Finite size scaling on the phase diagram of QCD

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Fluctuations, Correlations and RHIC Low Energy Runs
BNL USA
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Outline

The context

Experimental observations

Many interesting new phenomena: jet quenching, elliptic flow, strange chemistry, fluctuations of conserved quantities ...

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Does thermodynamics apply to the fireball? Yes, for chosen observables. Does QCD describe this thermodynamics? Yes. Improvements ongoing for both answers.

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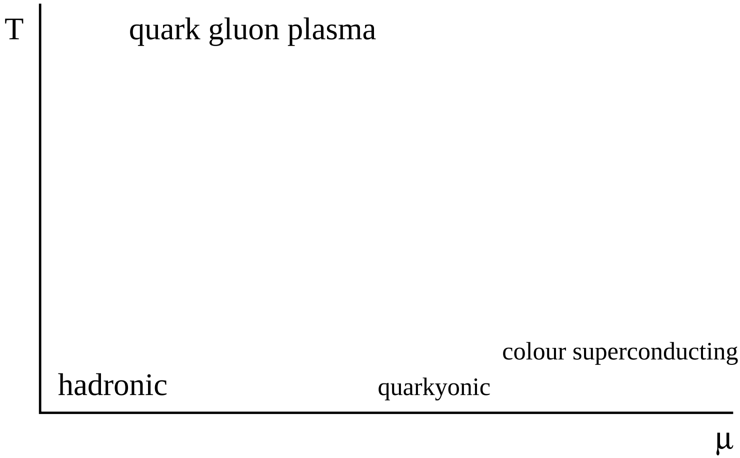

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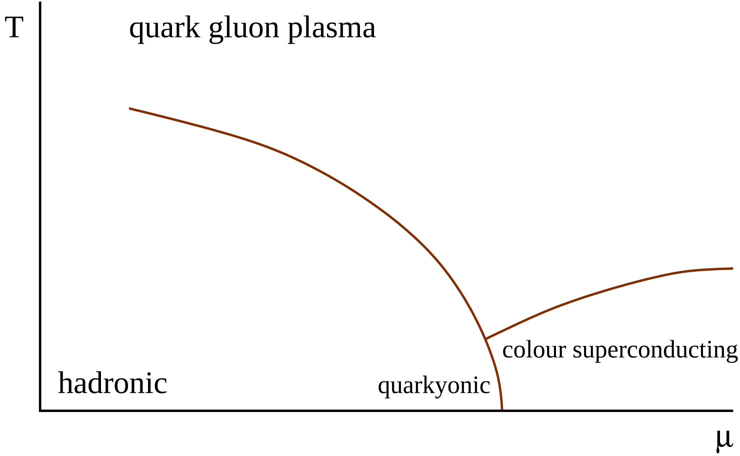

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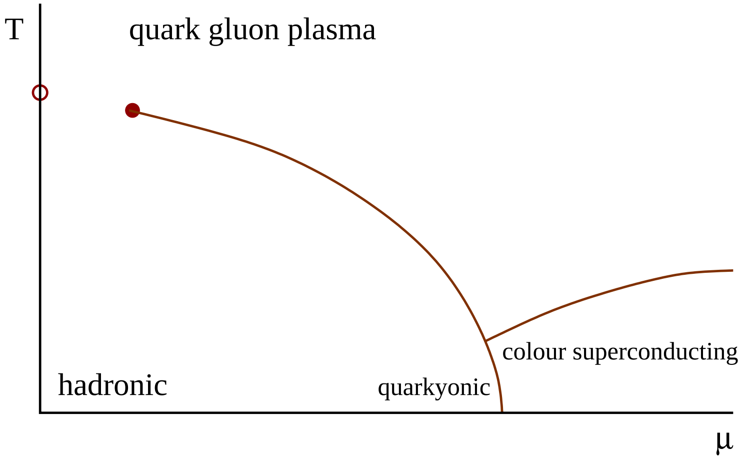

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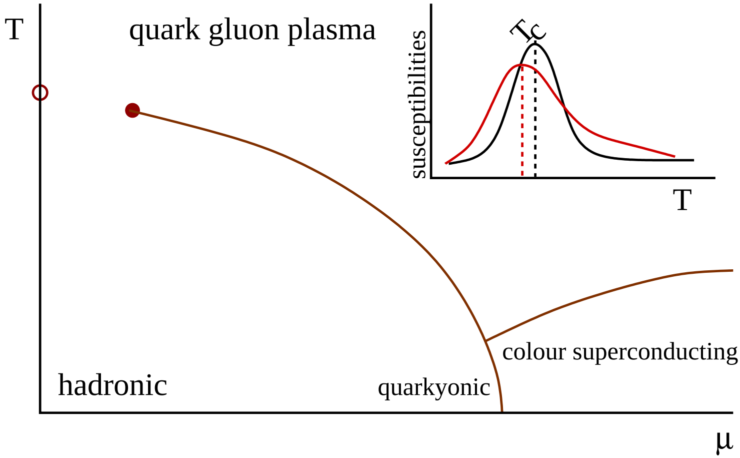

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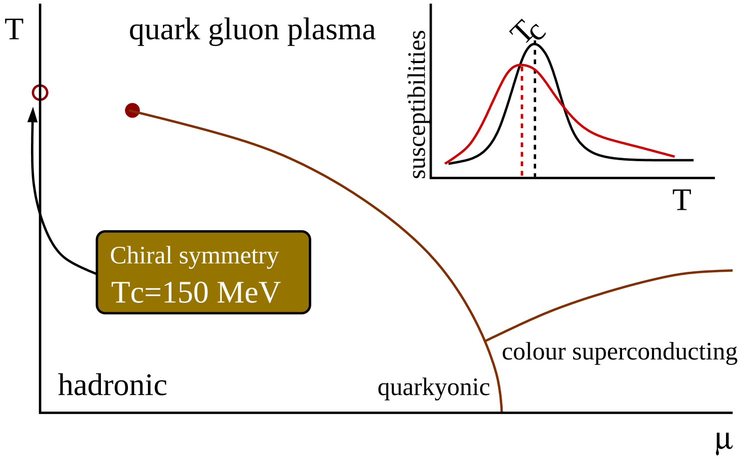

Y. Aoki *et al.*, Phys. Lett. B 643 (2006) 46

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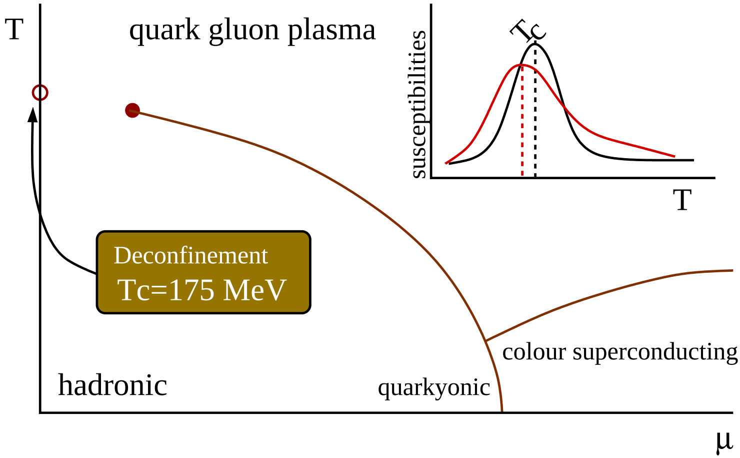

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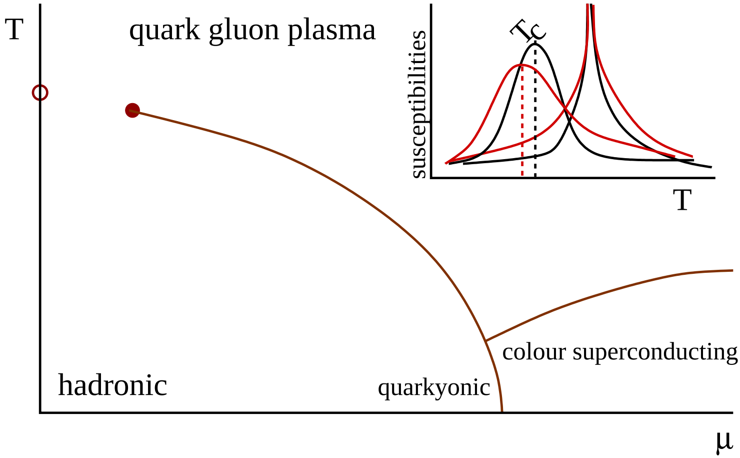

Gavai and Gupta, Phys. Rev. D 71 (2005) 110414, D 78 (2008) 114503

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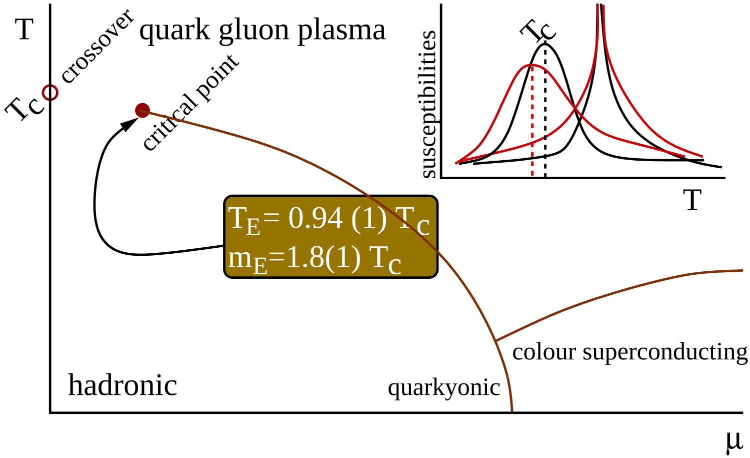

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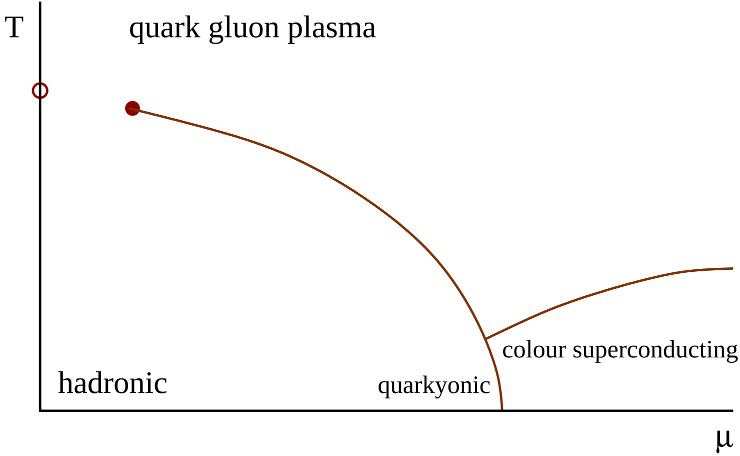

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Outline

Thermodynamics and fluctuations

Observations

In a single heavy-ion collision, each conserved quantity (B, Q, S) is exactly constant when the full fireball is observed. In a small part of the fireball they fluctuate: from part to part and event to event.

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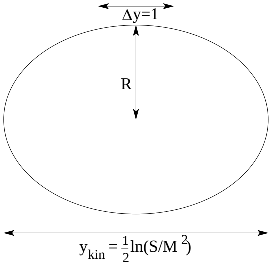
Comparison

When $V_{obs} \ll V_{fireball}$, Gaussian as $V_{obs}/\xi^3 \to \infty$. Finite size effects are mainly controlled by NLS. Otherwise system is in the critical regime.

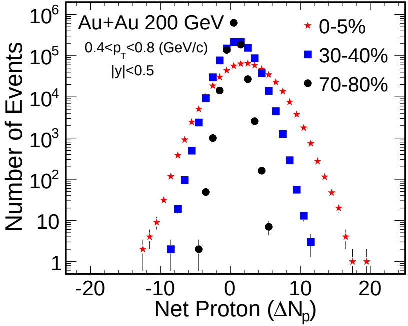
Typical sizes



$\sqrt{S} = 200$ GeV

1. Freezeout occurs at $T \simeq 150$ MeV, where $\xi T < 0.5$.
2. If, $R = 10$ fm, then $V_{fireball}/\xi^3 = \mathcal{O}(10^3)$.
3. $V_{fireball}/V_{obs} \simeq y_{kin} = 2\ln(\sqrt{S}/M_p) \simeq 10$.
4. As a result, $V_{obs}/\xi^3 = \mathcal{O}(10^2)$.

Event-to-event fluctuations



STAR arxiv:1004.4959

Central rapidity slice taken. p_T of 400–800 MeV. Important to check dependence on impact parameter. Protons observed: okay if isospin fluctuations small.

STAR 2010; Asakawa, Kitazawa: 2011

Grand canonical thermodynamics

When $V_{obs}/\xi^3 \to \infty$ and $V_{fireball}/V_{obs} \to \infty$, then thermodynamics in the grand canonical ensemble works; all distributions of conserved quantities are Gaussian.

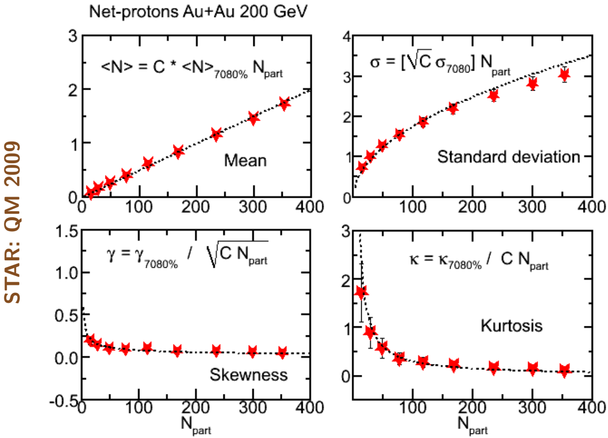
For a Gaussian the only non-vanishing cumulants are the mean, $[B]$, and the variance $[B^2]$. Observation of any other non-vanishing cumulant $[B^n]$ is a finite size effect. Since these cumulants are given by the NLS,

$$[B^n] = (VT^3)T^{n-4}\frac{\partial^n P(T,\mu)}{\partial\mu^n},$$

QCD determines finite size effects as well as the thermodynamic limit.

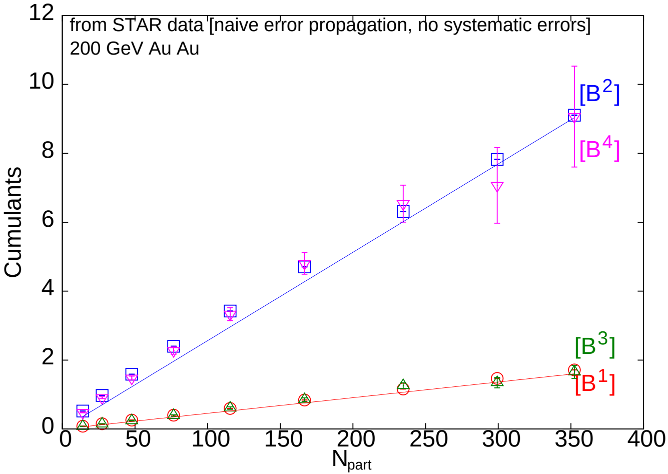
Test of lack of criticality: trivial volume dependence of cumulants, *i.e.*, all cumulants scale as V.

Shape of distribution



Combinations of cumulants: $\sigma^2 = [B^2]$, $S = [B^3]/\sigma^3$, $\kappa = [B^4]/\sigma^4$, change with volume (proxy: N_{part}).

Evolution of shape



Central limit theorem follows. Scaling implies $\xi^3 \ll V_{obs}$ at some $\sqrt{S}$.

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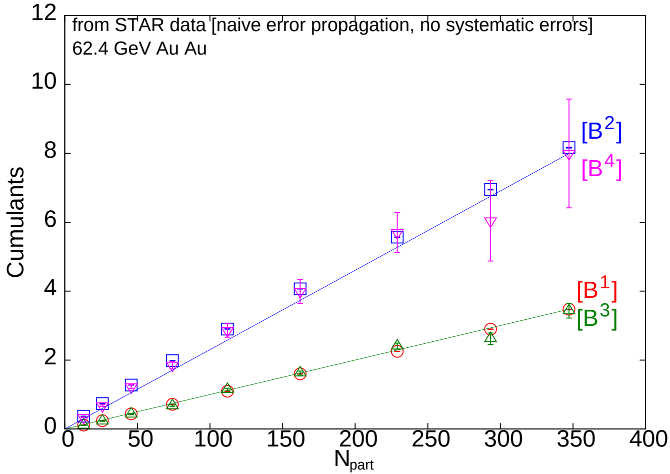

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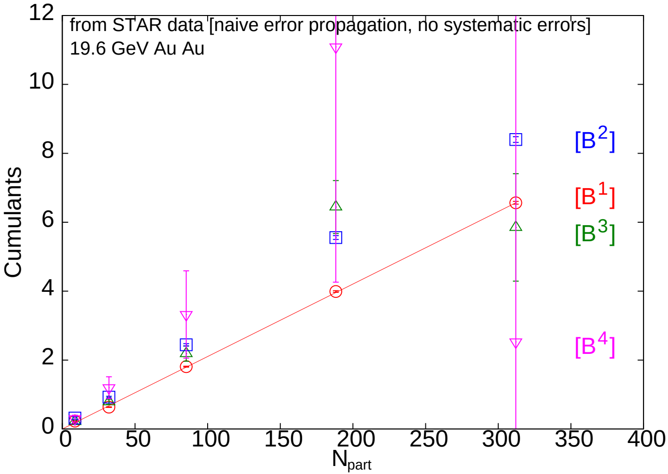

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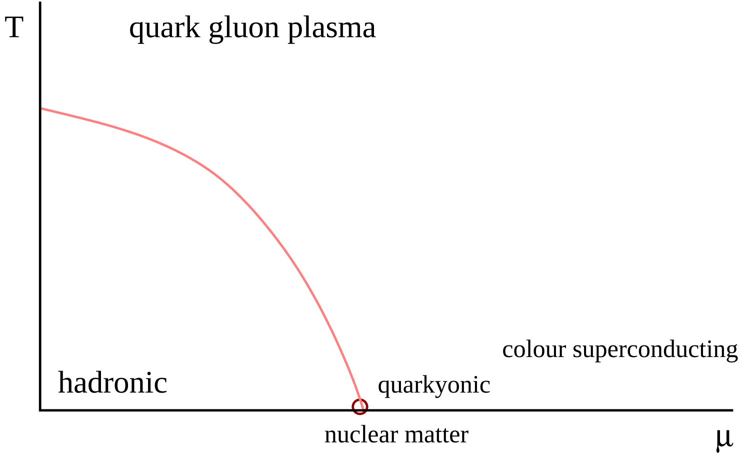
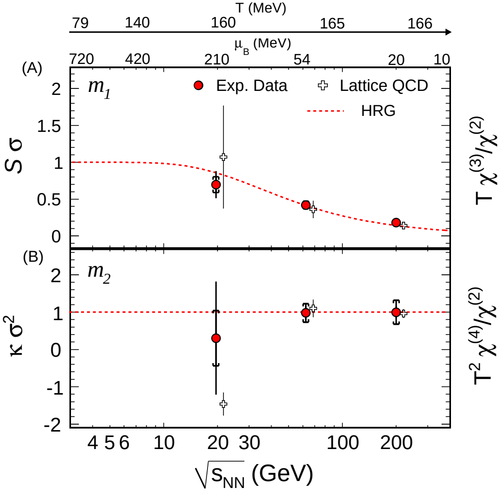
The freezeout curve



Hadron gas models: Becattini, Braun-Munzinger, Stachel, Cleymans, Redlich, ...

Checking the match



Science, 332 (2011) 1525

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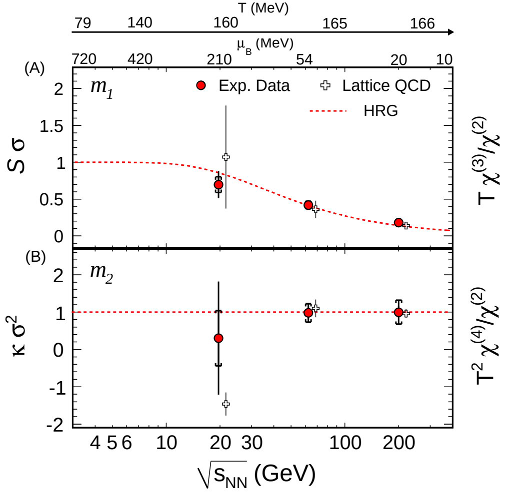

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$T_c = 175^{+1}_{-7}$ MeV

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Effect of flow

Out of control diffusion

If fireball static, then control of diffusion requires the hierarchy $\xi^3 \ll V_{obs} \ll V_{fireball}$. When $\sqrt[3]{V_{obs}} \simeq \xi$ then microscopic physics of transport controls observed distributions. This happens in the critical regime. Also turbulent?

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Other scales?

But plasma ball is not static, and new length scales become important.

Diffusion-advection phenomena

Entropy content in B or S small compared to entropy content of full fireball. Coupled relativistic hydro and diffusion equations can then be simplified to diffusion-advection equation.

Which is more important— diffusion or advection? Examine Peclet's number

$$\mathrm{Pe} = \frac{\lambda v}{D} = \frac{\lambda v}{\xi c_s} = M\frac{\lambda}{\xi}.$$

When $\mathrm{Pe} \ll 1$ diffusion dominates. When $\mathrm{Pe} \gg 1$ advection dominates. Crossover between these regimes when $\mathrm{Pe} \simeq 1$.

Bhalerao and SG, 2009

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Advective length scale

New length scale: defines when advection becomes comparable to diffusive evolution—

$$\lambda \simeq \frac{\xi}{M}.$$

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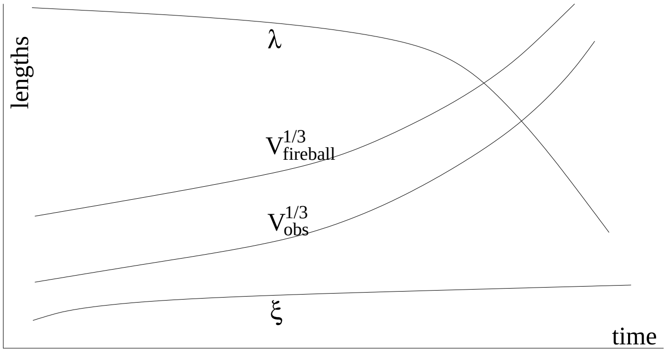
Peclet phenomenology



λ remains fairly constant until time R_0/c_s then falls rapidly as expansion becomes fully 3d. So freezeout time is not very strongly dependent on rapidity window.

Bhalerao and SG

Finite volumes: density sets a scale

When the total number of baryons (baryons + antibaryons) detected is B_+, the volume per detected baryon is $\zeta^3 = V_{obs}/B_+$. If $\zeta \simeq \xi$ then system may not be thermodynamic: controlled when $V_{obs}/\xi^3 \to \infty$.
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On cumulant order

In central Au Au collisions, the measurement of $[B^6]$ involves $\zeta/\xi \simeq 2$. Could it be used to study transport? Probe this by separating out samples with large B_+ and studying their statistics.

Outline

Length scales in a fireball

1. Scale of the persistence of memory, $V_{fireball}$. When $V_{fireball}/V_{obs} \gg 1$ then total charge of the system forgotten. May not hold at small $\sqrt{S}$.
2. Shortest length scale ξ, defined by transport: the diffusion constant. Scale at which baryon number transport becomes important.
3. Scale of observation volume, V_{obs}. Set by the detector. Thermodynamics and finite size scaling applicable for $V_{obs}/\xi^3 \gg 1$. Comparison to lattice works when $\xi^3 \ll V_{obs} \ll V_{fireball}$.
4. Peclet scale, $\lambda = \xi/M$ (where M is the Mach number). Controls freeze out of fluctuations.
5. Volume per unit baryon number, $\zeta^3 = V_{obs}/B_+$. Events with $\zeta \simeq \xi$, may not be observed at thermodynamic frequency because of slow diffusion.

Backup: predictions from QCD

- Lagrangian has free parameters: cutoff a, quark masses $m_u \simeq m_d \ll \Lambda_{QCD}$, $m_s \simeq \Lambda_{QCD}$, $\cdots$
- Compute enough quantities from QCD: $m_\pi(a, m_{ud}, m_s, \cdots)$, $m_K(a, m_{ud}, m_s, \cdots)$, $f_K(a, m_{ud}, m_s, \cdots)$, $f_\pi(a, m_{ud}, m_s, \cdots)$, $m_\rho(a, m_{ud}, m_s, \cdots)$, $m_p(a, m_{ud}, m_s, \cdots)$, $T_c(a, m_{ud}, m_s, \cdots)$, $T_E(a, m_{ud}, m_s, \cdots)$, $\mu_E(a, m_{ud}, m_s, \cdots)$
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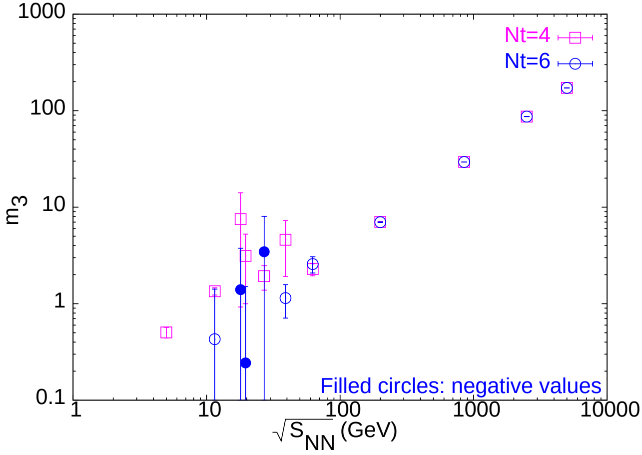
- Most universal part of the solution: Moore's law

Backup: Predictions along the freezeout curve



Gavai, SG (2010)

Lattice predictions along the freezeout curve of HRG models using $T_c = 170$ MeV.