QCD at finite density

Sourendu Gupta

ILGTI: TIFR

QCD in [the] medium
Kolkata, India
October 4, 2010

Getting around the sign problem

Connecting to experiments

Future

Outline

Getting around the sign problem

Connecting to experiments

Future

The physics challenge

Statement of the problem

No flavour changing currents in QCD; so each flavour is a conserved charge. Can introduce a chemical potential for each. So phase diagram of QCD is 4-dimensional: T, μ_u, μ_d and μ_s. Equivalently: T, μ_B, μ_Q and μ_S. Gavai and SG, 2005.

Apparent road blocks

Lattice should predict the phase diagram, but there are sign problems. Models do not capture the correct physics: hence give wrong phase diagram or no prediction (many possible phase diagrams).

The sign problem

Core of the problem

Gauge action positive; not changed by introduction of flavour chemical potentials.
Fermion determinant contains sign problem:

$$\det(D + m + \mu\gamma_0)^* = \det(D + m - \mu^*\gamma_0)$$

Cannot be free of sign problems when μ is real non-zero.

Importance sampling fails: no Monte Carlo procedure.

As yet unused fact

Problem could be representation dependent; clever reformulation may resolve the problem: for example, by changing to new variables.

Reweighting

- Glasgow: generate ensemble at one point in phase diagram, reweight to another point; problem of overlap.
- Finite temperature reweighting; overlap problem smaller. Fodor and Katz, 2001
- Taylor-expand the quark determinant inside the path-integral; amounts to differential reweighting. Bielefeld Swansea, 2002
- Gaussian approximation to the phase of the determinant; used to reweight configurations. Ejiri, 2007

No major methodological developments since 2007: brick wall of thermodynamic limit.

Madhava-Maclaurin (Taylor) series expansion

The pressure in a grand canonical ensemble allows a Madhava-Maclaurin series expansion:

$$P(T, \mu) = P(T) + \frac{\mu^2}{2!}\chi^{(2)}(T) + \frac{\mu^4}{4!}\chi^{(4)}(T) + \cdots$$

The coefficients are evaluated at $\mu = 0$ where there is no sign problem.

Evaluate the susceptibilities $\chi^{(n)}$ directly as expectation values of operators.
Gavai, SG, 2003
Evaluate the susceptibilities by constructing the pressure (or its derivatives) at series of imaginary chemical potentials and then fitting extrapolating functions to the data.
Cosmai *et al.*, Falcone *et al.*: Lattice 2010

Series Analysis

Series analysis for spin models

Analysis of series for critical behaviour since 1960s. Well-developed when series coefficients are exactly known. First step: evaluate radius of convergence. Then check whether singularity is due to physical parameter values.

Domb and Green, vol 2

Series analysis for $\mu \neq 0$ QCD

Similar idea, but needs to be adapted to specific problem. Series coefficients have statistical errors; coefficients are volume dependent. Some subtleties.

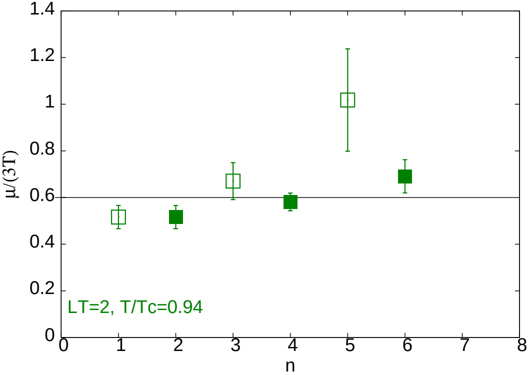
Gavai, SG, 2004, 2008

Finite volume effects in series analysis

1. Increasing order of series expansion and finite volume scaling closely tied together.
2. Susceptibility never diverges on finite volume, but grows higher and sharper with increasing volume. Major effect: growth of peak; minor effect: shift of peak.
3. Series expansion of such a sequence of functions should show lack of divergence for each volume if pushed to large enough order.
4. At finite order, signal of eventual divergence should build up.
5. With increasing volume, there should be a plateau of stability for radius of convergence before radius diverges.

Gavai, SG, 2004, 2008

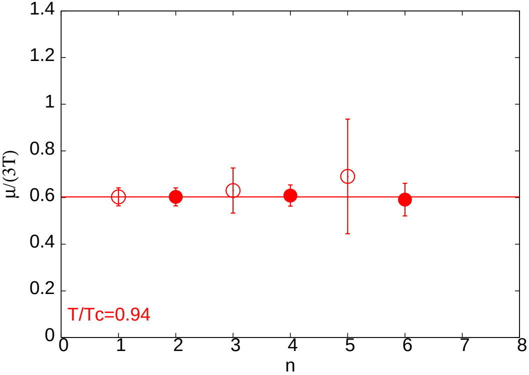
Finite volume effects



Gavai, SG, 0806.2233

Filled symbols: $r_n = \sqrt{(n+3)!\chi^{(n+1)}/(n+1)!\chi^{(n+3)}}$
Unfilled symbols: $r_n = ((n+2)!\chi^{(2)}/2!\chi^{(n+2)})^{1/n}$
$LT \geq 4$ and $Lm_\pi \geq 5$; plateau develops.

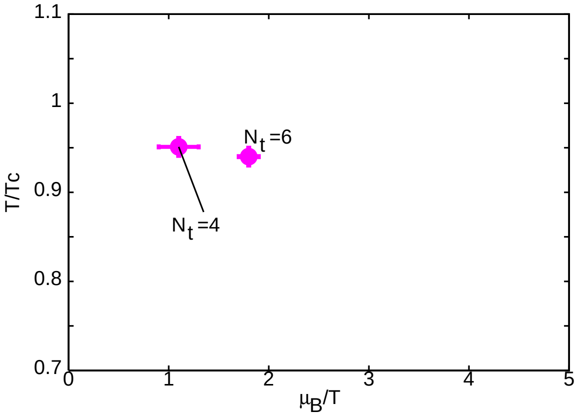
Finite volume effects



Gavai, SG, 0806.2233

Filled symbols: $r_n = \sqrt{(n+3)!\chi^{(n+1)}/(n+1)!\chi^{(n+3)}}$
Unfilled symbols: $r_n = ((n+2)!\chi^{(2)}/2!\chi^{(n+2)})^{1/n}$
$LT \geq 4$ and $Lm_\pi \geq 5$; plateau develops.

The critical end point

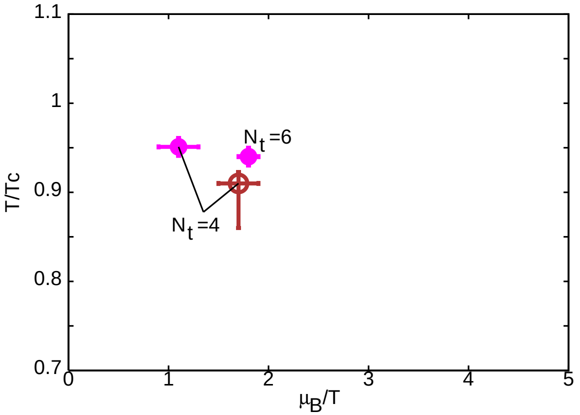


Cutoff dependence and the effect of strange quarks.

Staggered: $N_f = 2$, $m_\pi = 230$ MeV, $LT \geq 4$ Gavai, SG, 0806.2233

P4: $N_f = 2 + 1$, $m_\pi = 220$ MeV, $LT = 4$ Schmidt, 2010

The critical end point

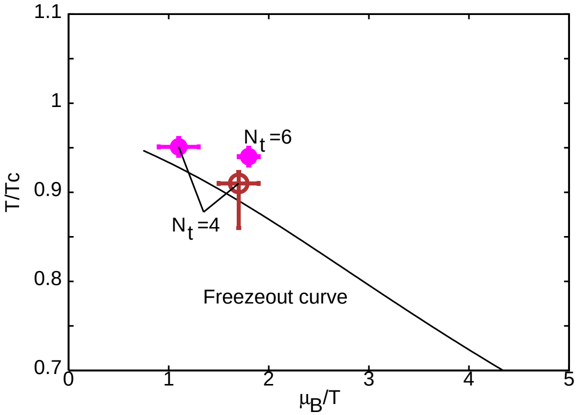


Cutoff dependence and the effect of strange quarks.

Staggered: $N_f = 2$, $m_\pi = 230$ MeV, $LT \geq 4$ Gavai, SG, 0806.2233

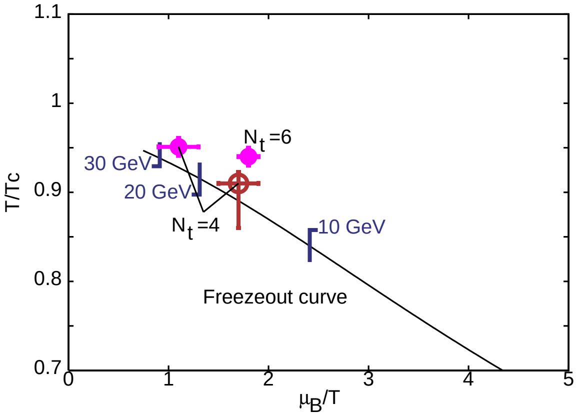
P4: $N_f = 2 + 1$, $m_\pi = 220$ MeV, $LT = 4$ Schmidt, 2010

The critical end point



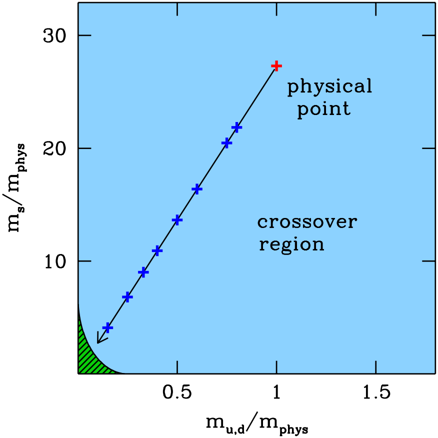
Cutoff dependence and the effect of strange quarks.
Staggered: $N_f = 2$, $m_\pi = 230$ MeV, $LT \geq 4$ Gavai, SG, 0806.2233
P4: $N_f = 2 + 1$, $m_\pi = 220$ MeV, $LT = 4$ Schmidt, 2010

The critical end point



Cutoff dependence and the effect of strange quarks.
Staggered: $N_f = 2$, $m_\pi = 230$ MeV, $LT \geq 4$ Gavai, SG, 0806.2233
P4: $N_f = 2 + 1$, $m_\pi = 220$ MeV, $LT = 4$ Schmidt, 2010

Strange quarks are not very important!



In $N_f = 2+1$:

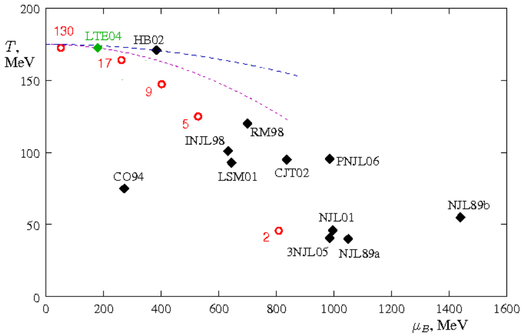
$$m_\pi^{crit} \begin{cases} = 0.07 m_\pi & (N_t = 4) \\ < 0.12 m_\pi & (N_t = 6) \end{cases}$$

Endrodi etal, 0710.0988

Similarly for $N_f = 3$.

Karsch etal, hep-lat/0309121

Model predictions

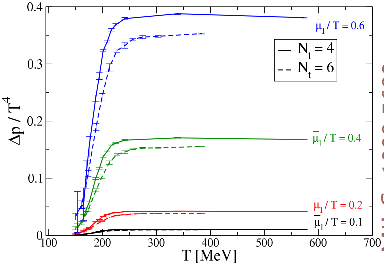


Stephanov, hep-ph/0701002

Accurate models of hadronic phase needed; open problem for 70 years. Scattered results, not predictive.

The pressure

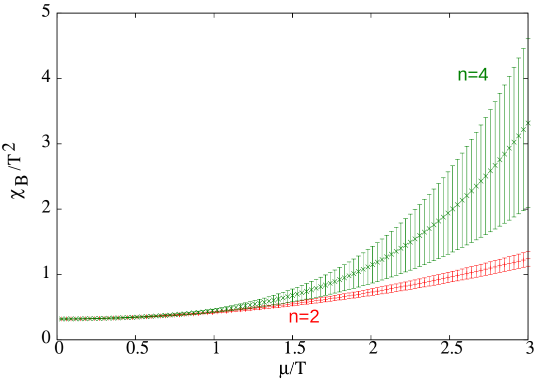
$$\Delta p = p(T,\mu) - p(T,0) = \frac{\mu^2}{2!}\chi^{(2)}(T) + \frac{\mu^4}{4!}\chi^{(4)}(T) + \cdots$$



MILC, 1003.5682

At CEP all terms equally important; finite sum wrong. Must resum.

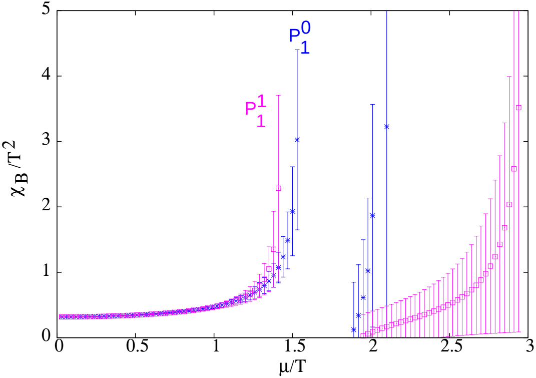
Extrapolating measurements



Gavai, SG, 0806.2233

Infinite series diverges, but truncated series finite and smooth: sum is bad. Resummations needed to reproduce critical divergence. Padé resummation useful.

Extrapolating measurements



Gavai, SG, 0806.2233

Infinite series diverges, but truncated series finite and smooth: sum is bad. Resummations needed to reproduce critical divergence. Padé resummation useful.

Outline

Getting around the sign problem

Connecting to experiments

Future

Gaussian Fluctuations

Normal fluctuations are Gaussian

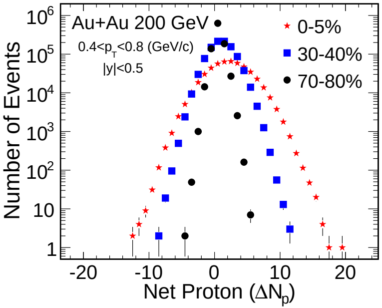
Suggestion by Stephanov, Rajagopal, Shuryak: measure the width of momentum distributions.
Better idea: use conserved charges, because at any normal (non-critical) point in the phase diagram:

$$P(\Delta B) = \exp\left(-\frac{(\Delta B)^2}{2VT\chi_B}\right). \qquad \Delta B = B - \langle B\rangle.$$

Why Gaussian?

At any non-critical point the appropriate correlation length (ξ) is finite. If the number of independently fluctuating volumes ($N = V/\xi^3$) is large enough, then net B has Gaussian distribution: **central limit theorem** (CLT).

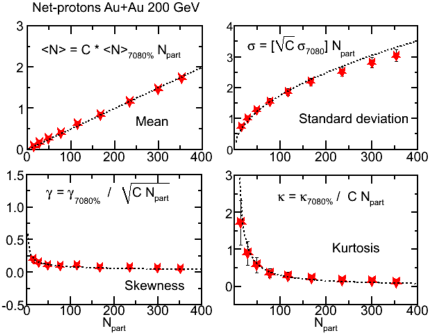
Event distributions of conserved charges



STAR, 1004.4959

- Fluctuations of conserved quantities are Gaussian: provided large volume and equilibrium
- Proton number a substitute for baryon number: how good?
- Is this Gaussian due (entirely or largely) to thermal fluctuations?

Look beyond Gaussian



STAR: QM 2009, Knoxville

- Higher cumulants scale down with larger powers of V.
- N_{part} is a proxy for V.
- Cumulants observed to scale correctly as N_{part}.
- Can one connect to QCD?

SG, 0909.4630

How to compare experiment with lattice QCD

The cumulants of the distribution are related to Taylor coefficients—

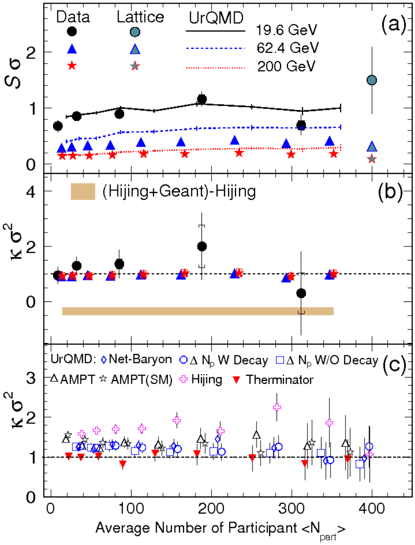
$$[B^2] = T^3 V \left(\frac{\chi^{(2)}}{T^2}\right), \quad [B^3] = T^3 V \left(\frac{\chi^{(3)}}{T}\right), \quad [B^4] = T^3 V \chi^{(4)}.$$

V is unknown, so direct measurement of QNS not possible. Define variance $\sigma^2 = [B^2]$, skew $\mathcal{S} = [B^3]/\sigma^3$ and Kurtosis, $\mathcal{K} = [B^4]/\sigma^4$. Construct the ratios

$$m_1 = \mathcal{S}\sigma = \frac{[B^3]}{[B^2]}, \qquad m_2 = \mathcal{K}\sigma^2 = \frac{[B^4]}{[B^2]}, \qquad m_3 = \frac{\mathcal{K}\sigma}{\mathcal{S}} = \frac{[B^4]}{[B^3]}.$$

These are comparable with experiment provided lattice data **extrapolated to relevant T and μ**: use Padé approximants.
SG, 0909.4630

Extrapolate lattice data to finite μ



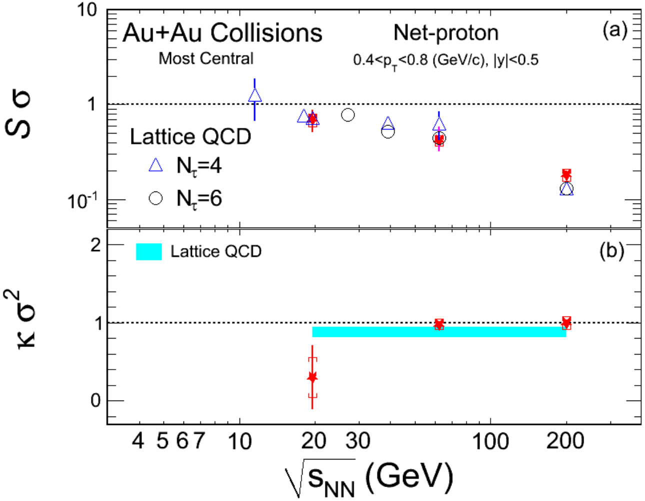
STAR Collaboration, 1004.4959 (2010)

Surprising agreement with lattice QCD:

- implies non-thermal sources of fluctuations are very small
- T does not vary across the freezeout surface.
- tests QCD in non-perturbative thermal region

Gavai, SG, 1001.3796

Experiment vs lattice QCD



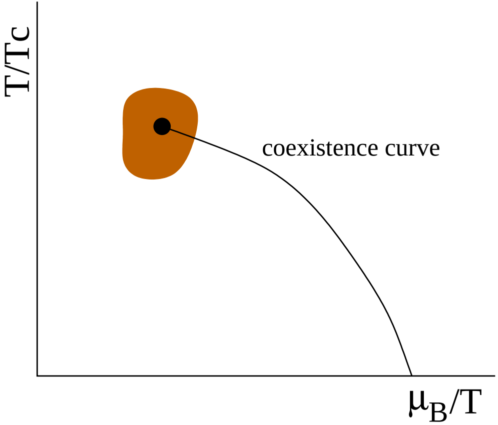
Outline

Getting around the sign problem

Connecting to experiments

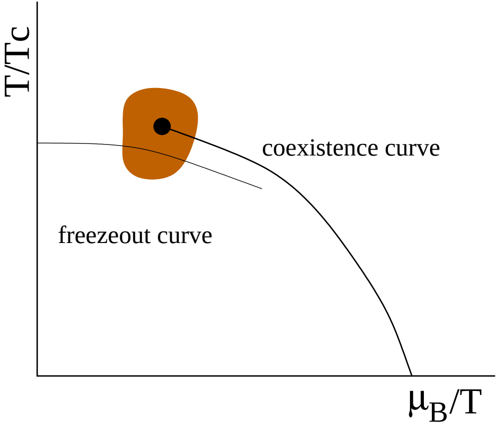
Future

How to find the critical point: be lucky!



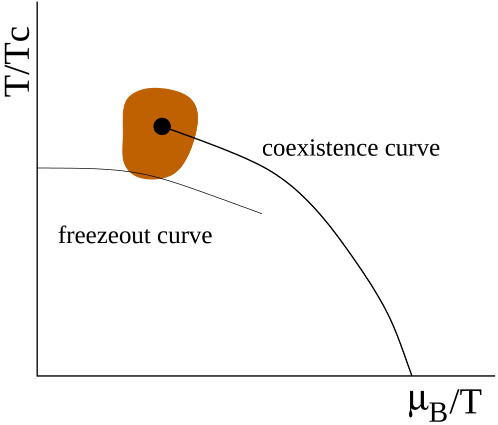
Scan along the freezeout curve: any signal of critical region?
Lattice says yes. Hadron resonance gas: no.
Gavai and SG, arXiv:1001.3796

How to find the critical point: be lucky!



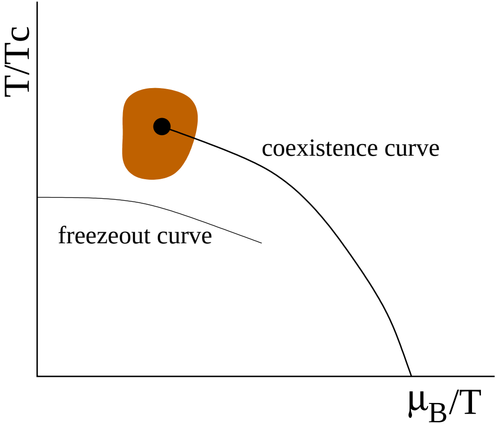
Scan along the freezeout curve: any signal of critical region?
Lattice says yes. Hadron resonance gas: no.
Gavai and SG, arXiv:1001.3796

How to find the critical point: be lucky!



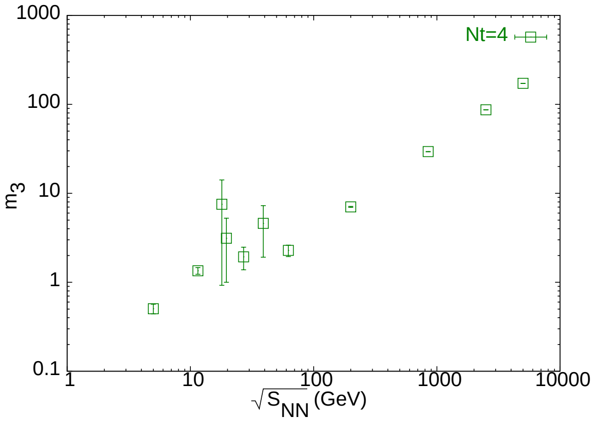
Scan along the freezeout curve: any signal of critical region?
Lattice says yes. Hadron resonance gas: no.
Gavai and SG, arXiv:1001.3796

How to find the critical point: be lucky!



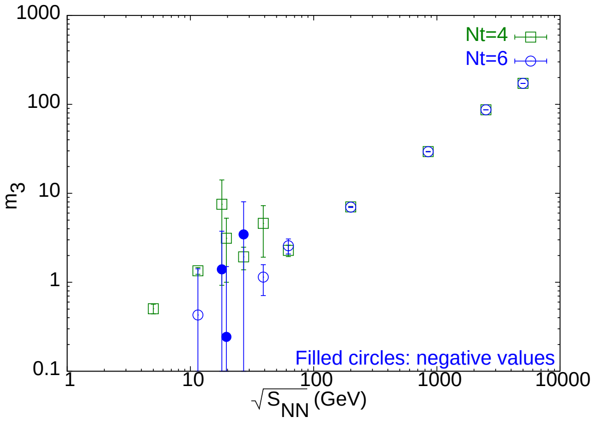
Scan along the freezeout curve: any signal of critical region?
Lattice says yes. Hadron resonance gas: no.
Gavai and SG, arXiv:1001.3796

The energy scan at RHIC



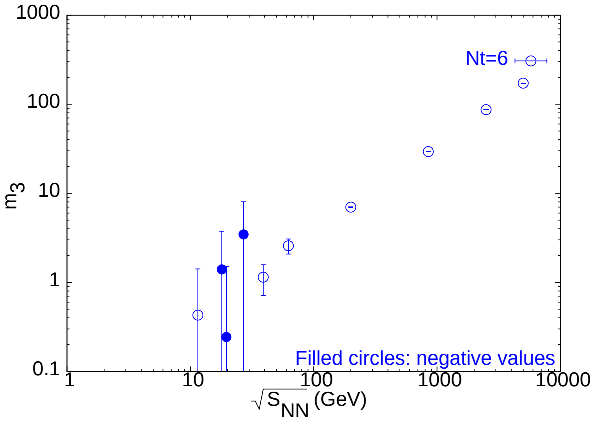
Gavai and SG, arXiv:1001.3796

The energy scan at RHIC



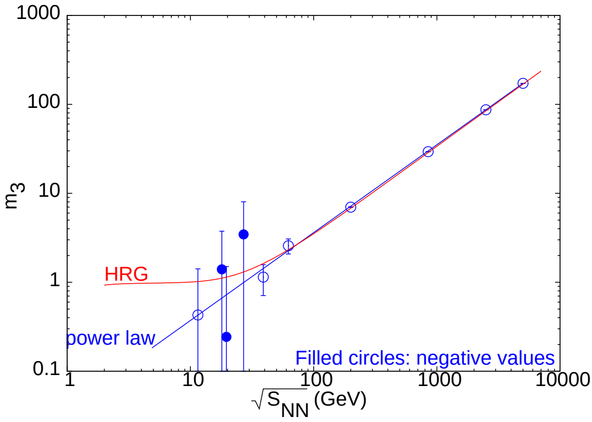
Gavai and SG, arXiv:1001.3796

The energy scan at RHIC



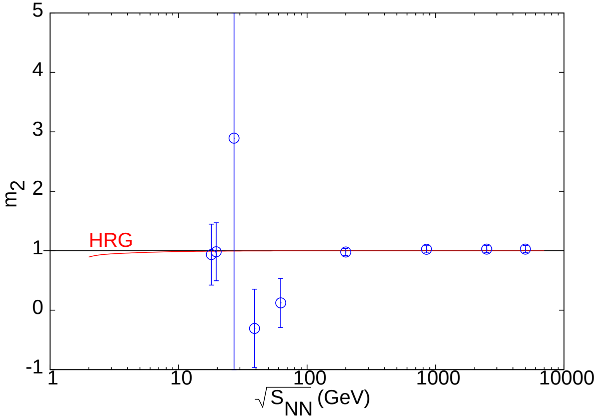
Gavai and SG, arXiv:1001.3796

The energy scan at RHIC



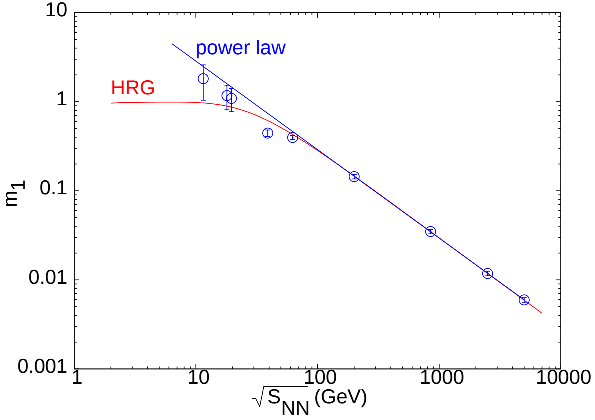
Gavai and SG, arXiv:1001.3796

The energy scan at RHIC



Gavai and SG, arXiv:1001.3796

The energy scan at RHIC



Gavai and SG, arXiv:1001.3796

Summary

1. Sign problem for QCD effectively solved: operator and imaginary μ results agree; different actions give results in the same ball park. Lattice spacing effects— final frontier.
2. Extrapolation of physics predictions to finite chemical potential need resummation; Padé approximants are one way.
3. Models still unreliable; need more accurate description of the hadron phase of QCD.
4. Path to comparison with experimental data open. Hadron gas models blind to critical point. Lattice needs smaller spacing. Window of opportunity for models.

Open questions

1. Are there other (non-thermal) sources of fluctuations in the data? Partly answered by comparison with MCs. But that does not eliminate (for example) volume fluctuations. Can one do something about this?
2. Can one separate various kinds of long-ranged correlations in the experiment? Need to remove late stage (hadronic) correlations.
3. Is there any other way to directly look for long correlation lengths in collider experiments? For example, change acceptance volume and test for failure of CLT.
4. Lattice predicts connections between χ_B, χ_Q and χ_S Gavai, SG, 2005. Need to see these in experiments.
5. Is there any way to explore larger parts of the phase diagram in the laboratory?