# Searching for a QCD critical point

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1. What we want: primordial fluctuations
2. Primordial fluctuations from QCD
3. Evolution of fluctuations
4. Summary

# Outline

# Ensembles

- Event ensembles at colliders: created from the full recorded data set through various selection criteria. Values of conserved quantities can fluctuate from one event to another
- Thermodynamic ensembles: defined by letting certain conserved quantities fluctuate (grand canonical) while keeping others fixed (canonical).
- Relation between the two: the system is a small part of a big (ion-ion) collider event, the heat-bath is the remainder. Is the event a thermostat? Maybe, but need experimental check.
- **This is the most important new thing that STAR can do with the existing data set**.

# Thermodynamic fluctuations at a normal point

- For any system in thermodynamic equilibrium, the fluctuations of a conserved quantity ($Q$, $B$ or $S$) are Gaussian:

$$P(Q) \propto \exp\left(-\frac{Q^2}{2VT\chi_Q}\right), \quad \text{so} \quad \langle \Delta Q^2 \rangle = VT\chi_Q$$

  Bias free experimental measurement of $\chi_Q$ etc possible: connection to QCD possible. Asakawa, Heinz and Muller, Phys.Rev.Lett. 85, 2072, 2000; Jeon and Koch, Phys.Rev.Lett. 85, 2076, 2000.

- What $V$? Which $T$? Parameters have to be specified at the time that the fluctuations were set up. Need hydro and diffusion to evolve to final state: **later**.
- Is the experimental distribution Gaussian? Is there skew or Kurtosis in the present data? If so then there are non-thermal effects. Understand the origin of this non-thermal behaviour: jets, flow, etc.

# Thermodynamic fluctuations at a critical point

- At a normal point the baryon-baryon correlation length ($\xi$) is small (about 0.2 fm). Therefore $V \gg \xi^3$: many "independently fluctuating volumes" hence distributions are Gaussian: central limit theorem.
- Near a critical point $\xi$ diverges (leading to divergence of $\chi^{(2)}$). When $V \simeq \xi^3$, there is single "critically correlated volume" undergoing fluctuations. This destroys Gaussian behaviour.
- At the critical $\sqrt{S}$, different collider events are different samplings of this critical system. The event-to-event distribution is far from Gaussian and the Kurtosis is large.
- The shape of the distribution (mean, variance, skew, kurtosis, *etc.*) can all be predicted from QCD (non-linear susceptibilities). Once non-thermal effects are removed, all measurements of these quantities can make contact with basic theory.

# Which distribution should one measure?

- The distribution of $B$ is the most direct measurement. Since neutrons are not visible to the detector, it has been suggested that net proton number be used as a proxy. Hatta and Stephanov
- Fluctuations of $Q$ are correlated with $B$. The linkage of $B$ and $Q$ is given by the two numbers—

$$C_{BQ|B} = \frac{1}{2} \qquad \text{and} \qquad C_{BQ|Q} \simeq \frac{1}{5}.$$

  Therefore, critical behaviour in fluctuations of $B$ also show up in the fluctuations of $Q$. Gavai, SG
- $Q$ is much easier to measure than $B$. Systematic errors (eg, is $N_p$ always proportional to $B$?) much easier to control in $Q$.
- Strangeness is dominated by $K$. Do uncharged strange particles give significant bias in the measurements?

# Outline

# What is a quark number susceptibility?

The derivatives of the pressure with respect to a chemical potential are quark number susceptibilities:

$$\chi^{(n)}(\mu, T) = \frac{d^n P(\mu, T)}{d\mu^n}.$$

The first derivative is called the quark number density. All higher derivatives are called quark number susceptiblities. Note that these quantities are dimensional.

The Taylor series expansion is useful:

$$P(\mu, T) = P(0, T) + \frac{1}{2!}\chi^{(2)}(0, T)\mu^2 + \frac{1}{4!}\chi^{(4)}(0, T)\mu^4 + \cdots$$

Odd terms are zero. The series coefficients need to be evaluated at $\mu = 0$: can be determined on the lattice. **Second derivative of the series gives a series expansion for $\chi^{(2)}(\mu, T)$.**

# The critical end point

At the critical end point $\chi^{(2)}(\mu^E, T^E)$ diverges. Series no longer summable. Use standard tests for divergence of series: successive terms become comparable:

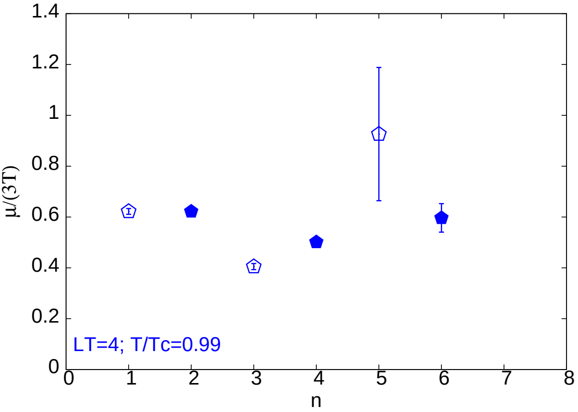
$$\frac{1}{(n-2)!}\chi^{(n)}(0,T^E)(\mu^E)^{n-2} = \frac{1}{n!}\chi^{(n+2)}(0,T^E)(\mu^E)^n.$$

Therefore the estimate of the critical end point is

$$\mu^E = \sqrt{n(n-1)\frac{\chi^{(n)}(0,T^E)}{\chi^{(n+2)}(0,T^E)}},$$

at $T^E$ should be independent of $n$.
Term $n=2$ closely related to Kurtosis; not exactly the same.

# $N_t = 6$: Radius of convergence




Filled symbols: $(n!\chi^{(2)}/\chi^{(n+2)})^{1/n}$.
Open symbols: $\sqrt{n(n+1)\chi^{(n+1)}/\chi^{(n+3)}}$.

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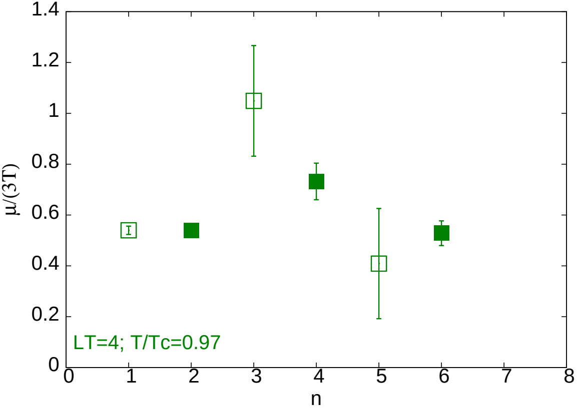


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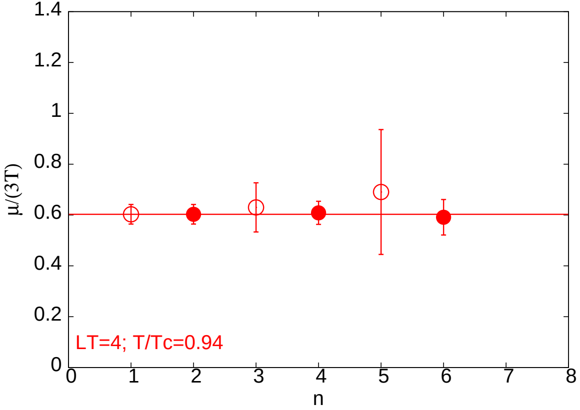


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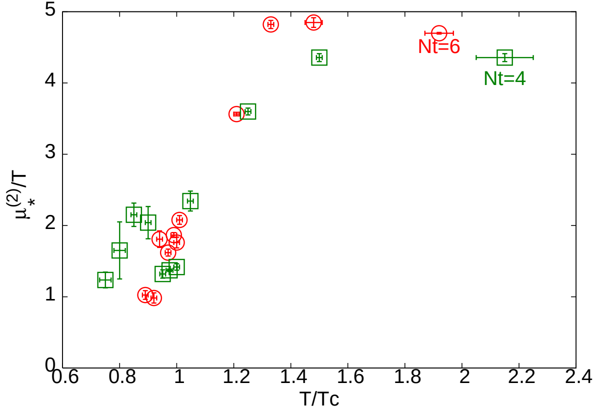
# What are “systematic errors” for lattice

1. Quark masses have to be realistic: the $T = 0$ value of $m_\pi = 140$ MeV. We use a quark mass that gives $m_\pi = 235$ MeV.
2. The volume has to be large in terms of the pion's Compton wavelength: $Lm_\pi \gg 1$. The volume must also be large in units of the thermal wavelength: $LT \gg 1$.
3. The lattice spacing $a$ should be taken to zero. We have used $a = 1/4T$ and $a = 1/6T$.
4. For $a = 1/6T^E$, $LT = 4$ and $m_\pi = 235$ MeV we have $\mu^E/T^E = 1.8 \pm 0.1$. For $a = 1/4T^E$, $LT = 4$ and $m_\pi = 235$ MeV we found $\mu^E/T^E = 1.3 \pm 0.3$. In the limit $L \to \infty$ we had $\mu^E/T^E = 1.1 \pm 0.1$, *i.e.*, roughly 17% decrease. Changing to $m_\pi = 140$ MeV will also decrease $\mu^E/T^E$.
5. **Race between beam and CPU to find the position of the QCD critical end point?**

# Close to Kurtosis: the radius of convergence




The ratio $\mu^*(T)/T = \sqrt{2\chi^{(2)}(0, T^E)/T^2\chi^{(4)}(0, T^E)}$. Lattice spacing dependence quantifies possible systematic errors: $LT = 4$ and $m_\pi = 235$ MeV is kept fixed.

# Outline

# Diffusion

1. Number density fluctuations must begin to diffuse as time passes. For current RHIC dataset $\mu/T$ is small, hence number densities can be considered a small perturbation over energy density. Therefore: diffusion in the background of an expanding fluid flow.
2. The diffusion equation:
$$\frac{\partial n}{\partial t} = \mathcal{D}\nabla^2 n.$$
This is a non-causal equation. Kelly, 1965
3. The causal "second order" diffusion equation is Kelly's equation:
$$\tau_R \frac{\partial^2 n}{\partial t^2} + \frac{\partial n}{\partial t} = \mathcal{D}\nabla^2 n.$$
4. We convert them to relativistic equations, and investigate solutions in the background of a longitudinal flow. Bhalerao and SG, 2009.
5. Extension to fully coupled diffusion + hydro is possible. Extension to three-dimensional flow straightforward.

# Ideal fluid

No dissipation: evolution equation for conserved charge densities is the continuity equation. For longitudinal background flow this gives:

$$\frac{dn(\tau,\eta)}{d\tau} = -\frac{n(\tau,\eta)}{\tau} \quad \text{with solution} \quad \tau n(\tau,\eta) = \tau_0 n(\tau,\eta).$$

This is just the conservation law for the conserved charge. The volume element expands linearly with $\tau$ in longitudianl flow, so the integral over space-time rapdity, $\eta$, of $\tau n$ is conserved. We call this Bjorken attenuation.

## Note for experiments

The charge in a bin, $Q(\tau_f,\eta)$, is to be identified with $\tau_f \Delta\eta n(\tau_f,\eta)$. $Q$ is conserved.

# First order diffusion

In a longitudinally expanding background, the diffusion equation becomes

$$\frac{dn(\tau,\eta)}{d\tau} = -\frac{n(\tau,\eta)}{\tau} + \frac{\mathcal{D}}{\tau^2}\frac{\partial^2 n}{\partial\eta^2}.$$

Linear equations, solve by Fourier transforming in $\eta$:

$$\tau n(\tau,k) = \tau_0 n(\tau,k) \exp\left[-\frac{\mathcal{D}k^2}{\tau_0}\left(1-\frac{\tau_0}{\tau}\right)\right].$$

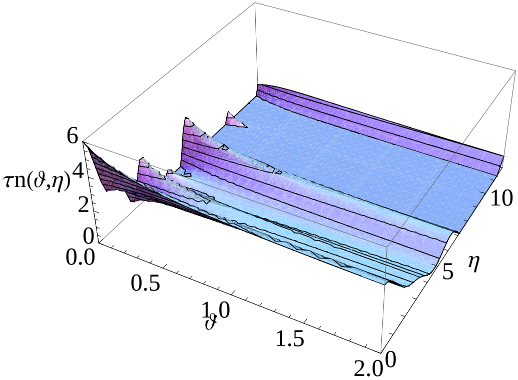
## Note for experiments

The corresponding experimental observable is

$$\overline{P}(\tau_f,k) = \left|\sum_{j=1}^{N_t} q_j \mathrm{e}^{-ik\eta_j}\right|^2,$$

where the sum is over tracks. This is to be compared to the power spectrum: $|\tau_f n(\tau_f,k)|^2$.

# First order diffusion




Usual intuition: diffusion destroys structure, the sharpest structures are destroyed fastest.

# Second order diffusion

Equations are similar to a damped oscillator. In a longitudinal flow background one can write this as

$$\partial_\vartheta \begin{pmatrix} n \\ \nu \end{pmatrix} = -M \begin{pmatrix} n \\ \nu \end{pmatrix}, \qquad M = \begin{pmatrix} 1 & ik \\ ic_s^2 k & \mathrm{e}^{\vartheta} \end{pmatrix},$$

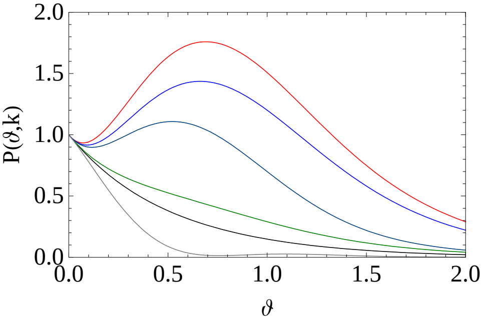
where $\vartheta = \log(\tau/\tau_R)$. Matrix M is not normal. The equations are not autonomous.

Construct a power spectrum $P = |n|^2 = x^\dagger A x$. Then

$$\frac{\partial P}{\partial \vartheta} = -x^\dagger \mathcal{M} x, \qquad \mathcal{M} = \begin{pmatrix} 2 & ik \\ -ik & 0 \end{pmatrix},$$

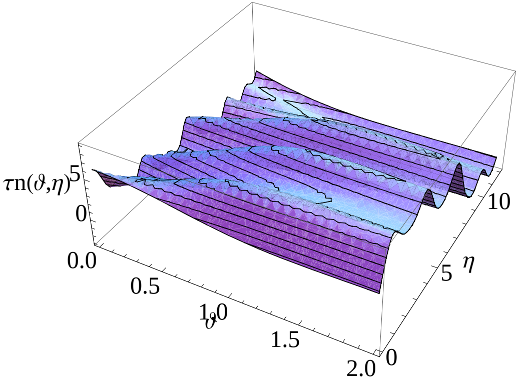
Numerical range of $\mathcal{M}$ has indefinite sign: hence **transient amplification possible and generic**.

# Transient amplification of the power spectrum



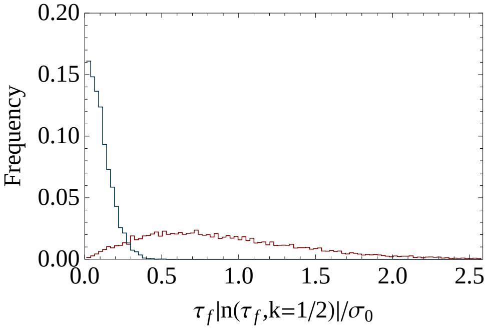

One draw from Gaussian random ensemble of initial conditions. $\vartheta = 2$ corresponds to $\tau = 7.4\tau_R$. Spectral sum rule: $\tau_R = \mathcal{D}/c_s^2$. Freezeout likely for $2 \leq \vartheta < 3$.

# Transient amplification of the profile



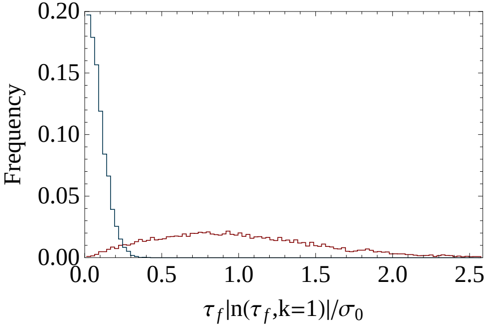

One draw from Gaussian random ensemble of initial conditions. Profile of initial $n$ same as for the first order example before.

# Experimental signature



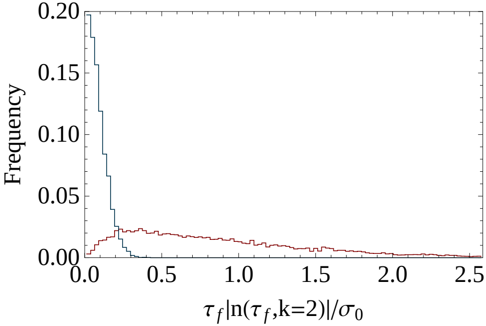

Initial conditions: drawn from unit Gaussian.
Final distribution for $k = 1/2$.

# Experimental signature




Initial conditions: drawn from unit Gaussian.
Final distribution for $k = 1$.

# Experimental signature




Initial conditions: drawn from unit Gaussian.
Final distribution for $k = 2$.

# Some interesting points

1. For $k = 0$ the influence of hydrodynamics goes away: conserved quantity.
2. Net charge within a fixed window is not conserved.
3. If there is power at large $k$ then Fick diffusion has not set in.
4. Conversely, if Fick diffusion has set in, then the transport coefficient $\mathcal{D}$ can be bounded provided $\tau_f$ can be determined independently:

$$\mathcal{D} \le \tau_f \sinh^2 \Delta\eta.$$

# Outline

# Summary

- The critical point of QCD is likely within reach of collider energy scans. Its precise location is a race between beam and CPU: experiment and lattice computation.
- Experimental signatures involve event-to-event distributions of conserved charges, for example, $B$, $Q$ and $S$. Since uncharged particles are not observed, $B$ and $S$ have to be replaced by proxies. However, all distributions see the critical behaviour.
- Kurtosis of the net charge within an acceptance window may be an important observable.
- Power spectrum of the experimental event-to-event distributions of $B$, $Q$, and $S$ are important for understanding the process of diffusion, and the extraction of initial distributions from the final distributions.

## An advertisement

TIFR is planning to expand research directions: one of the new directions under consideration is experimental and theoretical heavy-ion physics and allied topics in extreme QCD.