Can STAR test non-perturbative QCD?

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ILGTI: TIFR

STAR Collaboration Meeting
Goa University
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Outline

The set of questions

Can STAR test any non-perturbative predictions of QCD?

In heavy-ion collisions QCD often enters indirectly: as the result of a long secondary computation such as hydro. Instead, can one get directly at QCD?

Can STAR test the existence of a critical point of QCD?

Do heavy-ion experiments have anything to say about the phase diagram? Or are they just dirtier versions of proton-proton collisions?

Outline

Non-linear susceptibilities

Taylor expansion of the pressure in μ_B

$$P(T,\mu_B+\Delta\mu_B)/T^4 = \sum_n \frac{1}{n!}\left[\chi^{(n)}(T,\mu_B)T^{n-4}\right]\left(\frac{\Delta\mu_B}{T}\right)^n$$

has Taylor coefficients called non-linear susceptibilities (NLS).
When $\mu_B = 0$ they can be computed directly on the lattice, otherwise reconstructed from such computations.
(Gavai, SG: 2003, 2010)
Cumulants of the event-to-event distribution of baryon number are directly related to the NLS:

$$[B^2] = T^3V\left(\frac{\chi^{(2)}}{T^2}\right), \quad [B^3] = T^3V\left(\frac{\chi^{(3)}}{T}\right), \quad [B^4] = T^3V\chi^{(4)}.$$

V unknown, can be removed by taking ratios.
(SG: 2009)

Tests and assumptions

$$m_1: \quad \frac{[B^3]}{[B^2]} = \frac{\chi^{(3)}(T,\mu_B)/T}{\chi^{(2)}(T,\mu_B)/T^2}$$

$$m_2: \quad \frac{[B^4]}{[B^2]} = \frac{\chi^{(4)}(T,\mu_B)}{\chi^{(2)}(T,\mu_B)/T^2}$$

$$m_3: \quad \frac{[B^4]}{[B^3]} = \frac{\chi^{(4)}(T,\mu_B)}{\chi^{(3)}(T,\mu_B)/T}$$

Also for cumulants of electric charge, Q, and strangeness, S.

1. Two sides of the equation equal if there is thermal equilibrium and no other sources of fluctuations.
2. Right hand side computed in the grand canonical ensemble (GCE). Can observations simulate a grand canonical ensemble? What T and μ_B?
3. Why should hydrodynamics and diffusion be neglected?

Why thermodynamics and not dynamics?

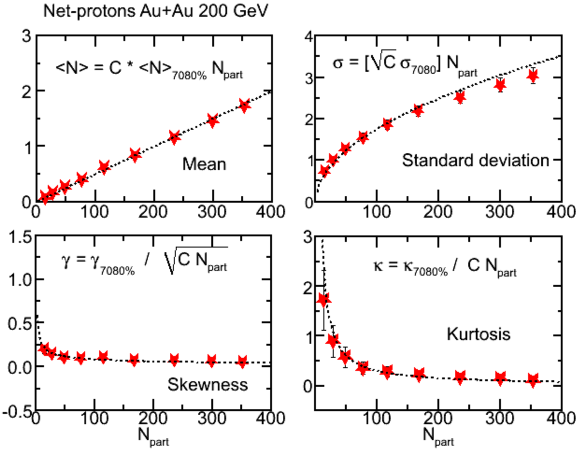
Chemical species may diffuse on the expanding background of the fireball, so why should we neglect diffusion and expansion?

First check whether the system size, ℓ, is large enough compared to the correlation length ξ: Knudsen's number $K = \xi/\ell$. If $K \ll 1$, ie, $\ell \gg \xi$ then central limit theorem will apply.
Next, compare the relative importance of diffusion and advection through a dimensionless number (Peclet's number):

$$\mathcal{W} = \frac{\ell^2}{t\mathcal{D}} = \frac{\ell v_{flow}}{\mathcal{D}} = \frac{\xi v_{flow}}{K\mathcal{D}} = \frac{v_{flow}}{Kc_s} = \frac{M}{K}.$$

When $\mathcal{W} \ll 1$ diffusion dominates. After chemical freeze-out K is small but Mach's number $M \simeq 1$, so flow dominates: fluctuations are frozen in. So detector observes thermodynamic fluctuations <u>at chemical freeze out</u>.
(Bhalerao, SG: 2009)

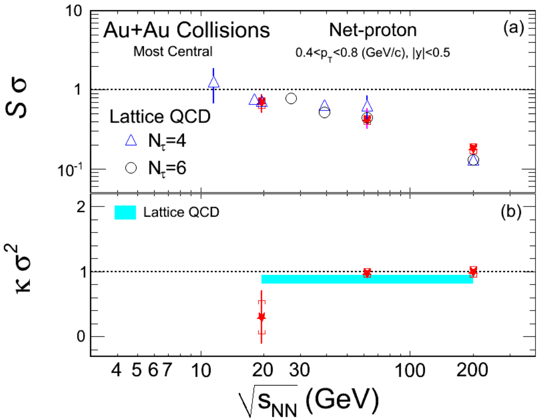
STAR measurements: 2009



$\ell \gg \xi$ ($K \ll 1$) tested and found true.

STAR Collaboration: QM 2009, Knoxville

STAR measurements: 2010

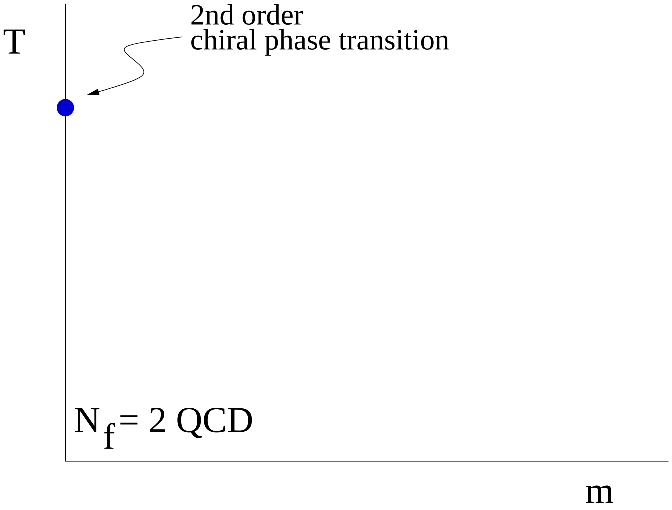


First ever agreement between lattice and experiment!

STAR Collaboration: 2010

Outline

The two flavour phase diagram at $\mu = 0$



Pisarski and Wilczek, PR D 29, 338 (1984)

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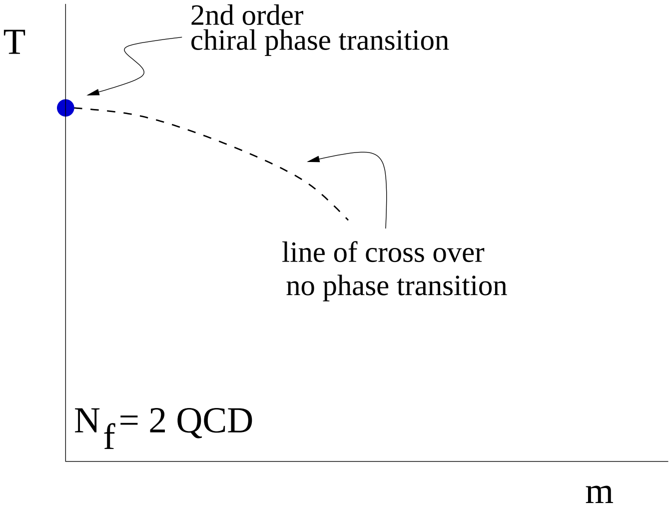

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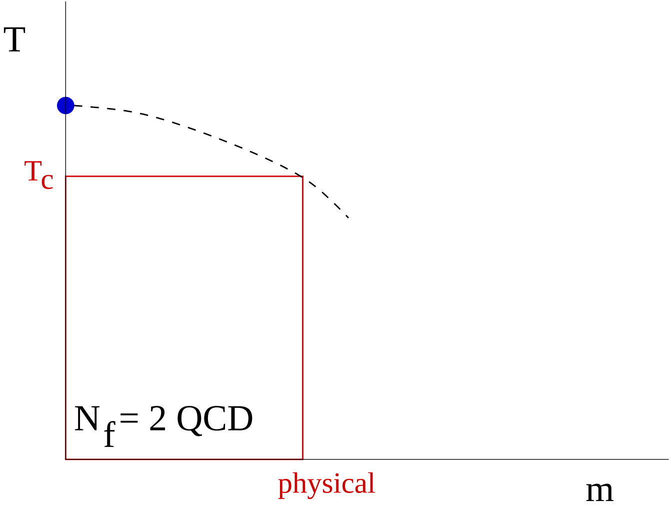

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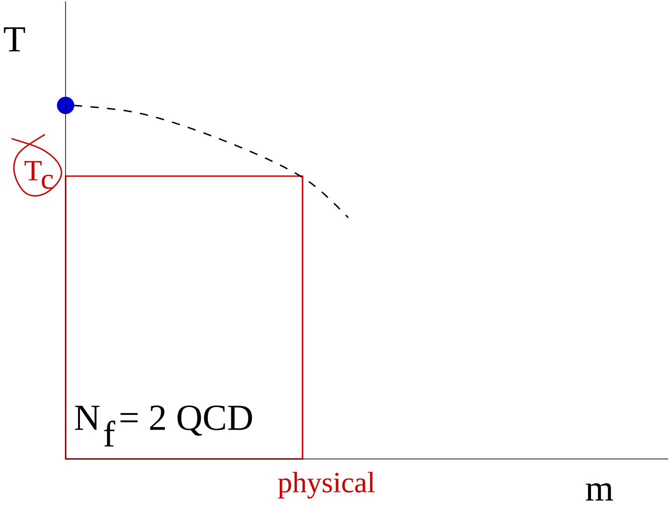

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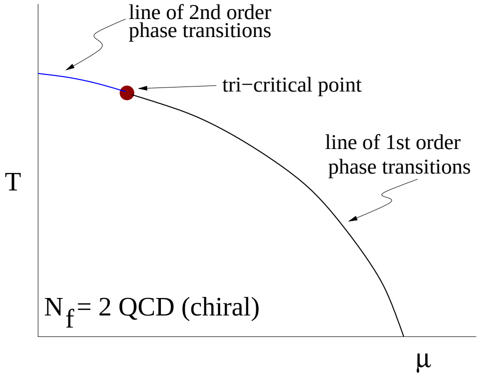

Rajagopal, Stephanov, Shuryak hep-ph/9806219 hep-ph/9903292

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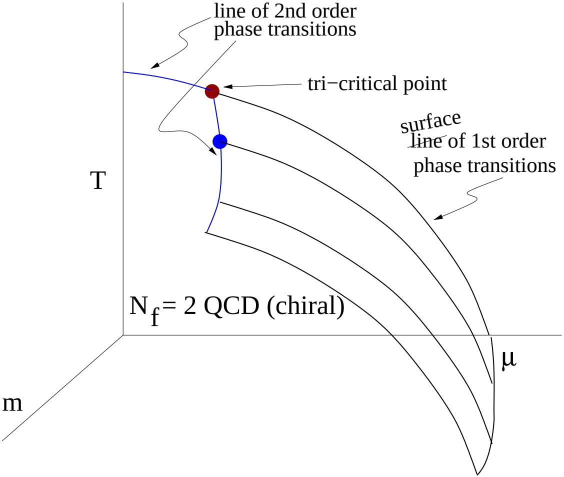

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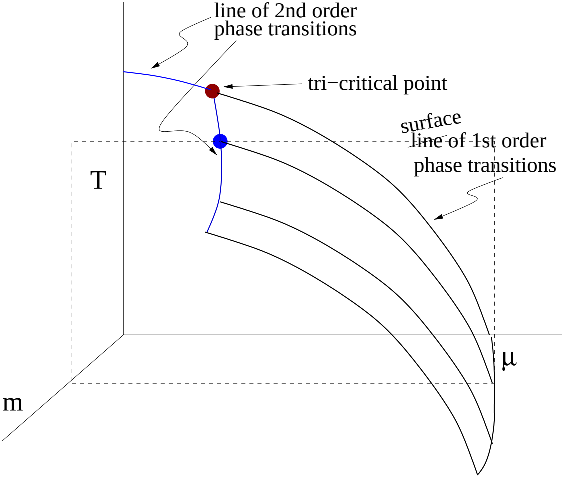

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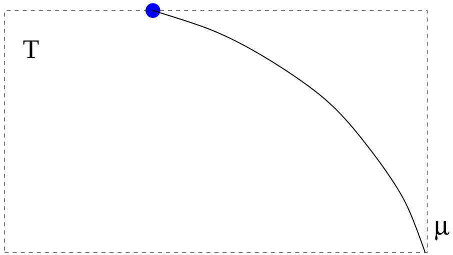

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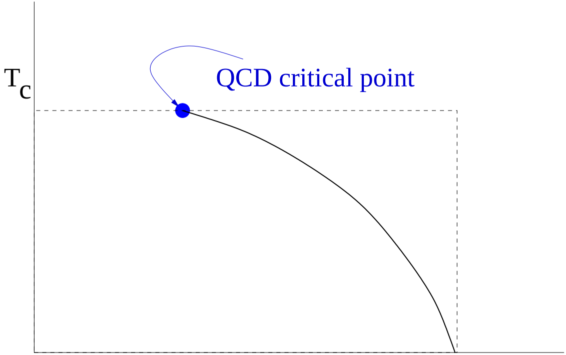

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Lattice computation of the critical end point

Taylor expansion of pressure gives the Taylor expansion of the quark number susceptibility (QNS):

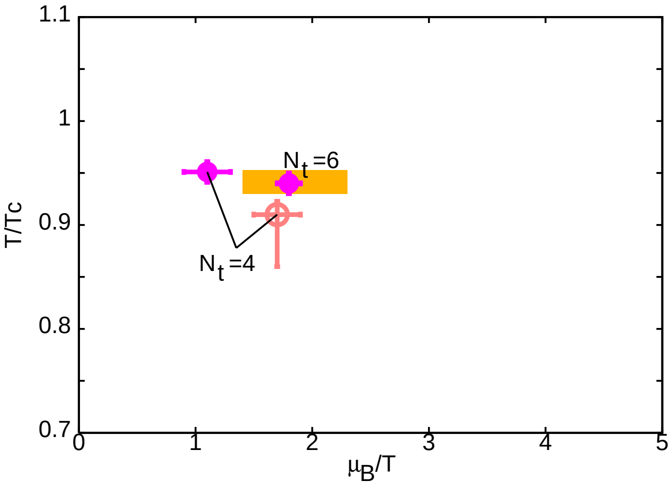
$$\frac{\chi^{(2)}(T,\mu_B)}{T^2} = \sum_{n=2}^{\infty} \frac{1}{(n-2)!}\chi^{(n)}(T)T^{n-4}\left(\frac{\mu_B}{T}\right)^{n-2}$$

This diverges at the critical end point (T^E, μ_B^E). Look for divergence of the series.

Gavai, SG: 2005, 2008. Schmidt: 2010

Do lattice simulations at finite temperature and $\mu_B = 0$; measure NLS; find the radius of convergence of the series. If regularities found in the radius of convergence (FSS), then this gives the critical end point.

Results



Composite figure: Lattice review 2010.

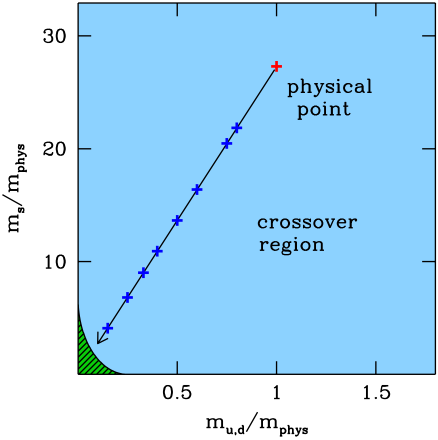
Become a lattice pundit in 2 minutes

When you see a lattice computation ask the following:

- How large is the lattice? Is $\zeta = LT$ varied? In ILGTI computations these are varied from $\zeta = 2$ to 4, 6. RBRC chooses $\zeta = 4$. Older computations often used $\zeta \leq 2$: too small.
- How heavy are the quark masses? In QCD tuning the quark mass changes m_π. ILGTI and RBRC (now) tune bare quark masses to give $m_\pi = 230$ MeV.
- How small is the lattice spacing? What quark action is used? ILGTI uses lattice spacing $1/a \simeq 1200$ MeV with staggered quarks; RBRC uses $1/a \simeq 800$ MeV with P4 quarks. The results should agree in the continuum ($1/a \to \infty$); they are beginning to agree now.

Is the strange quark an issue?

Since $m_{u,d} \ll T$ light flavours are important. But $m_s \simeq T$. Do strange quarks matter? Lattice computations say unquenching the strange quark may not be a big issue.



In $N_f = 2 + 1$:

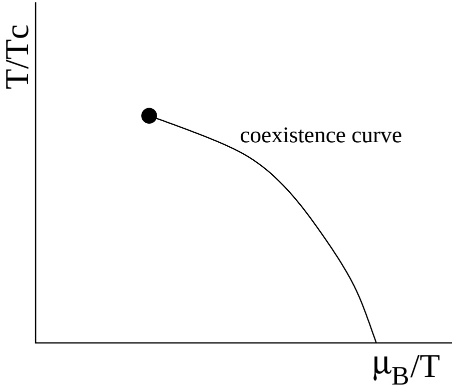
$$m_\pi^{crit} \begin{cases} = 0.07 m_\pi & (N_t = 4) \\ < 0.12 m_\pi & (N_t = 6) \end{cases}$$

Endrodi etal, 0710.0988

Similarly for $N_f = 3$.

Karsch etal, hep-lat/0309121

Look for critical divergences?



Eigendirections of RG: t and h. Unknown. Model results?
If $\chi_B^{(2)} \simeq 1/|\mu - \mu_B|^{\phi}$ then $\chi_B^{(n)} \simeq 1/|\mu - \mu_B|^{(\phi+n-2)}$. Critical index ϕ close to Ising magnetic exponent.

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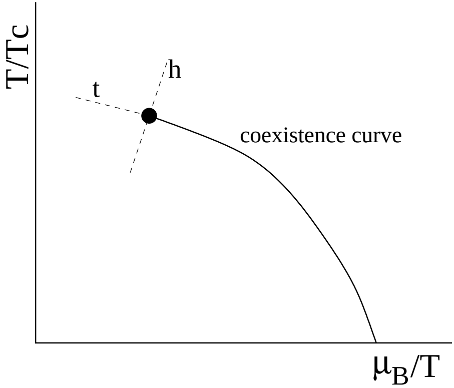

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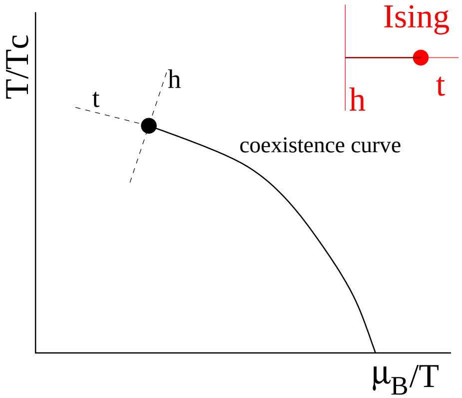

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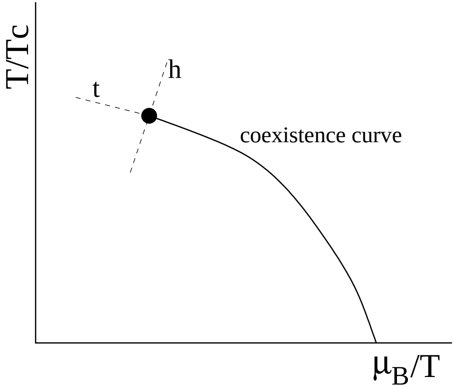

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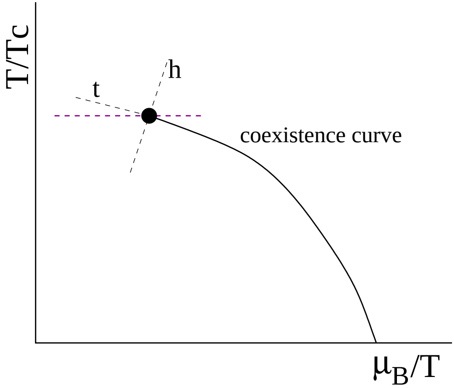

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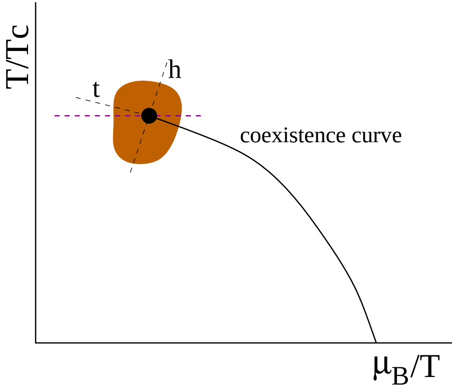

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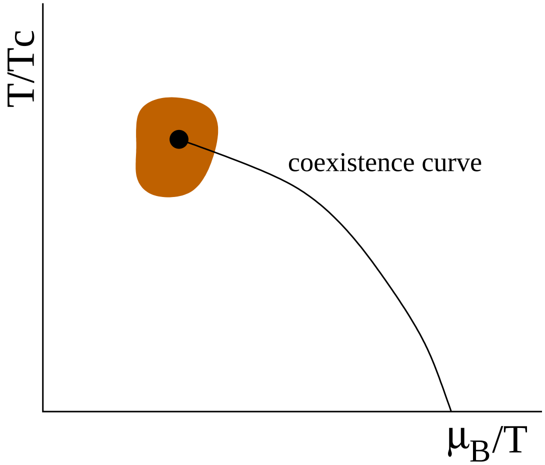
Finite size effects damp divergences

1. System size limits correlation lengths near the critical point: $\ell \simeq \xi$. The Knudsen number is never small near the CEP, so central limit theorem will stop working. Check the scaling of σ^2, $\mathcal{S}$ and $\mathcal{K}$ and see whether there are violations of the central limit theorem. (SG: 2009)
2. As a result, the Peclet number need not be large, and diffusion may play an important role even close to kinetic freeze-out. Then fluctuations of conserved quantities may not be comparable to thermal equilibrium values at chemical freeze-out!
3. Another way of saying this is: critical divergences are limited due to finite size effects: no singularities, hence no direct measurement of the critical exponents. System drops out of equilibrium due to finite lifetime. (Stephanov: 2008; Berdnikov, Rajagopal: 1998)

Can experiments see the critical point?



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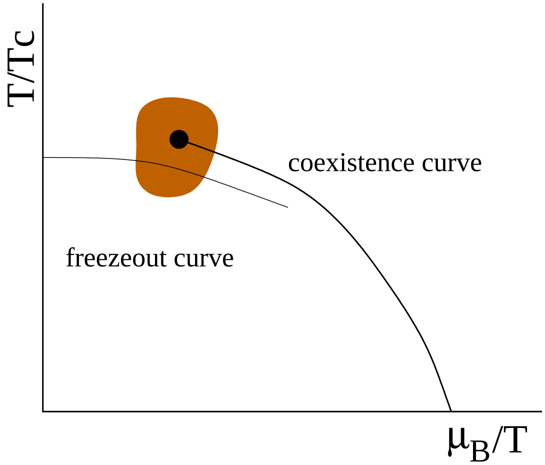

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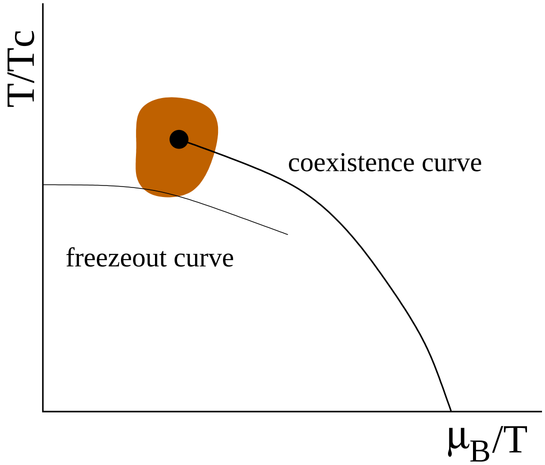

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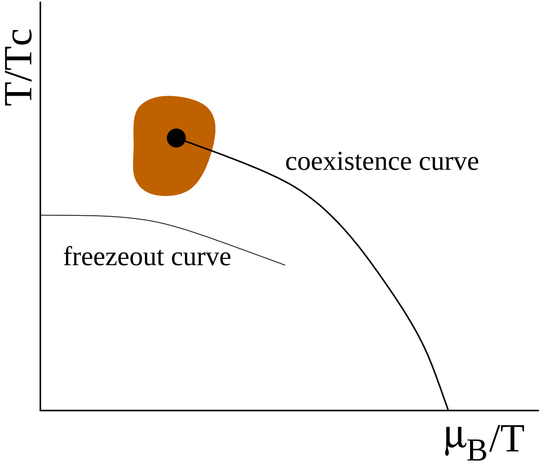
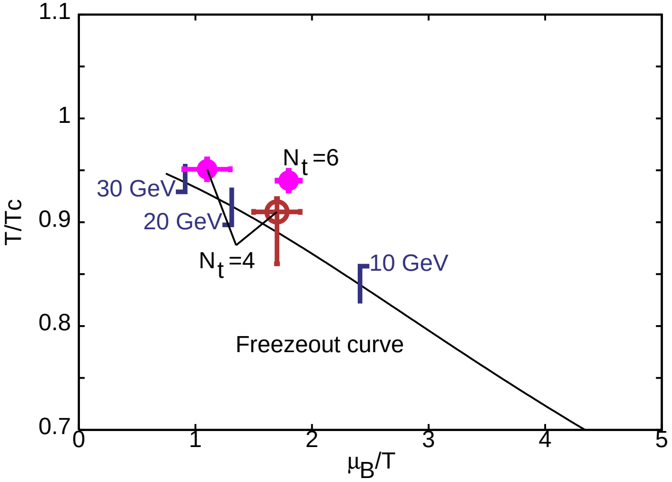
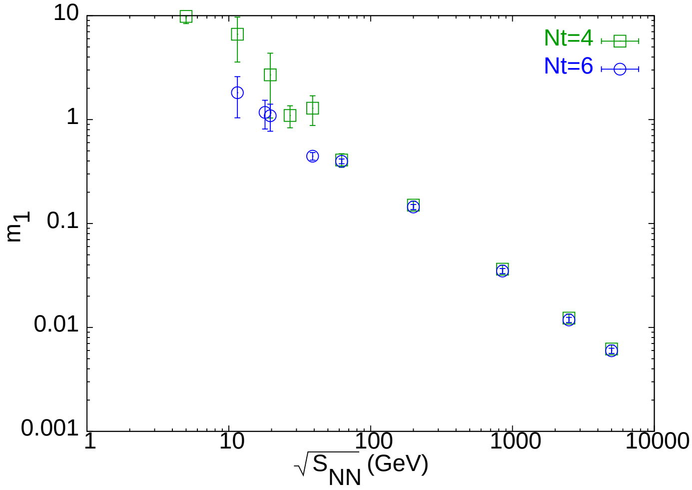


We are so lucky



Look at m_1, m_2 and m_3 along the freeze-out curve.

$m_1 = \mathcal{S}/\sigma$



Gavai, SG: 2010

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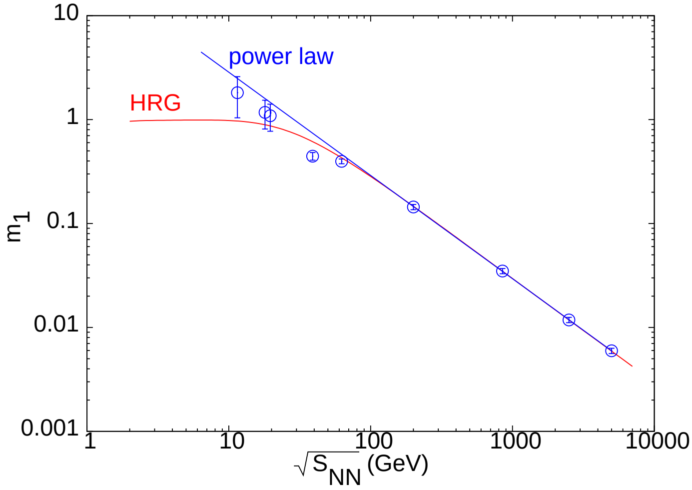

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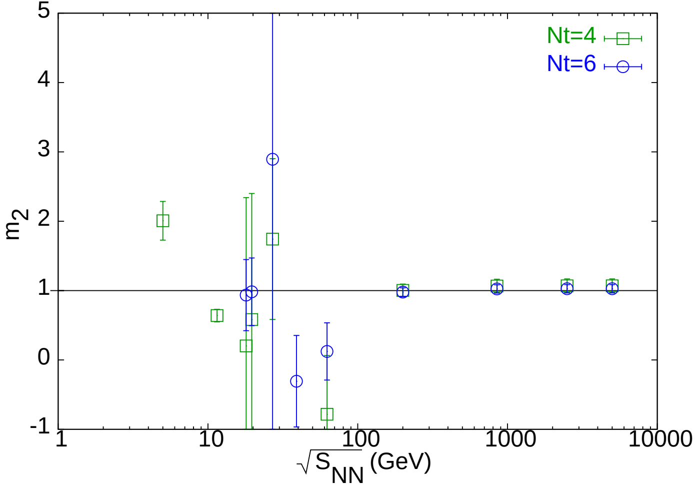
$m_2 = \mathcal{K}\sigma^2$



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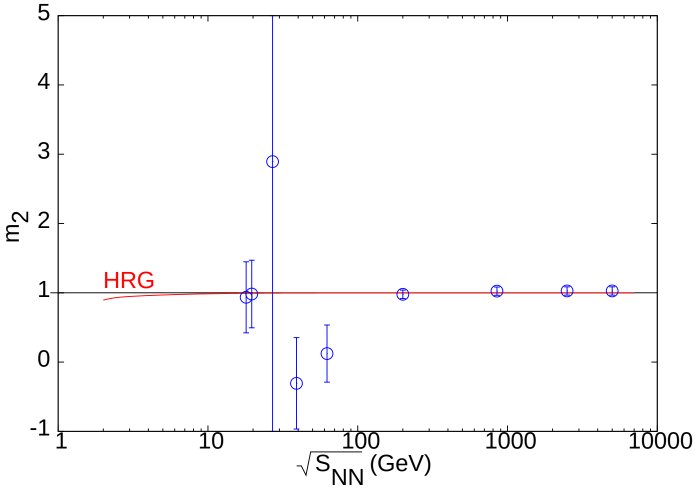

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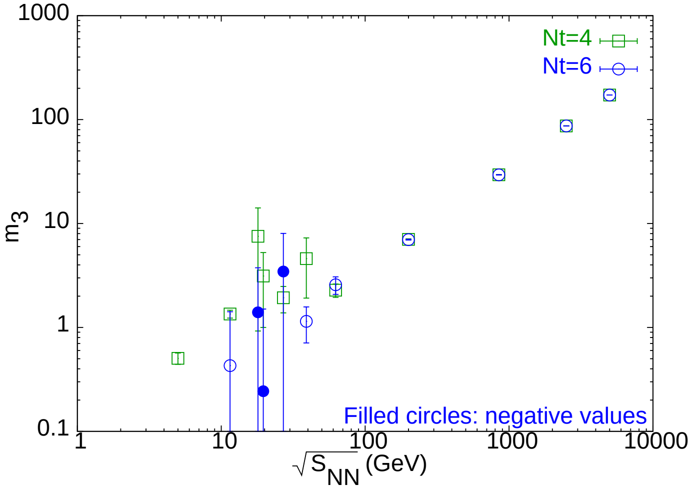

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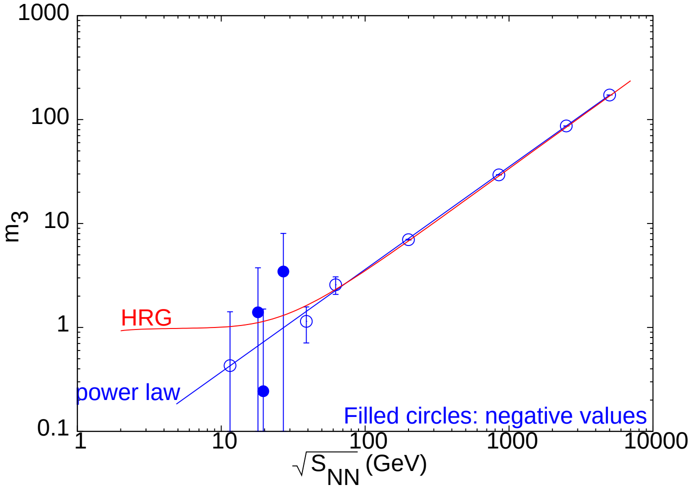


Gavai, SG: 2010

Outline

CEP search scheme

- Exciting result from STAR: first comparison of lattice predictions with experiment. First direct test of QCD in a medium!
- Straightforward arguments lead us to believe that there are strong finite size effects near a critical point which lead to departures from thermal equilibrium.
- Check for large finite size effects: does the central limit theorem break down? At each $\sqrt{S}$ repeat scaling with $N_{\rm part}$ analysis already performed by STAR at $\sqrt{S} = 200$ GeV.
- Lattice QCD predicts that near the CEP the Kurtosis, $\mathcal{K}$, turns negative. Does experiment see this? m_1 should remain positive and both m_2 and m_3 should turn negative.
- Does thermal equilibrium break down? Is there a departure from the lattice computation?

Outline

The meaning of errors and error propagation

One usually uses the central limit theorem to say that the error in a measurement, Δx, is related to the variance of the measurement, $\sigma^2(x)$, and the number of measurements, N, by the formula

$$(\Delta x)^2 = \frac{\sigma^2(x)}{N}.$$

This works if the distribution of the estimates of the measurements is Gaussian. The error is the 65% confidence limit.

It is very tempting to use standard error analysis for products and ratios of numbers. If $t = x/y$ and then one usually writes

$$\left(\frac{\Delta t}{t}\right)^2 = \left(\frac{\Delta x}{x}\right)^2 + \left(\frac{\Delta y}{y}\right)^2.$$

What does this mean?

The error formula for ratios is meaningless

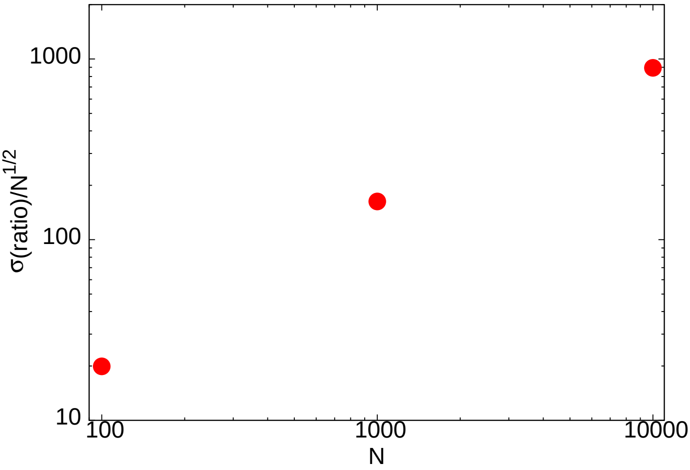
One can prove that the ratio (or product) of two Gaussian distributed numbers is not Gaussian. In fact the ratio has infinite variance! So the formula for error propagation gives some Δt which has no meaning as a confidence limit. In fact, since the variance is infinite, such a Δt is perfectly meaningless. (Geary: 1930)

You do not believe this?

Do a Monte Carlo experiment. Draw two numbers x and y each from a Gaussian distribution. Take the ratio t. Repeat this process N times so you have N values of x, y and t. Now compute the variance of each quantity. As you increase N you find $\sigma^2(x)$ and $\sigma^2(y)$ are finite numbers but $\sigma^2(t)$ grows without bound.

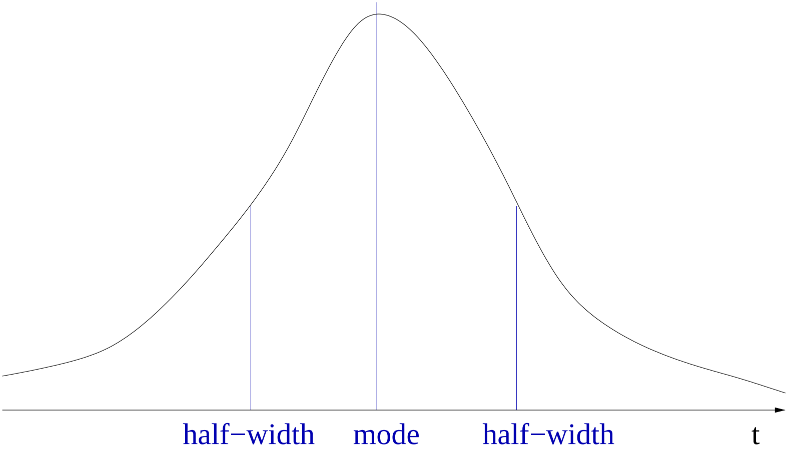
Do not use the error propagation formula for ratios. Other methods are possible.

The variance of a ratio grows with statistics



Result of a simple Monte Carlo test.

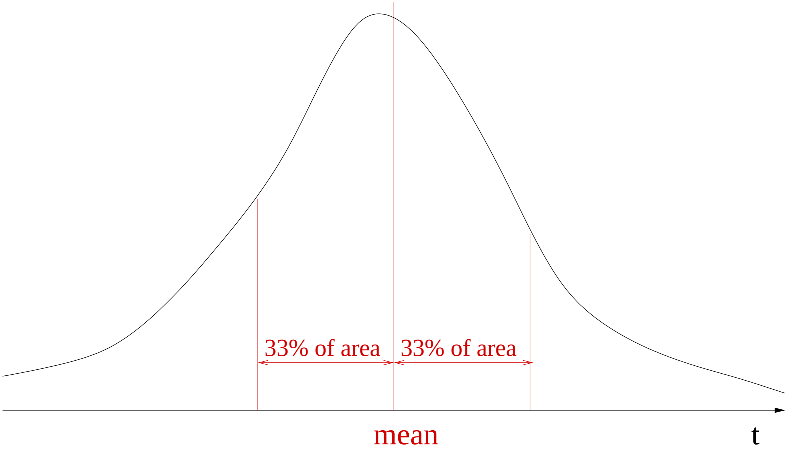
Alternatives to variance



Measures of error exist even when variance is infinite.

Phase transitions and statistics are very closely linked!

Alternatives to variance



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