Finite size scaling on the phase diagram of QCD

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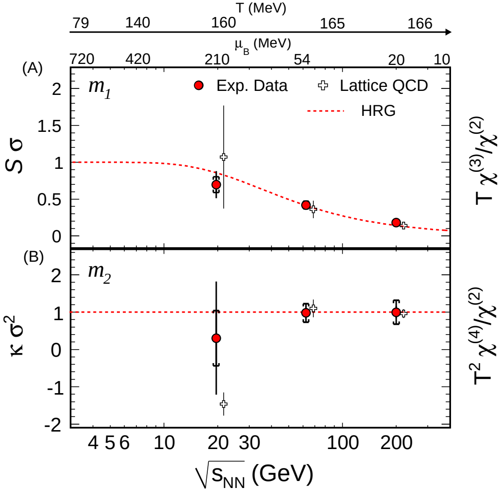
Critical Point and the Onset of Deconfinement
Wuhan China
November 9, 2011

Outline

1. The story till now
2. New approaches to standing questions
3. Systematic errors and intrinsic scales
4. Summary

Fluctuations of conserved quantum numbers



Gavai, SG (2010); STAR (2010); GLMRX, Science (2011)

Madhava-Maclaurin series method from Mumbai

Series expansion of pressure ($t = T/T_c$ and $z = \mu_B/T$):

$$\frac{1}{T}^4 P(t,z) = \frac{P(T)}{T^4} + \frac{\chi^{(2)}(T)}{T^2}\frac{z^2}{2!} + \chi^{(4)}(T)\frac{z^4}{4!} + T^2\chi^{(6)}(T)\frac{z^6}{6!} + \cdots ,$$

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Derivatives give the successive "susceptibilities":

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$$\chi^{(3)}(t,z) = \chi^{(4)}z + T^2\chi^{(6)}\frac{z^3}{3!} + T^4\chi^{(8)}\frac{z^5}{5!} + \cdots,$$

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Series diverge at the critical point: can be used to estimate the position of the critical point:

$$z_* = 1.8 \pm 0.1 \qquad \text{lattice cutoff 1.2 GeV}$$

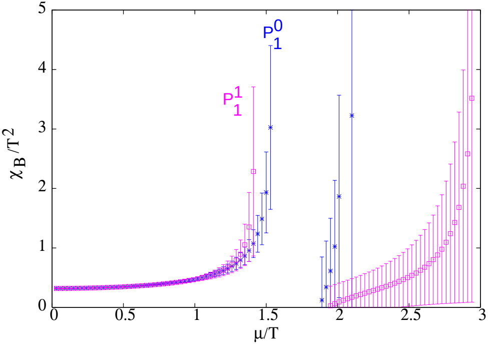
Gavai, SG (2008)

Also tested for 3d Ising Model

Moore, York (2011)

Physical quantities at $\mu \neq 0$

Sum the series! Not enough to sum a finite number of terms when the series diverges. Must sum all orders: possible when radius of convergence can be estimated. Method tried: Padé resummation.



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Two earlier suggestions

If the critical point is far from the freezeout curve over a certain range of energy, then m_1 decreases with increasing $\sqrt{S_{NN}}$ (since z decreases) and m_3 increases. Using these two measurements and comparing with lattice predictions, it is possible to estimate the freezeout conditions: T/T_c and μ_B/T. This method is independent of the usual one in which hadron yields are interpreted through a resonance gas picture [15]. Comparison of the two methods then allows us to estimate T_c by inverting the argument of the previous paragraph. Mutual agreement of the values of T_c

so derived at different $\sqrt{S_{NN}}$ would constitute the first firm experimental proof of thermalization. If this proof holds then one also obtains the simplest and most direct measurement of T_c found till now. Since such a thermometric measurement can be made reliably with data at large $\sqrt{S_{NN}}$, where μ_B is small, it would remain a valid measurement whether or not a critical point is found in the low energy scan at RHIC.

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The first strategy

Use the chemical freezeout curve and the agreement of data and prediction along it to measure

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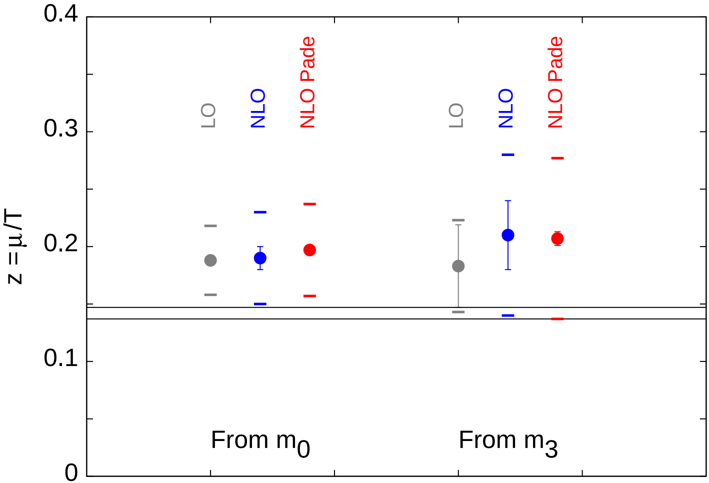
Using the Madhava-Maclaurin expansion,

$$m_0 = \frac{[B^2]}{[B]} = \frac{\chi^{(2)}(t,z)/T^2}{\chi^{(1)}(t,z)/T^3} = \frac{1+\mathcal{O}\left(\frac{z}{z_*}\right)^2}{z\left[1-3\left(\frac{z}{z_*}\right)\right]}$$

$$m_3 = \frac{[B^4]}{[B^3]} = \frac{\chi^{(4)}(t,z)}{\chi^{(3)}(t,z)/T} = \frac{1+\mathcal{O}\left(\frac{z}{z_*}\right)^2}{z\left[1-10\left(\frac{z}{z_*}\right)\right]}$$

Match lattice predictions and data (including statistical and systematic errors) assuming knowledge of z_*.

The second strategy: μmetry



A third strategy

Fit m_0 and m_3 simultaneously to get both z and z_*. Since z_* is the position of the critical point: high energy data already gives information on the critical point!

Indirect experimental estimate of the critical point

From the highest RHIC energy using both statistical and systematic errors:

$$\frac{\mu^E}{T^E} \geq 1.7$$

Compatible with current lattice estimates: but no lattice input.

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Reduction of systematic errors on m_0 and m_3 can give estimates of both upper and lower limits on the estimate of the critical point. Cross check the BES result by high energy RHIC/LHC data.

Three signs of the critical point

At the critical point $\xi \to \infty$.

1: CLT fails

Scaling $[B^n] \simeq V$ fails: fluctuations remains out of thermal equilibrium. Signals of out-of-equilibrium physics in other signals.

2: Non-monotonic variation

At least some of the cumulant ratios m_0, m_1, m_2 and m_3 will not vary monotonically with $\sqrt{S}$. If no critical point then $m_{0,3} \propto 1/z$ and $m_1 \propto z$.

3: Lack of agreement with QCD

Away from the critical point agreement with QCD observed. In the critical region no agreement.

Outline

Length scales in thermodynamics

Persistence of memory?

B, Q, S is exactly constant in full fireball volume $V_{fireball}$. In a part of the fireball they fluctuate. When $V_{obs} \ll V_{fireball}$ then global conservation unimportant. Change acceptance to change V_{obs}.

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The central limit theorem

When $\xi^3 \ll V_{obs}$, then thermalization possible: by diffusion of energy, B, Q, and S to/from V_{obs} to rest of fireball. Many "fluctuation volumes" implies that thermodynamic fluctuations are Gaussian (central limit theorem).

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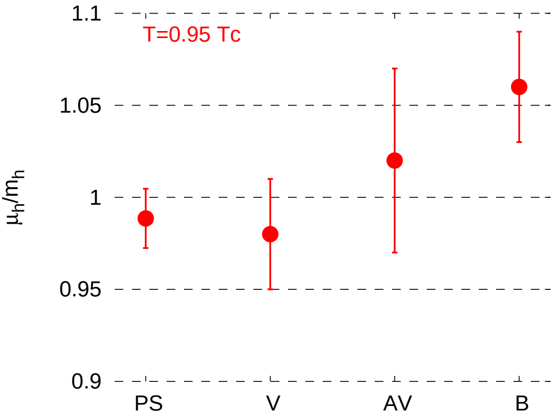
Finite size scaling

Since V_{obs} is finite, departure from Gaussian. Finite size scaling possible: if equilibrium then relate QCD predictions to finite volume effects.

Correlation lengths



Correlation length in thermodynamics is defined through a static correlator: same as screening lengths. Implies $\xi^3 \ll V_{obs}$; check.

Padmanath *et al.*, 2011

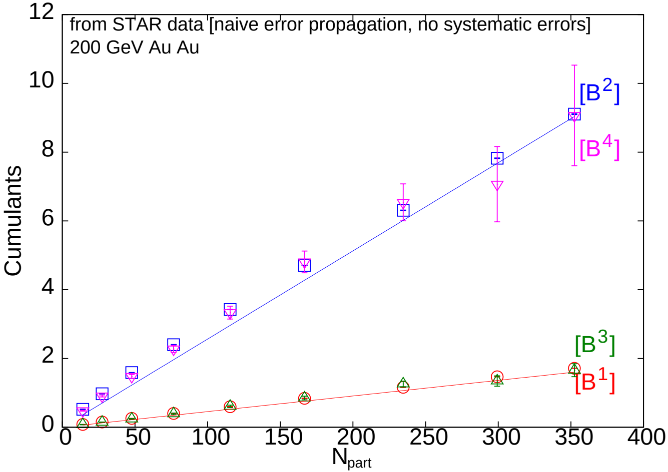
Grand canonical thermodynamics

For a Gaussian the only non-vanishing cumulants are the mean, $[B]$, and the variance $[B^2]$. Observation of any other non-vanishing cumulant $[B^n]$ is a finite size effect. Since these cumulants are given by the NLS,

$$[B^n] = (VT^3)\ \ T^{n-4}\chi^{(n)}(T),$$

QCD determines finite size effects as well as the thermodynamic limit.
Away from criticality: linear volume dependence of all cumulants.
Alternatively: scaling of standard deviation ($\sqrt{[B^2]} \propto \sqrt{V}$), skewness ($\mathcal{S} = [B^3]/[B^2]^{3/2} \propto 1/\sqrt{V}$) and Kurtosis ($\mathcal{K} = [B^4]/[B^2]^2 \propto /V$).
SG, CPOD 2009

Evolution of shape



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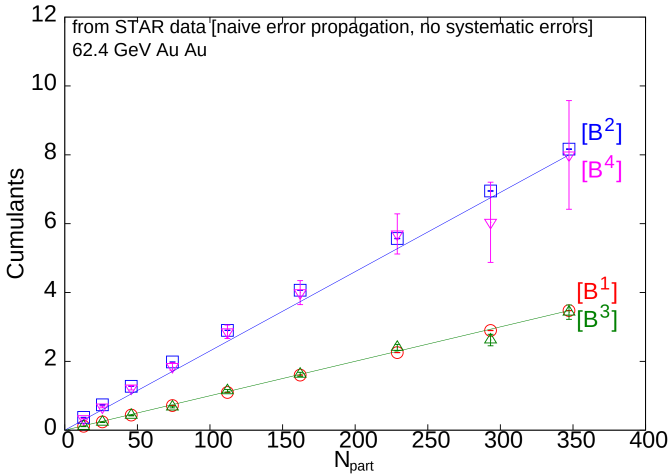

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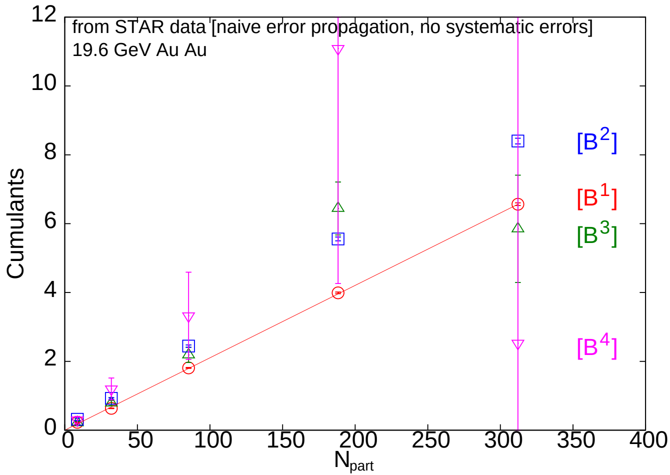

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Coupling diffusion to flow

Entropy content in B or S small compared to entropy content of full fireball. Coupled relativistic hydro and diffusion equations can then be simplified to diffusion-advection equation.
Which is more important— diffusion or advection? Examine Peclet's number

$$\mathrm{Pe} = \frac{\lambda v}{D} = \frac{\lambda v}{\xi c_s} = M \frac{\lambda}{\xi}.$$

When $\mathrm{Pe} \ll 1$ diffusion dominates. When $\mathrm{Pe} \gg 1$ advection dominates. Crossover between these regimes when $\mathrm{Pe} \simeq 1$.

Bhalerao and SG, 2009

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Advective length scale

New length scale: determines when flow overtakes diffusive evolution—

$$\lambda \simeq \frac{\xi}{M}.$$

Bhalerao and SG, 2009

Finite volumes: density sets a scale

When the total number of baryons (baryons + antibaryons) detected is B_+, the volume per detected baryon is $\zeta^3 = V_{obs}/B_+$.
If $\zeta \simeq \xi$ then system may not be thermodynamic: controlled when $V_{obs}/\xi^3 \to \infty$.
Events which (by chance) have large B_+ take longer to come to chemical equilibrium: important to study these transport properties. Can one selectively study these rare events?

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On cumulant order

In central Au Au collisions, the measurement of $[B^6]$ involves $\zeta/\xi \simeq 2$. Could it be used to study transport? Probe this by separating out samples with large B_+ and studying their statistics.

Protons or baryons?

1. If $1/\tau_3$ is the reaction rate for the slowest process which takes $p \leftrightarrow n$, then the system reaches (isospin) chemical equilibrium at time $t \gg \tau_3$.
2. Once system is at chemical equilibrium, the proton/baryon ratio can be expressed in terms of the isospin chemical potential: μ_3. Since baryons are small component of the net isospin, μ_3 can be obtained in terms of the charge chemical potential μ_Q.
3. If not, then is it still possible to extract the shape of the E/E baryon distribution?

Asakawa, Kitazawa: 2011

Outline

Six scales to think of

1. Scale of the persistence of memory, $V_{fireball}$. When $V_{fireball}/V_{obs} \gg 1$ then overall conservation forgotten.
2. Shortest length scale ξ, controls scale at which diffusion of B becomes important.
3. Scale of observation volume, V_{obs}. Set by the detector. Comparison to lattice works when $\xi^3 \ll V_{obs} \ll V_{fireball}$.
4. Peclet scale, $\lambda = \xi/M$ (where M is the Mach number). Controls freeze out of fluctuations.
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