Phases of baryonic matter

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ILGTI: TIFR

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Outline

Inside the cave

QCD: theory of strong interactions

SU(3) gauge theory of interacting quarks and gluons. Theory of gluons classically scale free, quantum corrections generate a scale: Λ_{QCD}.

Inside the cave

QCD: theory of strong interactions

SU(3) gauge theory of interacting quarks and gluons. Theory of gluons classically scale free, quantum corrections generate a scale: Λ_{QCD}.

A theorist's reflex

Given Hamiltonian compute eigenstates, S-matrix elements: talks by Doi and Beane.
Compute physics in a heat-bath: $Z(T, \mu) = \mathrm{Tr}\ \exp[-\beta(H - \mu B)]$.
Thermodynamics and phase transitions straightforward (but tedious).

The physicist's reflex

" We didn't have flint when when I was a kid, we had to rub two sticks together. "

How many flavours

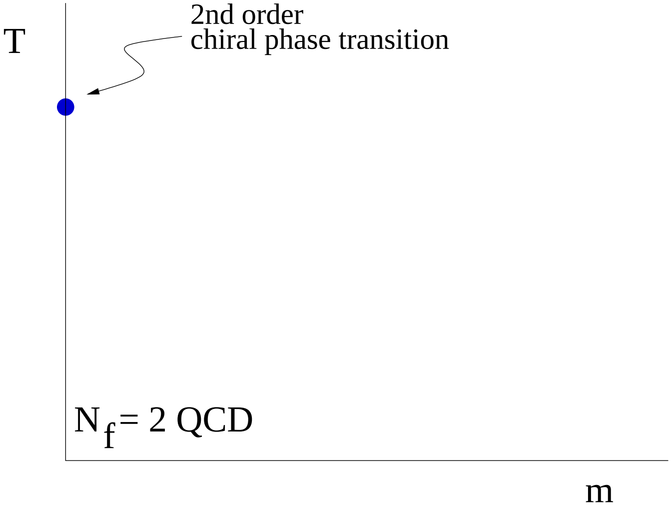
Decoupling

If some $m \gg \Lambda_{QCD}$ then that quark is not approximately chiral. In QCD two flavours are light ($m_{u,d} \ll \Lambda_{QCD}$) and one is medium heavy ($m_s \simeq \Lambda_{QCD}$). The rest are heavy ($m_{c,b,t} \gg \Lambda_{QCD}$).

What phase diagram?

Do we have a two flavour phase diagram or a three flavour phase diagram, or something else?

The two flavour phase diagram



Pisarski and Wilczek, PR D 29, 338 (1984)

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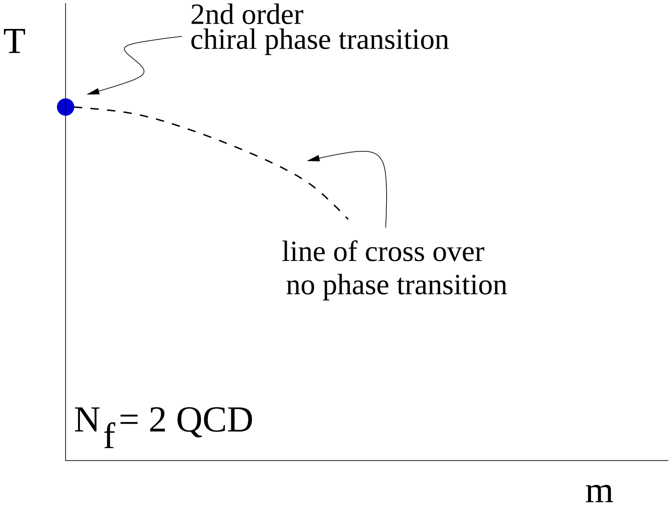

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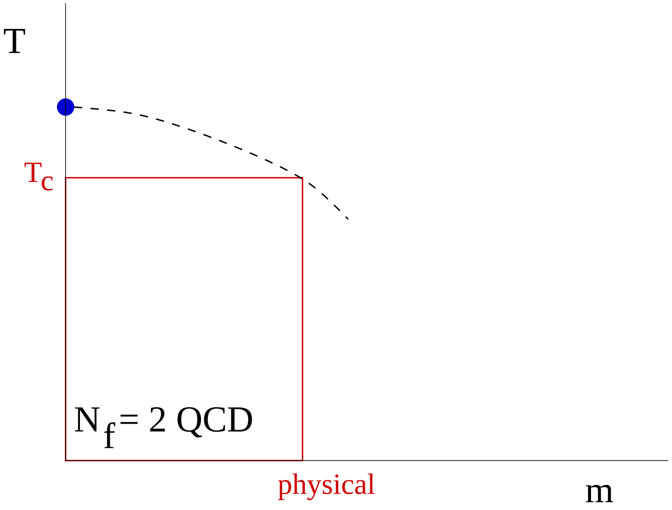

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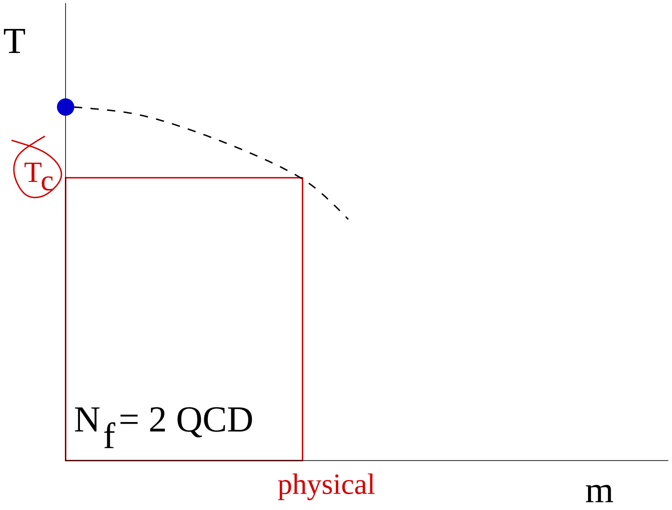
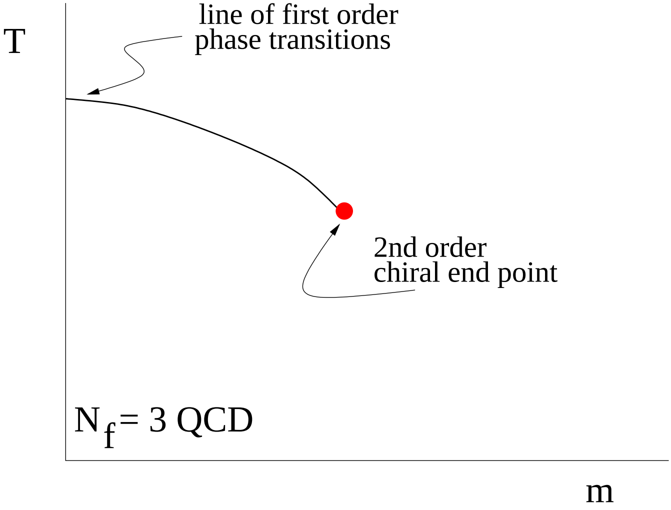


Pisarski and Wilczek, PR D 29, 338 (1984)

The three flavour phase diagram



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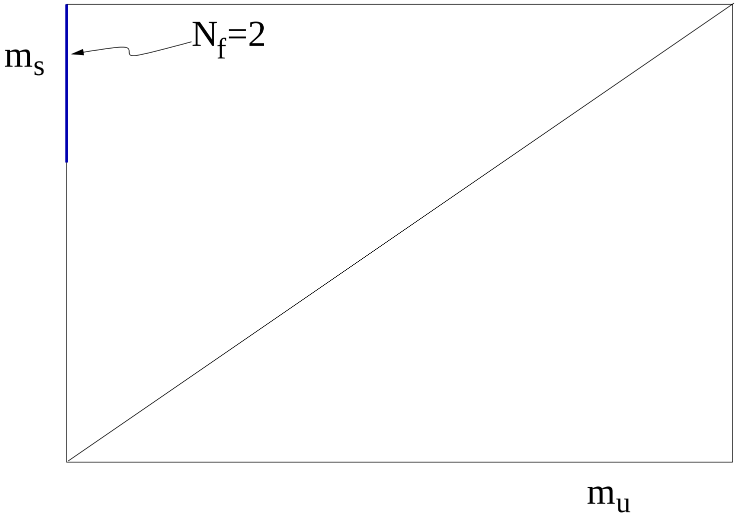
The Columbia plot



Brown et al, PRL 65, 2491 (1990)

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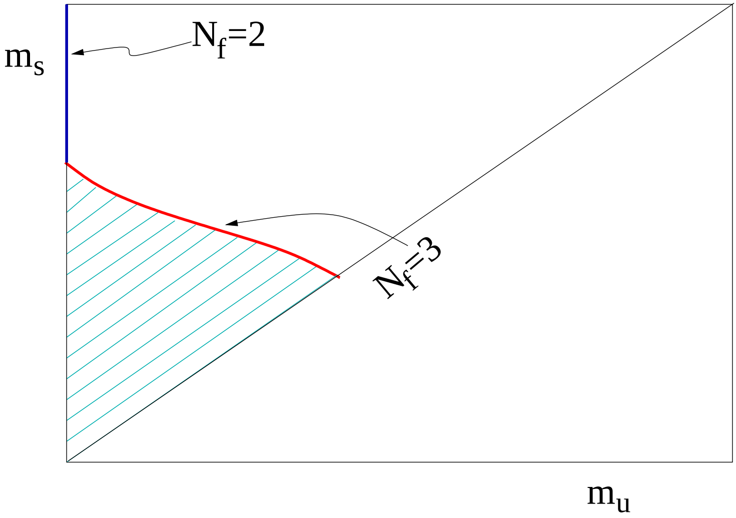

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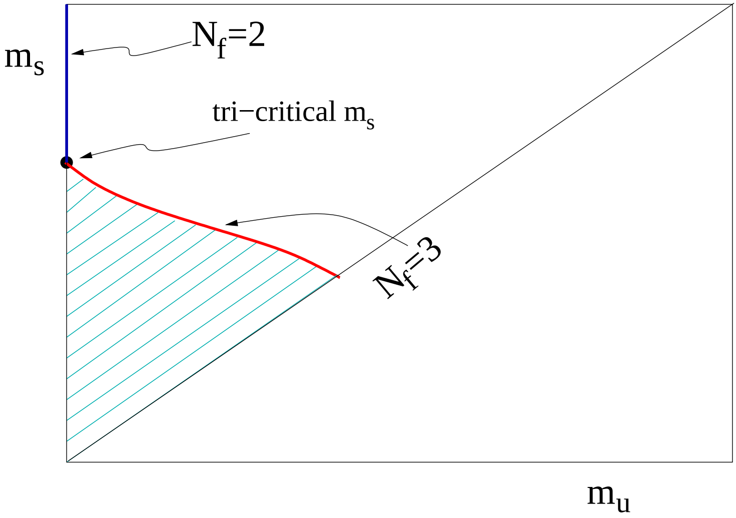

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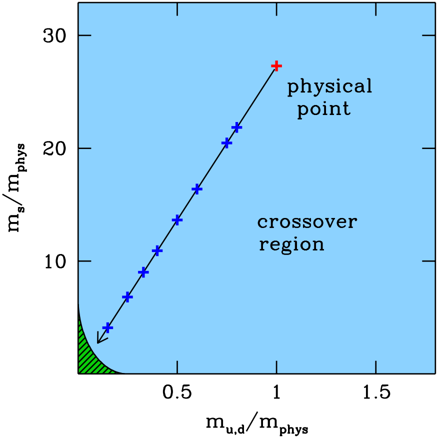
Lattice results for the Columbia Plot



In $N_f = 2+1$:

$$m_\pi^{crit} \begin{cases} = 0.07 m_\pi & (N_t = 4) \\ < 0.12 m_\pi & (N_t = 6) \end{cases}$$

Endrodi etal, 0710.0988 (2007)

Similarly for $N_f = 3$.

Karsch etal, hep-lat/0309121 (2004)

Broken flavour symmetry

1. Two independent lattice computations (now) agree on the position of the crossover temperature for physical quark mass ($m_\pi \simeq 140$ MeV):
$$T_c \simeq 170 \text{ MeV}.$$
Aoki etal, hep-lat/0611014 (2006); HotQCD, 2010.
2. No significant change in T_c as $m_{\pi^0}/m_{\pi^\pm}$ is changed from 1 to 0.78 (physical value bracketed). Gavai, SG, hep-lat/0208019 (2002)

$$\frac{T_c}{\Lambda_{\overline{MS}}} = \begin{cases} 0.49 \pm 0.02 & (m^2_{\pi^0}/m^2_{\pi^\pm} = 1) \\ 0.49 \pm 0.02 & (m^2_{\pi^0}/m^2_{\pi^\pm} = 0.78) \end{cases}$$

Both results extrapolated to the physical value of m_π/m_ρ.

Outline

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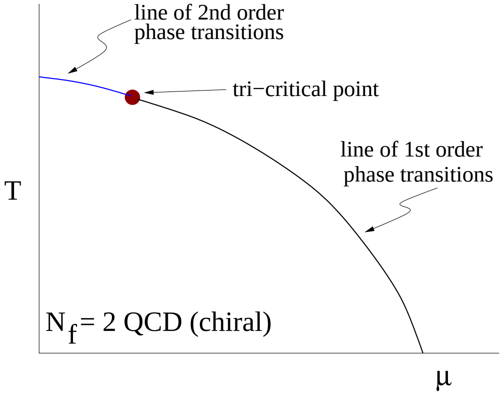

Rajagopal, Stephanov, Shuryak hep-ph/9806219 and hep-ph/9903292

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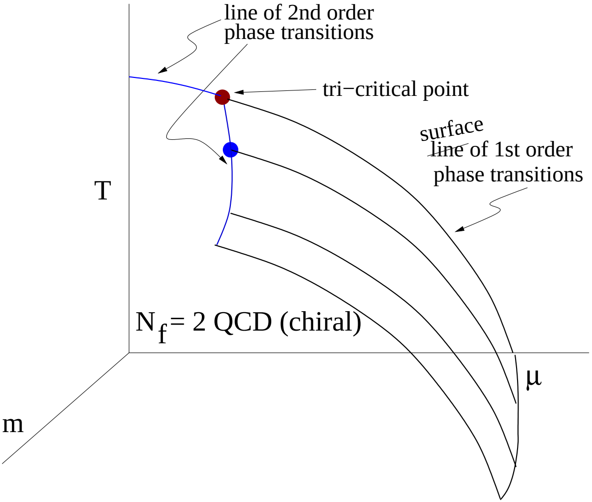

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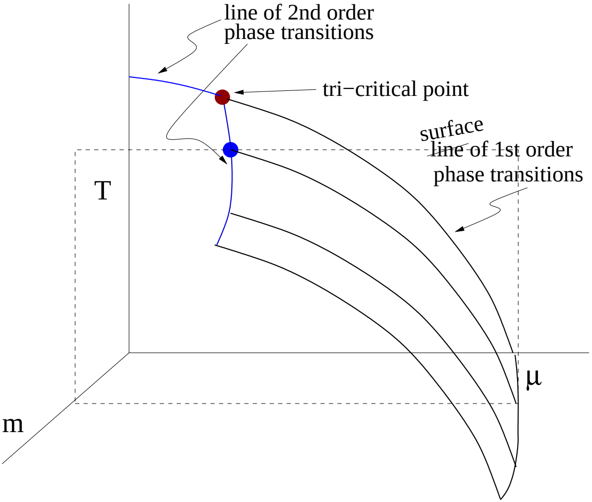

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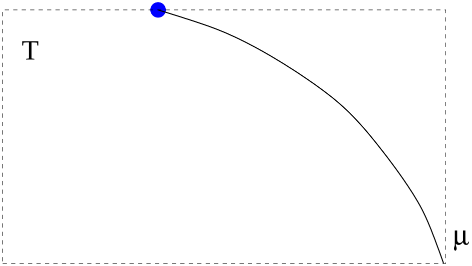

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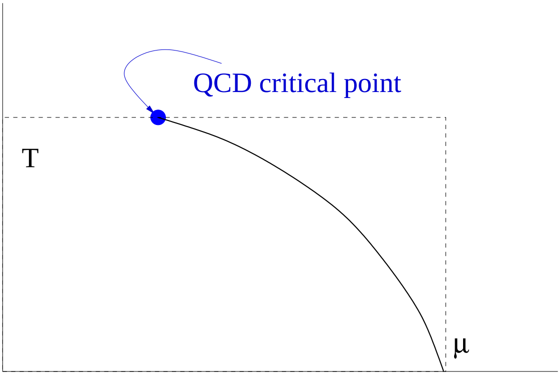

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Lattice setup

Lattice simulations impossible at finite baryon density: **sign problem**. Basic algorithmic problem in all Monte Carlo simulations: no solution yet.

Bypass the problem; make a Taylor expansion of the pressure:

$$P(T,\mu) = P(T) + \chi_B^{(2)}(T)\frac{\mu^2}{2!} + \chi_B^{(4)}(T)\frac{\mu^4}{4!} + \cdots$$

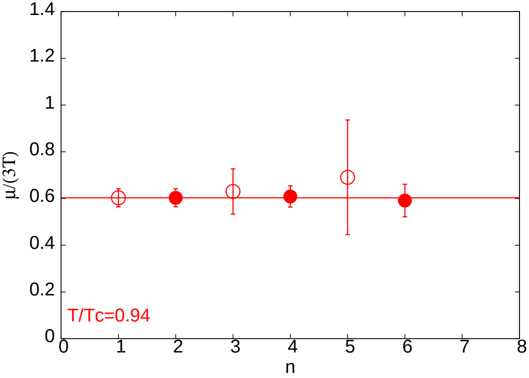
Series expansion coefficients evaluated at $\mu = 0$.
Implies

$$\chi_B^2(T,\mu) = \chi_B^{(2)}(T) + \chi_B^{(4)}(T)\frac{\mu^2}{2!} + \chi_B^{(6)}(T)\frac{\mu^4}{4!} + \cdots$$

Series fails to converge at the critical point.

, Gavai, SG, hep-lat/0303013 (2003)

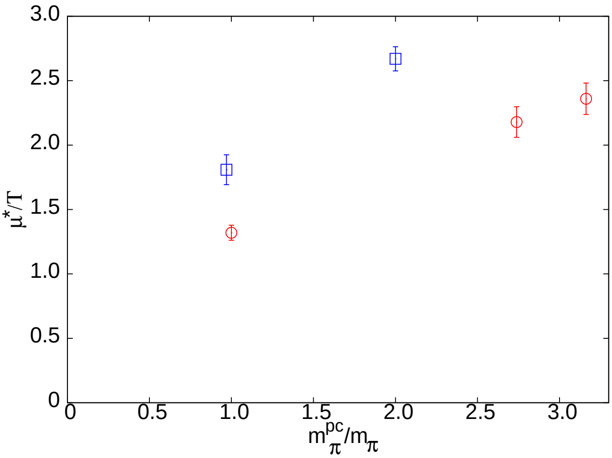
Series diverges



Radius of convergence of the series as a function of order ($a^{-1} = 1200$ MeV)

Gavai, SG, 0806.2233 (2008)

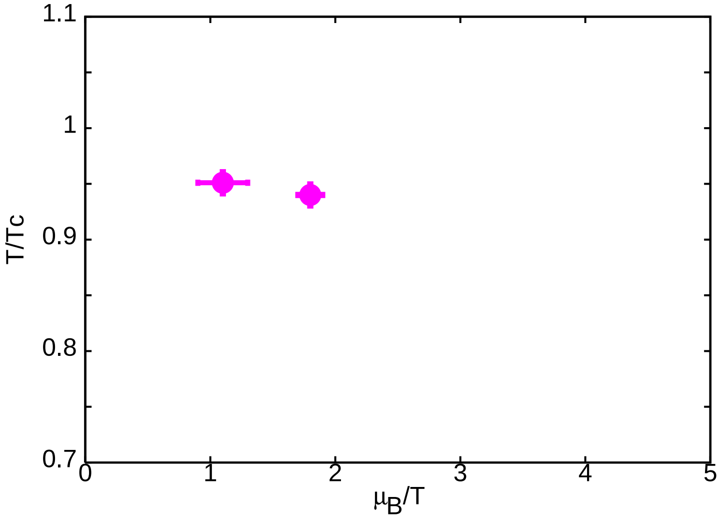
Dependence on quark mass



$a^{-1} = 800, 1200$ MeV

SG, hep-lat/0608022 (2006)

The critical point of QCD



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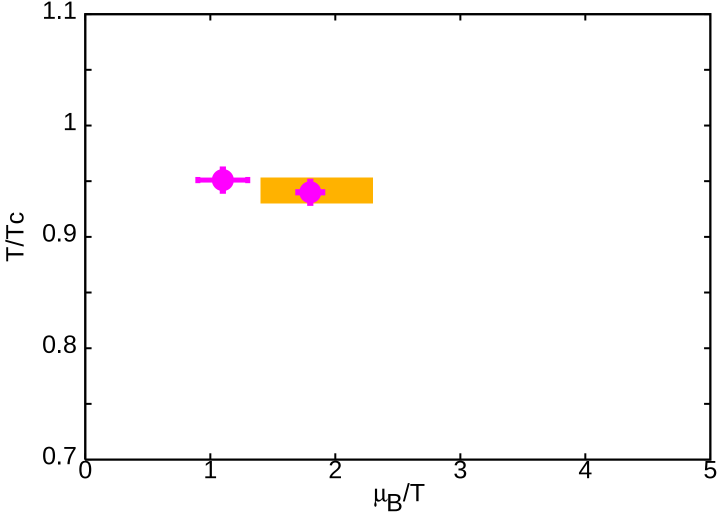
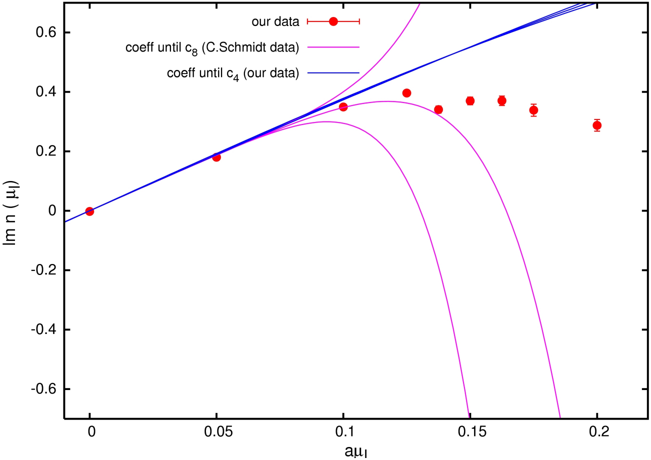
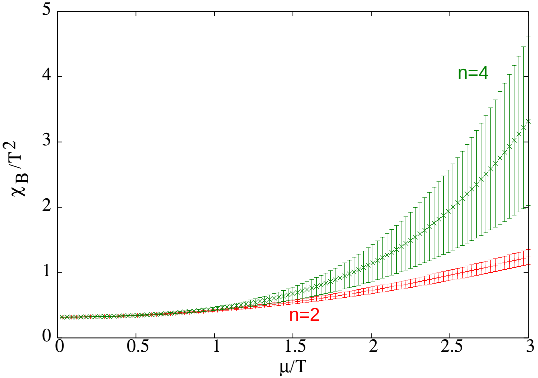


Direct test of extrapolation



Falcone, Laermann, Lombardo, Lattice 2010

Critical divergence: summation bad, resummation good



Infinite series diverges, but truncated series finite and smooth: sum is bad. Resummations needed to reproduce critical divergence. Padé resummation useful Gavai, SG, 0806.2233 (2008).

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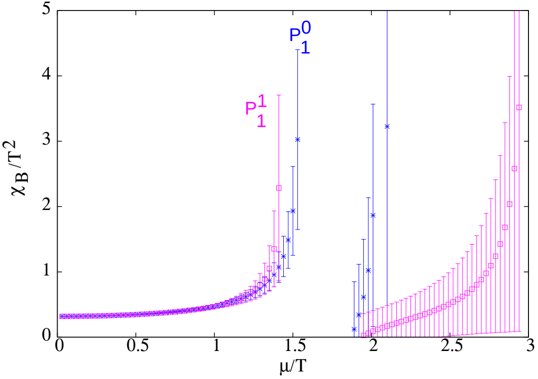

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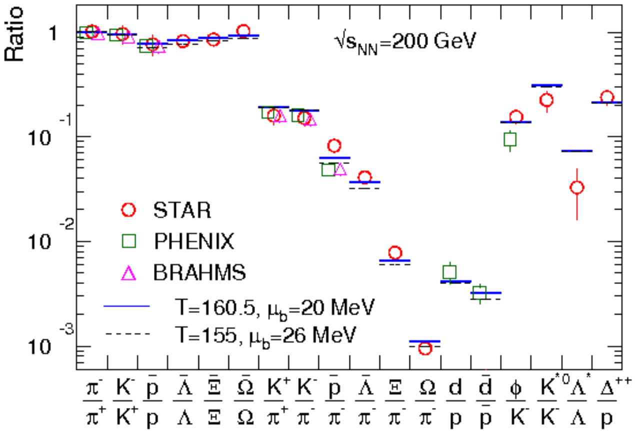
Outline

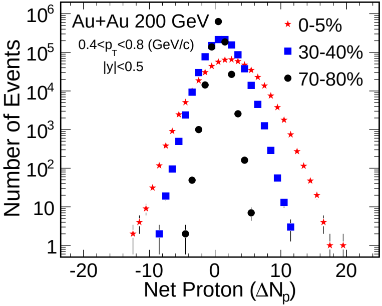
The fireball thermalizes



Thermal fit: $T = 160.5$ MeV, $\mu = 20$ MeV.

Andronic et al, nucl-th/0511071

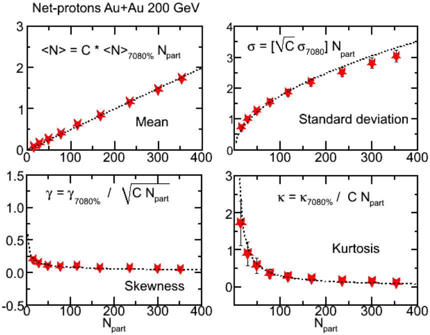
Event distributions of conserved charges



STAR, 1004.4959

- Fluctuations of conserved quantities are Gaussian: provided large volume and equilibrium
- Proton number a substitute for baryon number: how good?
- Is this Gaussian due (entirely or largely) to thermal fluctuations?

Look beyond Gaussian



STAR: QM 2009, Knoxville

- Higher cumulants scale down with larger powers of V.
- N_{part} is a proxy for V.
- Cumulants observed to scale correctly as N_{part}.
- Can one connect to QCD?

How to compare experiment with lattice QCD

The cumulants of the distribution are related to Taylor coefficients—

$$[B^2] = T^3 V \left(\frac{\chi^{(2)}}{T^2}\right), \quad [B^3] = T^3 V \left(\frac{\chi^{(3)}}{T}\right), \quad [B^4] = T^3 V \chi^{(4)}.$$

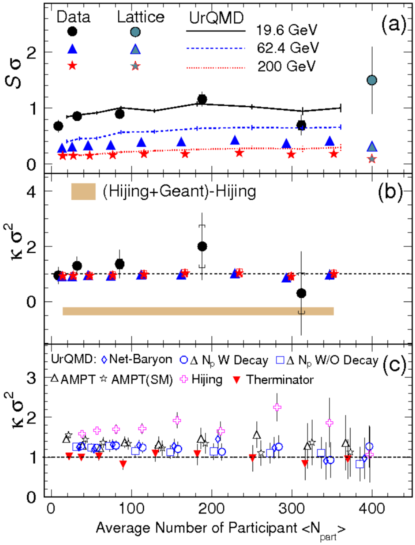
V is unknown, so direct measurement of QNS not possible. Define variance $\sigma^2 = [B^2]$, skew $\mathcal{S} = [B^3]/\sigma^3$ and Kurtosis, $\mathcal{K} = [B^4]/\sigma^4$. Construct the ratios

$$\mathcal{S}\sigma = \frac{[B^3]}{[B^2]}, \qquad \mathcal{K}\sigma^2 = \frac{[B^4]}{[B^2]}, \qquad \frac{\mathcal{K}\sigma}{\mathcal{S}} = \frac{[B^4]}{[B^3]}.$$

These are comparable with experiment provided lattice data extrapolated to relevant T and μ: use Padé approximants.

SG, 0909.4630

Extrapolate lattice data to finite μ



STAR Collaboration, 1004.4959 (2010)

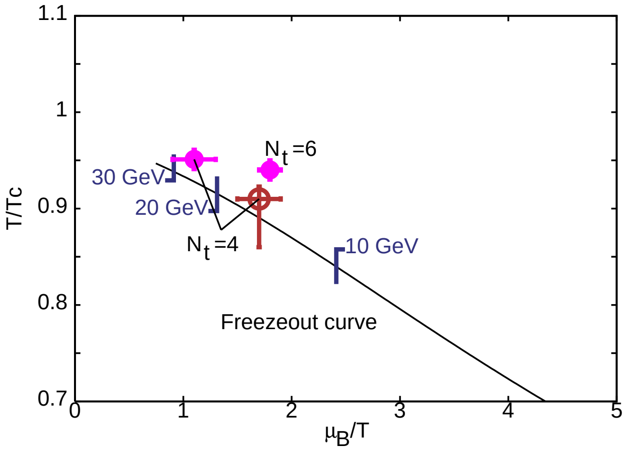
Surprising agreement with lattice QCD:

- implies non-thermal sources of fluctuations are very small
- T does not vary across the freezeout surface.
- tests QCD in non-perturbative thermal region

Gavai, SG, 1001.3796

Outline

The sign problem in QCD can be evaded



Lattice and experiments agree