Finite Chemical Potential in $N_t = 6$ QCD

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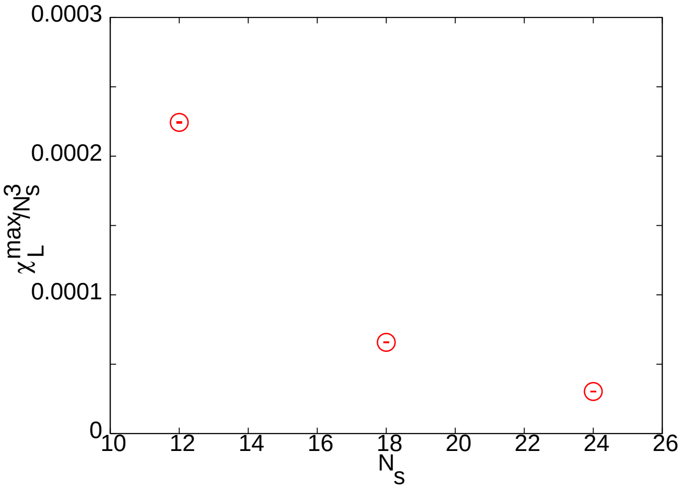
Simulation algorithm

- For $N_f = 2$ simulations with staggered quarks we used R-algorithm. Most runs used trajectory length of 1 MD time unit and $\delta T = 0.01$.
- Test case: $m/T_c = 0.1$, 6×24^3 lattice. Changed δT from 0.01 to 0.001. No change in bulk quantities: plaquettes, Re L, quark condensate.
- For same test case, changed trajectory length from 1 MD time unit to 3 MD time units. No change in bulk quantities, but with longer trajectories autocorrelation lengths decreased so that CPU time taken for generating decorrelated configs decreased.

Finite temperature cross over

- Cross-over coupling monitored using Polyakov Loop susceptibility: χ_L, an operator which enters fourth-order QNS: $(T/V)\langle O_{22}\rangle_c$, and an operator which enters eighth-order QNS: $(T/V)\langle O_{44}\rangle_c$. Measures consistent with each other within the precision of this work.
- Lattice sizes used: $LT = 2$, 3 and 4. Quark masses tuned so that $m/T_c = 0.1$, *i.e.*, $m_\pi/m_\rho \simeq 0.3$. Staggered quarks, Wilson action.
- For $m/T_c = 0.1$, we find $\beta_c = 5.425(5)$. Previous results bracketed this: for $m/T_c = 0.15$ one had $\beta_c = 5.438(40)$ (Gottlieb et al, PRL 59, 1987, 1513) and for $m/T_c = 0.075$ it was found that $\beta_c = 5.41$–5.43 (Bernard et al, PR D 45, 1992, 3854).
- 3-loop scaling works reasonably well between $N_t = 4$ and 6.
- Transition is not first order. Computations at larger volumes are required to distinguish cross over from second order transition.

Not first order



What is a QNS?

Taylor expansion of the pressure in $N_f = 2$ QCD is

$$P(T, \mu_u, \mu_d) = \sum_{n_u, n_d} \frac{1}{n_u! n_d!} \chi_{n_u, n_d}(T) \mu_u^{n_u} \mu_d^{n_d},$$

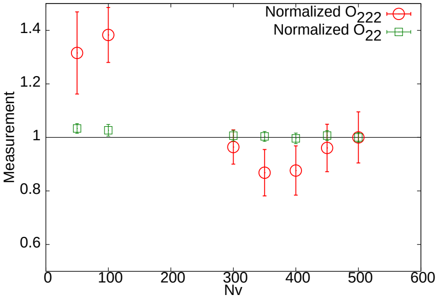
and, since the two quark flavours are degenerate, $\chi_{n_u,n_d} = \chi_{n_d,n_u}$. Diagonal QNS have either $n_u = 0$ or $n_d = 0$.

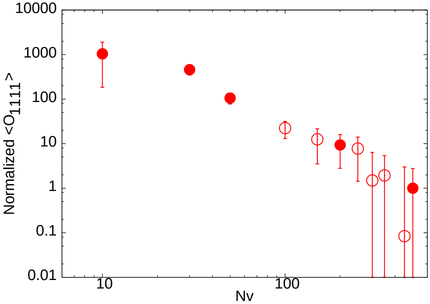
1. Expectation values of operators which can be written as (products of) quark loops with insertions of γ_0.
2. Single quark loop with N insertions (O_N) enters only into the diagonal QNS $\chi_{N,0}$.
3. Up to $N = 4$ (*i.e.*, $\langle O_2 \rangle$ and $\langle O_4 \rangle$) single loop has non-zero value for ideal fermion gas.

Statistics of measurement

1. More than 100 statistically independent gauge configurations at most couplings. Near T_c about 200 statistically independent gauge configurations at each coupling.
2. Noisy measurement of each trace. If we assume that each trace is Gaussian distributed, then the product of traces has long tails (hep-lat/0309014). Hence, very large number of noise vectors are needed for these measurements. We use 500 noise vectors per measurement.
3. To measure all QNS up to order 8, 20 fermion matrix inversions are required per random vector (hep-lat/0412035). Hence, 10000 inversions per gauge configuration.
4. CPU time required for each measurement is still one order of magnitude less than the time required to produce an independent gauge configuration.

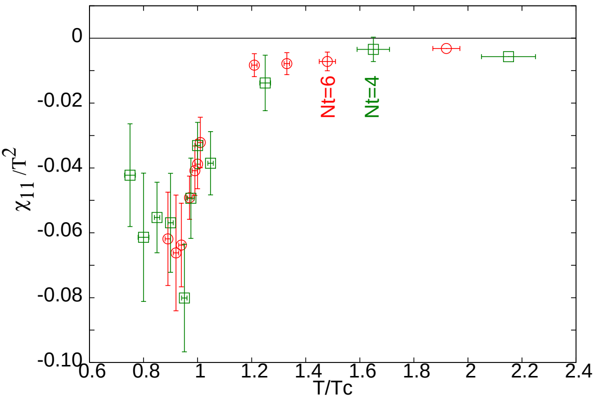
Number of noise vectors





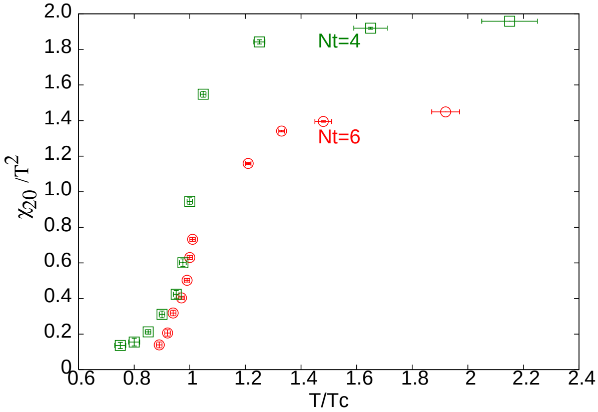
Choice of N_v sensitive to number of quark loops, not to order of QNS (*i.e.*, the number of γ_0 insertions). Since all operators are potentially comparable in size below T_c, choice is important.

Off-diagonal QNS



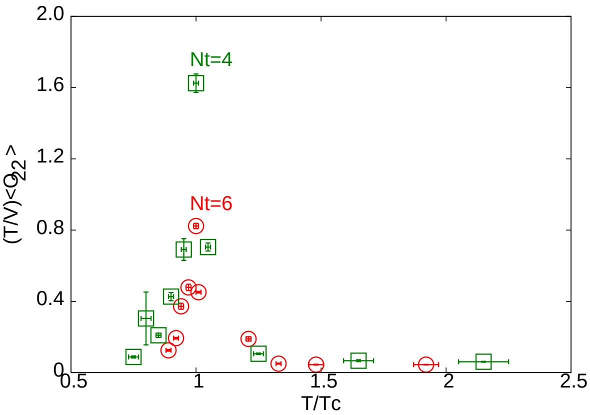
Sees only $\langle O_{11} \rangle$. No evidence for lattice spacing effects.

Diagonal QNS



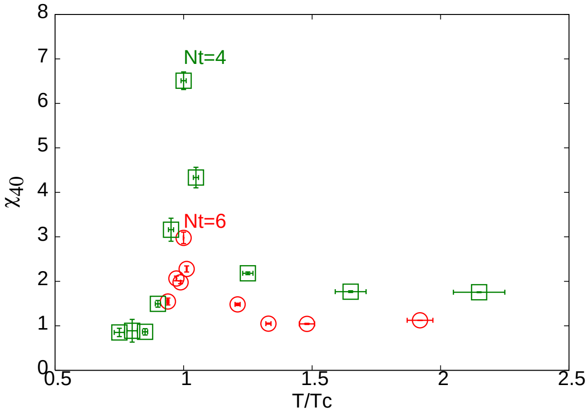
Sees $\langle O_{11}\rangle$ and $\langle O_2\rangle$. Second expectation value is cutoff dependent. Also, has a cross over. We look at its susceptibility $\langle O_{22}\rangle_c$ to identify T_c.

"Susceptibility" of QNS: $\langle O_{22} \rangle_c$— 4th order QNS



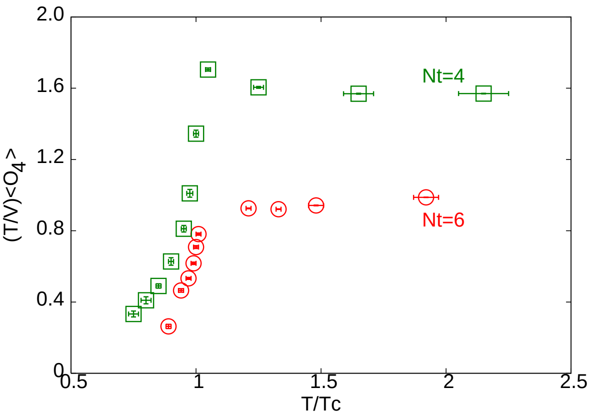
Peak at the same coupling as peak of χ_L. Within the 1% precision of T/T_c, the two quantities peak at the same coupling.

Diagonal fourth-order QNS



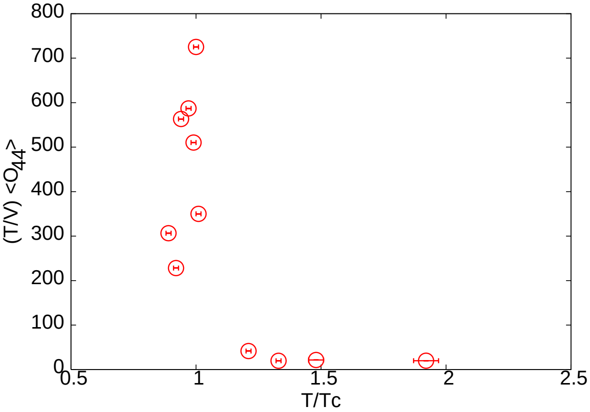
Non-zero for $T > T_c$. Has contribution from $\langle O_4 \rangle$, which has non-vanishing value for the ideal gas.

The operator O_4



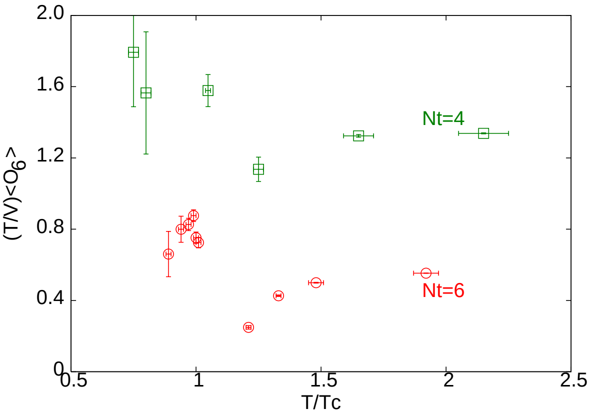
Rapid cross over from a small value in the hadronic phase to a non-vanishing value for the ideal gas.

"Susceptibility" of O_4: $\langle O_{44} \rangle_c$— 8th order QNS



This quantity peaks at the same coupling as χ_L and $\langle O_{22} \rangle_c$. Within the precision of our measurement there is no dependence of the cross over coupling on these observables.

The operator O_6— 6th order QNS



The operator expectation value $\langle O_6 \rangle$ has structure below T_c and hence its "susceptibility" cannot be used to probe the cross over coupling. Similar observation for $\langle O_8 \rangle$.

Radius of convergence

- If all coefficients of a series are positive, then the radius of convergence corresponds to a non-analyticity on the real axis.
- As a bound on the critical end point we take the radius of convergence at the smallest T where all the coefficients measured are positive.
- This gives

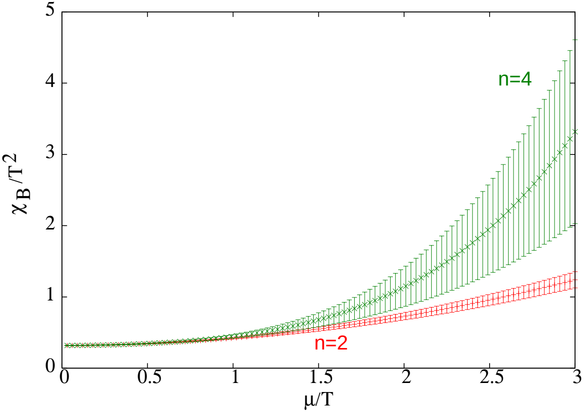
$$\mathbf{\frac{T^E}{T_c} = 0.94 \pm 0.01} \qquad \text{and} \qquad \mathbf{\frac{\mu_B^E}{T^E} = 1.8 \pm 0.1}$$

  with $Nf = 2$ when $m_\pi / m_\rho \simeq 0.3$ at a finite volume with $LT = 4$ and lattice cutoff of $a = 1/6T^E$.
- For a lattice cutoff of $a = 1/4T^E$ at the same renormalized quark mass and on the same volume we had found a similar value for T^E/T_c and $\mu_B^E/T^E = 1.3 \pm 0.3$. Extrapolation to $L \to \infty$ reduced this to 1.1 ± 0.1.
- Finite size scaling with $N_t = 6$ planned for the future.

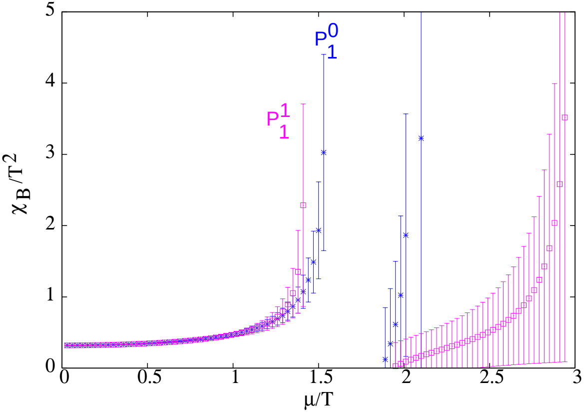
Fluctuations of Baryon number

$$P(\Delta B) = \exp\left(-\frac{(\Delta B)^2}{2VT\chi_B}\right).$$

Extrapolate χ_B to finite chemical potential.



Critical fluctuations



Use Padé approximants for the extrapolations: divergence at the critical end point. Error propagation requires care: see arXiv:0806.2233 [hep-lat].

Summary 1: finite temperature

1. Simulations of $N_f = 2$ QCD (staggered quarks, Wilson action) with renormalized quark mass $m_\pi/m_\rho \simeq 0.3$ with $N_t = 6$ and $LT = 2$, 3 and 4.
2. Thermodynamics stable against order of magnitude change in δT for the R-algorithm: between 0.01 and 0.001.
3. Finite temperature cross over located at $\beta_c = 5.425(5)$, consistent with previous computations at neighbouring masses. Measurements consistent with χ_L, $(T/V)\langle O_{22}\rangle_c$ and $(T/V)\langle O_{44}\rangle_c$ within precision of this computation.

Summary 2: finite chemical potential

1. Finite lattice spacing effects seen in many QNS. Consistent with the magnitude of effects previously observed in the quenched theory. Effects partially cancel in ratios, hence radius of convergence has relatively smaller lattice artifacts.

$$\frac{T^E}{T_c} = 0.94 \pm 0.01 \qquad \text{and} \qquad \frac{\mu_B^E}{T^E} = 1.8 \pm 0.1$$

   with lattice cutoff of $a = 1/6T^E$, compared to $\mu_B^E/T^E = 1.3 \pm 0.3$ at $a = 1/4T^E$ on the same size lattice.

2. Series expansion gives limited information on the values of physical quantities at finite μ_B. Need to use series resummations, for example, using Padé approximants.