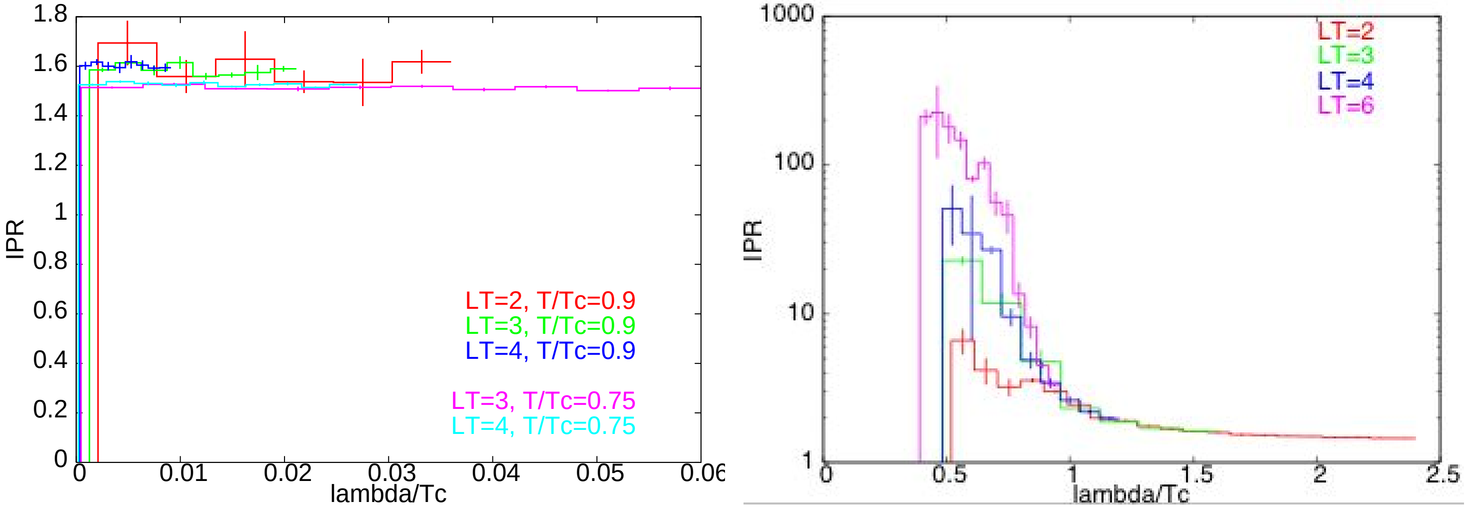
Lowest Dirac eigenvalues



The Banks-Casher formula and observations of the variation of $\langle\overline{\psi}\psi\rangle$ with T imply that a spectral gap should open up at finite temperature. Data (for staggered $N_f = 2$, $a = 1/4T$, varying spatial volumes) shows that this occurs slowly.

As $V \to \infty$, the most rapid change seems to occur after T_c. Crossover from chiral effective theory to dimensional reduced effective theory?

Localization of eigenvectors

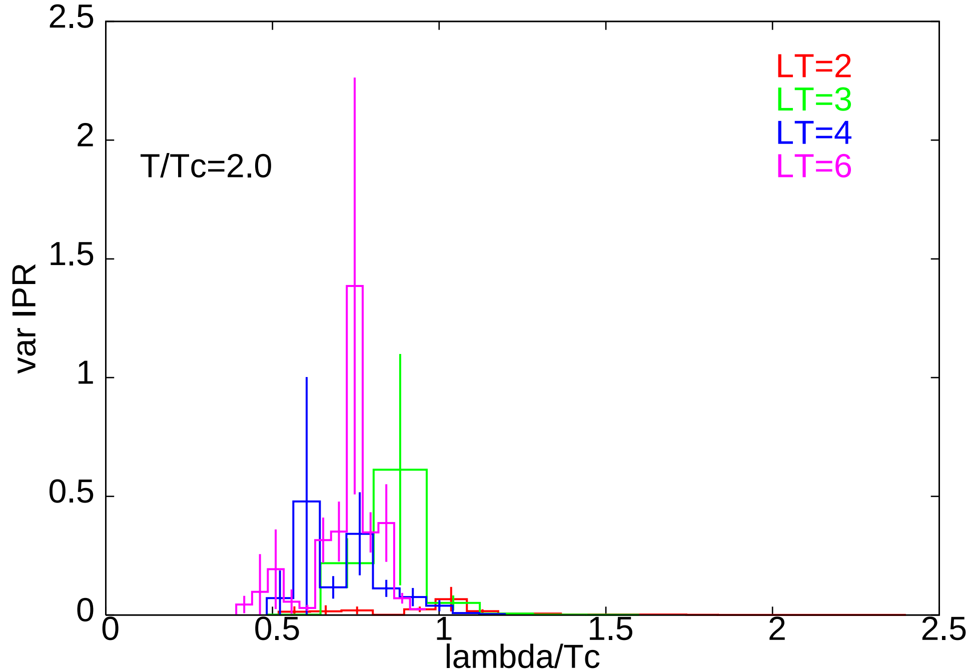


The local moments of normalized Dirac wavefunctions, $\psi_\alpha(r)$ are defined to be—

$$P_n^\gamma = V^{n-1} \sum_r p_\gamma^n(r), \quad , p_\gamma(r) = \sum_{\alpha\beta} \psi_\alpha^*(r) \gamma_{\alpha\beta} \psi_\beta(r),$$

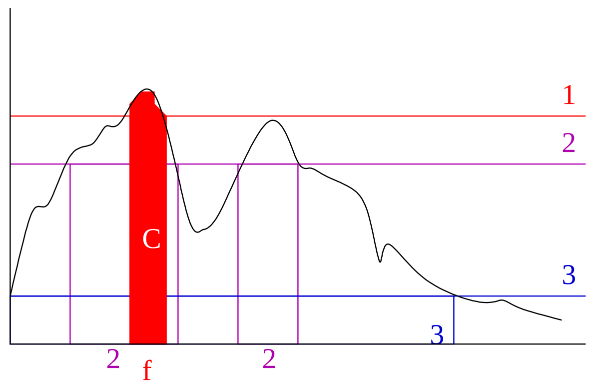
where α and β are Dirac-flavour indices. The case $n = 2$ is called Inverse Participation Ratio.

Large fluctuations



Strong config-to-config fluctuations in localization observed, $\Delta P_2/P_2 = \mathcal{O}(1)$.

Another notion of localization

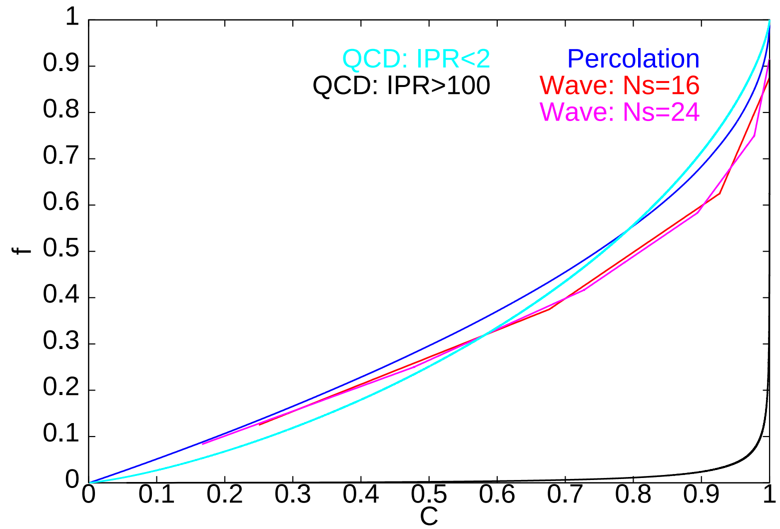


Localization is a matter of how quickly $p(r)$ falls off. Take a function value p and find the two measures $\mathcal{C}(p)$ (fraction of integral) and $f(p)$ (fraction of space occupied). Eliminating p one finds $f(\mathcal{C})$.

Also important is notion of connectivity of the set on which $p(r) > p$ has support. Is this simply connected, multiple regular pieces or some fractal geometry?

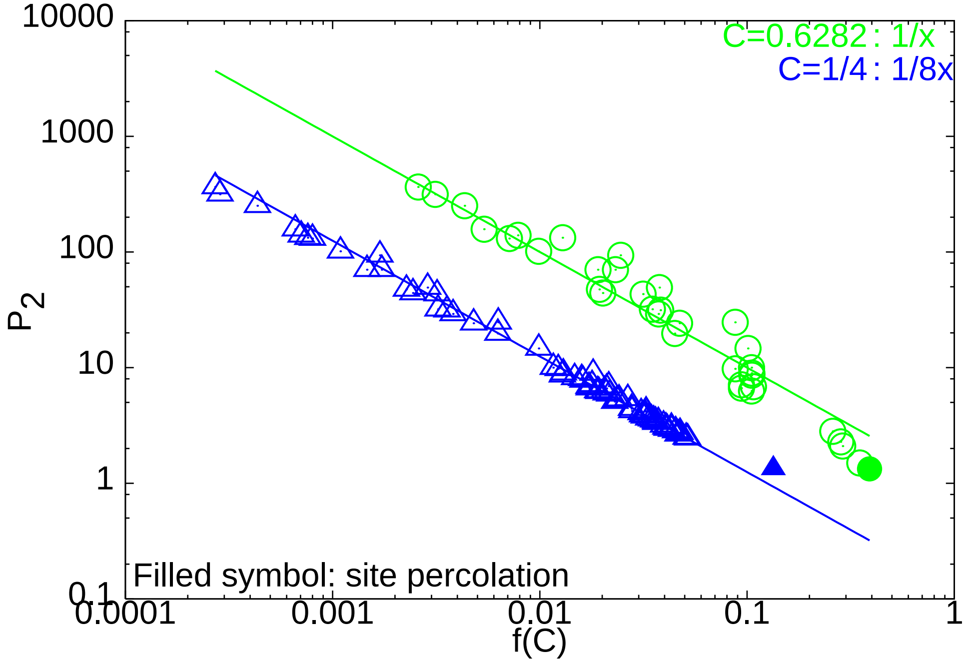
Horvath

Four models



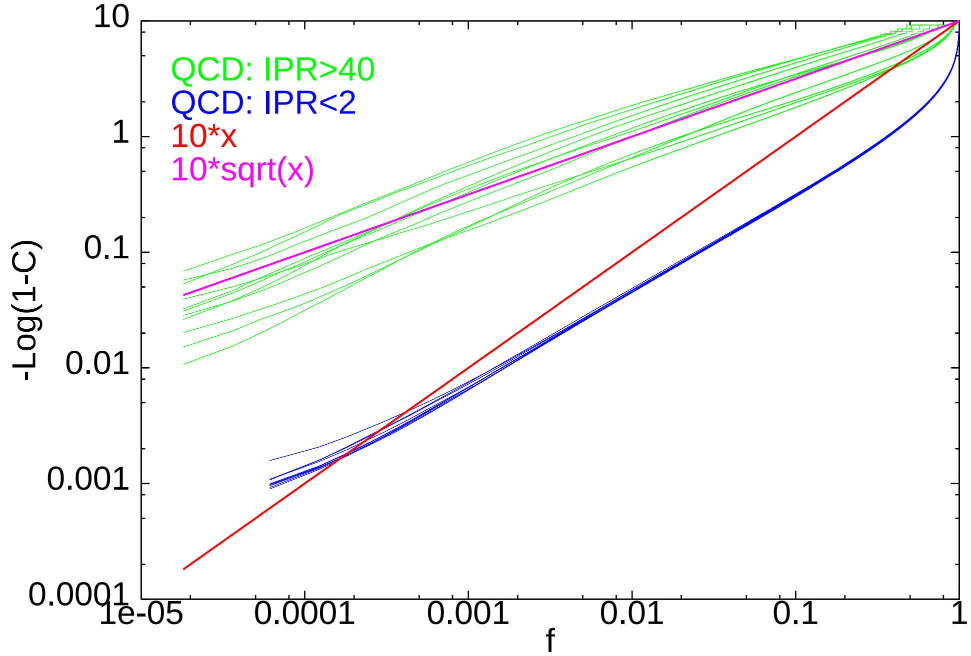
1. Constant function $p(r) = 1/V$ gives $P_2 = 1$ and singular $f(\mathcal{C})$
2. Delta function gives $P_2 = V$ and $f(\mathcal{C}) = 0$ (for $\mathcal{C} < 1$).
3. Standing wave $p(r) = \cos^2(kr)$ gives $P_2 = 1.5$ and $f(\mathcal{C})$ as shown.
4. Random function $p(r)$ on sites maps to site percolation problem. Gives $P_2 = 1.66$ and $f(\mathcal{C})$ as shown.

Equivalent notions of localization



The notions of $f(\mathcal{C})$ and P_2 (IPR) are statistically equivalent. Choose a value of $\mathcal{C} = \mathcal{C}_*$. The corresponding $f(\mathcal{C}_*)$ is correlates well with P_2.

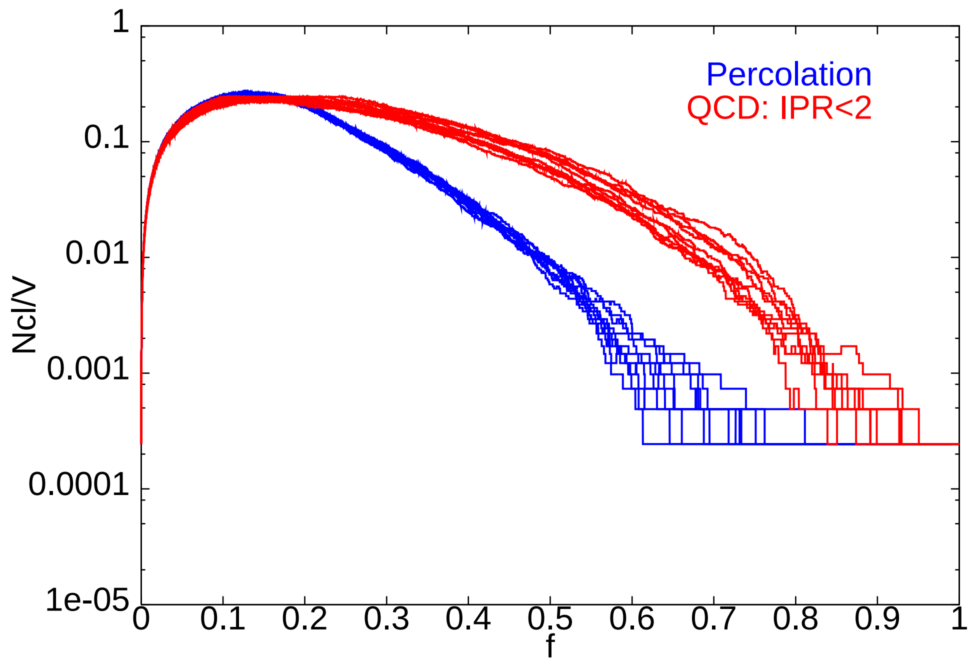
Rate of fall



Exponential fall far from peaks implies $\mathcal{C} \simeq 1 - g(R)\exp(-R^2)$ and $f \simeq R^d$. Check whether $-\log(1-\mathcal{C}) \propto \sqrt{f}$.

Localized states fall exponentially. Extended states have radically different behaviour.

Clusters?



In percolation problems, the number of clusters, N_{cl}, peaks near the transition. N_{cl}/V is an universal function of f. For extended states in QCD, the scaled cluster distribution follows this curve below the transition, but is more extended above: implying there are more holes inside the percolating cluster where smaller clusters can survive.

Mott's argument

Mott's argument about localization and existence of a mobility edge is more general than specific models like Anderson's—

1. If there is a localized and an extended state very close in energy, then they mix under any small perturbation of the Hamiltonian, thus removing localization.

2. Thus localization is robust only when a mobility edge forms, thus separating localized and extended states.

   Mott, 1965

The loophole: if there are localized states with spatial holes (fractal support), then the argument fails, since the lack of overlap can be arranged in space rather than in energy.

Perturbation of Dirac operator

Stability of Dirac eigenvectors under perturbation has to be examined in any case, whenever some property is not protected by topology. Use leading order perturbation theory for eigenfunctions—

$$|i\rangle = \sum_j \frac{\langle j|\delta M|i\rangle}{E_i - E_j}\,|j\rangle, \quad \text{where} \quad M(U+\delta U) = M(U) + \delta M$$

Examining estimators of the coefficients

$$C_{ij} = \frac{\sum_r \sqrt{p_i(r)p_j(r)}}{|E_i - E_j|}.$$

Why no phase information? δM contain arbitrary gauge fields, which randomize phase relations between $|i\rangle$ and $|j\rangle$.

Stability under perturbation

1. Estimate perturbation required to mix $|i\rangle$ and $|j\rangle$ as $1/C_{ij}$.

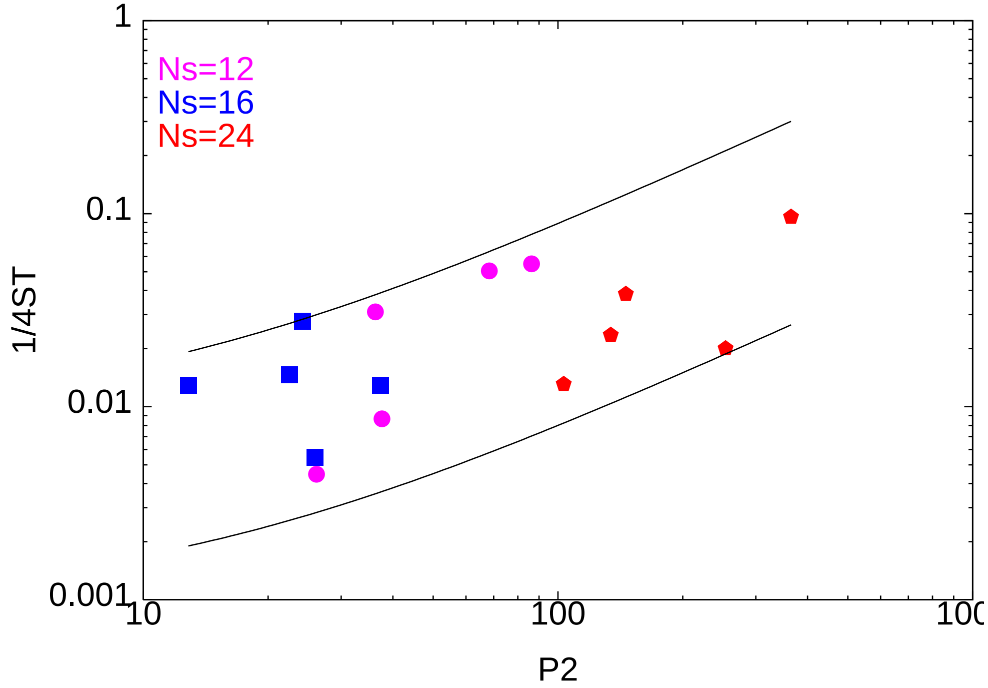
2. Measure of stability is $\mathcal{S}_i = \min_j C_{ij}$; more accurately, $\delta M_i = \mathcal{O}(1/\mathcal{S}_i)$.

3. Estimate $1/\mathcal{S}_i$ for each localized eigenvector $|i\rangle$.

4. Stability for a configuration is the minimum value of $\mathcal{S}_i$, i.e.,
   $\mathcal{S} = \max_i \min_j C_{ij}$.

$\mathcal{S}$ is a direct measure of the stability of localized eigenvalues, and easier to examine on finite lattices (discrete Dirac spectra) than the formation of mobility edges. Also, since the extended eigenvectors may have fractal support, Mott's argument may fail, and there could be stable localization without mobility edges.

Stability and localization



As one tunes up the magnitude of δM, the hardest eigenvalue to mix always turns out to be the one with the largest P_2 (IPR). There is a rough correlation between P_2 and $\mathcal{S}$ as shown. The small value of $1/\mathcal{S}$ indicates that leading order perturbation theory is not totally wrong. Localized eigenvalues are stable.

Summary

1. Screening correlators support evidence from QNS that there are quarks in the plasma. QNS indicates that they are not weakly interacting in the range 1–$2T_c$. What about screening? Small k versus large k? Lots of interesting questions here for weak-coupling-wallahs.
2. The minimum eigenvalue of the massless staggered Dirac operator (partially quenched) shows a crossover from a "chiral" effective theory to dimensionally reduced effective theory.
3. Localization observed at high temperature. Different measures of localization correlated. First evidence that extended eigenvectors could be full of holes, where some localized states could live.
4. Localized eigenvectors are stable under perturbation. Stable localized states are sandwiched (in energy) between other states which are extended. Breakdown of the Mott argument and support for holey solitude.