The phase diagram of strongly interacting matter

Sourendu Gupta

TIFR

Indian Institute of Science, Bangalore
November 25, 2011

Introduction

The Discovery of the Strong Interactions: a long story

1911: the discovery of the atomic nucleus

The scattering of α and β particles by matter and the structure of the atom, E. Rutherford, Phil. Mag. 21 (**1911**) 668–88.

1961: quarks underly the visible world

Axial Vector Current Conservation in Weak Interactions, Y. Nambu, Phys. Rev. Lett. 4 (**1960**) 380–2.
The Eighfold Way: A Theory of strong interaction symmetry Murray Gell-Mann CTSL-20, TID-12608, Mar. **1961**, 49 pp.
Derivation of strong interactions from a gauge invariance, Yuval Ne'eman, Nucl. Phys. 26 (**1961**) 222–9.

The Discovery of the Theory of Strong Interactions

Asymptotic freedom in QCD

Ultraviolet Behavior of Nonabelian Gauge Theories, D. J. Gross and F. Wilczek, Phys. Rev. Lett. 30 (**1973**) 1343–6.
Reliable Perturbative Results for Strong Interactions? H. D. Politzer, Phys. Rev. Lett. 30 (**1973**) 1346–9.

Confinement in QCD

Confinement of Quarks, K. G. Wilson Phys. Rev. D 10 (**1974**) 2445–59.

Quantum Chromo Dynamics: a hard problem

Asymptotic freedom: weak coupling expansion works at large energy. Strong coupling at low energy hard to tackle. Many models of strong coupling— quark models, Nambu Jona-Lasinio model, Skyrme model, PNJL model, AdS/CFT models.

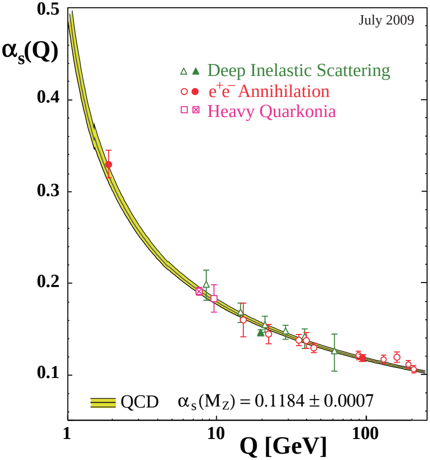
Inadequacy of models

"One may say that the reason why QCD is hard to solve is not just that it is strongly coupled in the IR, but that it is strongly coupled in the IR and weakly coupled in the UV."

David Mateos, Gauge-String duality applied to heavy ion collisions: Limitations, insights and prospects (plenary talk at Quark Matter 2011)

No model of hadron/quark physics simpler than a nonabelian gauge theory is known.

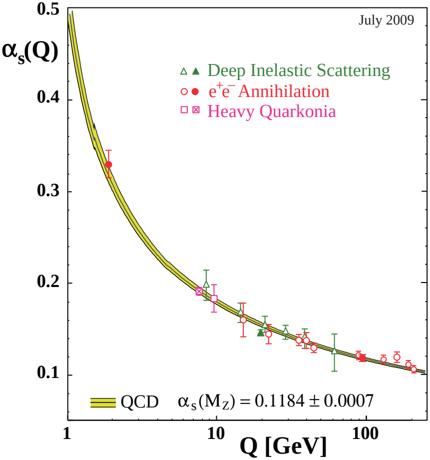
QCD works very well



K. Nakamura et al. (Particle Data Group)

QED coupling: $\alpha = \frac{e^2}{\hbar c}$. QCD coupling: analogous, runs with distance. Potential between static charges not Coulombic.

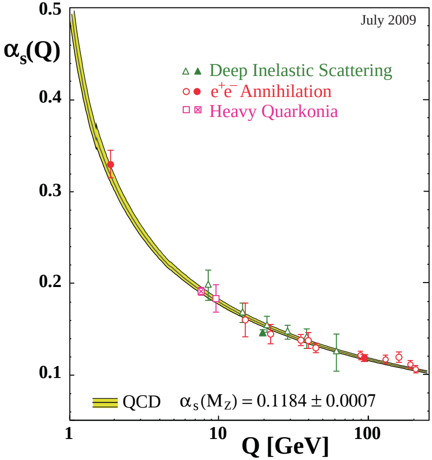
QCD works very well



K. Nakamura et al. (Particle Data Group)

QED coupling: $\alpha = \frac{e^2}{\hbar c}$.
QCD coupling: analogous, runs with distance. Potential between static charges not Coulombic.
Logarithmic corrections to potential. Log implies QCD has an intrinsic length scale: Λ_{QCD}.

QCD works very well



K. Nakamura et al. (Particle Data Group)

QED coupling: $\alpha = \frac{e^2}{\hbar c}$.
QCD coupling: analogous, runs with distance. Potential between static charges not Coulombic.
Logarithmic corrections to potential. Log implies QCD has an intrinsic length scale: Λ_{QCD}.
Trade this scale for any dimensional observable.

Predictions from QCD: renormalization

- Lagrangian has free parameters: cutoff a, quark masses $m_u \simeq m_d \ll \Lambda_{QCD}$, $m_s \simeq \Lambda_{QCD}$, $\cdots$
- Fix the free parameters using some of the predictions. Then the remaining are predictions.
- Compute enough quantities from QCD: $m_\pi(a, m_{ud}, m_s, \cdots)$, $m_K(a, m_{ud}, m_s, \cdots)$, $f_K(a, m_{ud}, m_s, \cdots)$, $f_\pi(a, m_{ud}, m_s, \cdots)$, $m_\rho(a, m_{ud}, m_s, \cdots)$, $m_p(a, m_{ud}, m_s, \cdots)$, $T_c(a, m_{ud}, m_s, \cdots)$, $T_E(a, m_{ud}, m_s, \cdots)$, $\mu_E(a, m_{ud}, m_s, \cdots)$

Predictions from QCD: renormalization

- Lagrangian has free parameters: cutoff a, quark masses $m_u \simeq m_d \ll \Lambda_{QCD}$, $m_s \simeq \Lambda_{QCD}$, $\cdots$
- Fix the free parameters using some of the predictions. Then the remaining are predictions.
- Compute enough quantities from QCD: $m_\pi(a, m_{ud}, m_s, \cdots)$, $m_K(a, m_{ud}, m_s, \cdots)$, $f_K(a, m_{ud}, m_s, \cdots)$, $f_\pi(a, m_{ud}, m_s, \cdots)$, $m_\rho(a, m_{ud}, m_s, \cdots)$, $m_p(a, m_{ud}, m_s, \cdots)$, $T_c(a, m_{ud}, m_s, \cdots)$, $T_E(a, m_{ud}, m_s, \cdots)$, $\mu_E(a, m_{ud}, m_s, \cdots)$

Predictions from QCD: renormalization

- Lagrangian has free parameters: cutoff a, quark masses $m_u \simeq m_d \ll \Lambda_{QCD}$, $m_s \simeq \Lambda_{QCD}$, $\cdots$
- Fix the free parameters using some of the predictions. Then the remaining are predictions.
- Compute enough quantities from QCD: $m_\pi(a, m_{ud}, m_s, \cdots)$, $m_K(a, m_{ud}, m_s, \cdots)$, $f_K(a, m_{ud}, m_s, \cdots)$, $f_\pi(a, m_{ud}, m_s, \cdots)$, $m_p(a, m_{ud}, m_s, \cdots)$, $m_\rho(a, m_{ud}, m_s, \cdots)$, $T_c(a, m_{ud}, m_s, \cdots)$, $T_E(a, m_{ud}, m_s, \cdots)$, $\mu_E(a, m_{ud}, m_s, \cdots)$

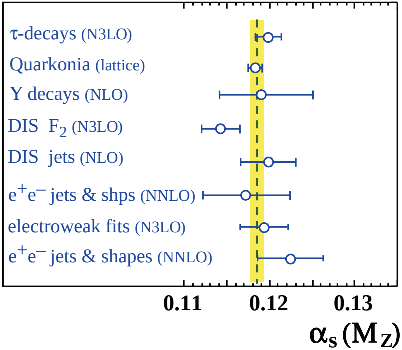
Predictions from QCD: renormalization

- Lagrangian has free parameters: cutoff a, quark masses $m_u \simeq m_d \ll \Lambda_{QCD}$, $m_s \simeq \Lambda_{QCD}$, $\cdots$
- Fix the free parameters using some of the predictions. Then the remaining are predictions.
- Compute enough quantities from QCD: $m_\pi(a, m_{ud}, m_s, \cdots)$, $m_K(a, m_{ud}, m_s, \cdots)$, $f_K(a, m_{ud}, m_s, \cdots)$, $f_\pi(a, m_{ud}, m_s, \cdots)$, $m_p(a, m_{ud}, m_s, \cdots)$, $m_\rho(a, m_{ud}, m_s, \cdots)$, $T_c(a, m_{ud}, m_s, \cdots)$, $T_E(a, m_{ud}, m_s, \cdots)$, $\mu_E(a, m_{ud}, m_s, \cdots)$
- Take the cutoff to infinity. Difficult on the lattice; many technical devlopments on how to get continuum predictions from large a— add RG irrelevant terms to the action, choose scale setting appropriately.

Predictions from QCD: renormalization

- Lagrangian has free parameters: cutoff a, quark masses $m_u \simeq m_d \ll \Lambda_{QCD}$, $m_s \simeq \Lambda_{QCD}$, $\cdots$
- Fix the free parameters using some of the predictions. Then the remaining are predictions.
- Compute enough quantities from QCD: $m_\pi(a, m_{ud}, m_s, \cdots)$, $m_K(a, m_{ud}, m_s, \cdots)$, $f_K(a, m_{ud}, m_s, \cdots)$, $f_\pi(a, m_{ud}, m_s, \cdots)$, $m_p(a, m_{ud}, m_s, \cdots)$, $m_\rho(a, m_{ud}, m_s, \cdots)$, $T_c(a, m_{ud}, m_s, \cdots)$, $T_E(a, m_{ud}, m_s, \cdots)$, $\mu_E(a, m_{ud}, m_s, \cdots)$
- Take the cutoff to infinity. Difficult on the lattice; many technical devlopments on how to get continuum predictions from large a— add RG irrelevant terms to the action, choose scale setting appropriately.
- Most universal part of the solution: Moore's law

Weak and strong coupling agree



K. Nakamura et al. (Particle Data Group), J. Phys. G 37, (2010) 075021

Weak coupling regime (non-lattice): $\alpha_S(M_Z) = 0.1186 \pm 0.0011$.
Strong coupling regime (lattice): $\alpha_S(M_Z) = 0.1189^{+0.0004}_{-0.0006}$.
Shintani, Lattice 2011, July 2011

Bulk Strongly Interacting Matter

Thermodynamics is the frame work

Bulk matter is described by thermodynamics: (almost) irrespective of the microscopic forces between constituents. Strongly interacting matter seems to be no different.

Conserved quantities

Energy and net electrical charge (Q), net baryon number (B), and net strangeness (S) are conserved. Grand canonical ensemble (GCE): trade them for temperature T and chemical potentials (μ_Q, μ_B and μ_S).

Thermodynamics is the frame work

Bulk matter is described by thermodynamics: (almost) irrespective of the microscopic forces between constituents. Strongly interacting matter seems to be no different.

Conserved quantities

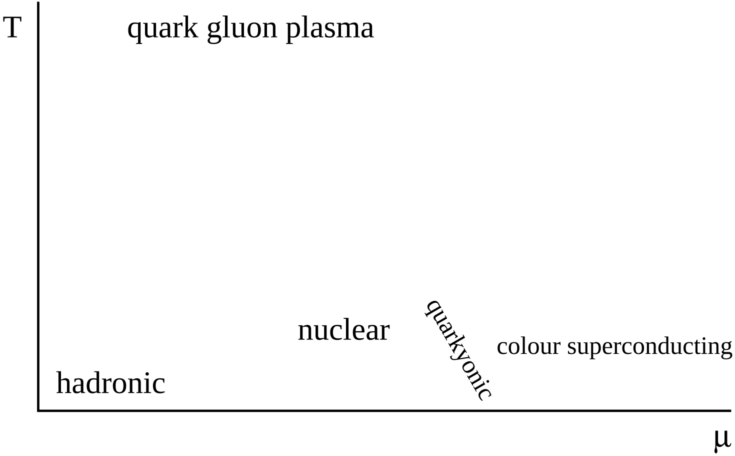
Energy and net electrical charge (Q), net baryon number (B), and net strangeness (S) are conserved. Grand canonical ensemble (GCE): trade them for temperature T and chemical potentials (μ_Q, μ_B and μ_S).

Phase transitions are possible: pion may no longer be a Nambu boson. Phase diagram requires simultaneous control of weak and strong interaction regimes.

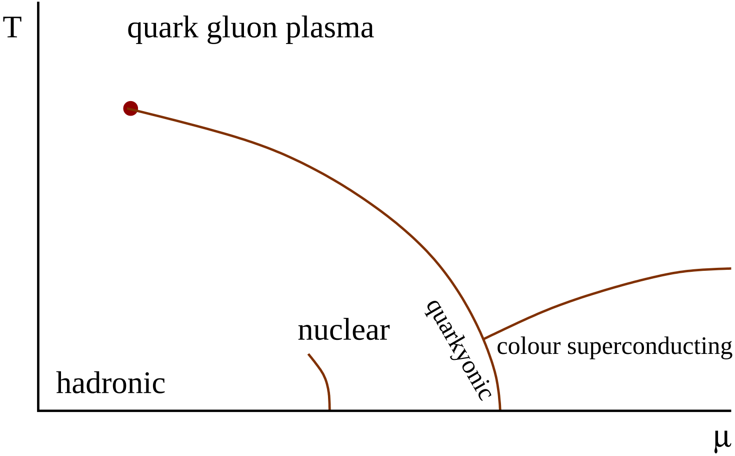
The phases of QCD

- The vacuum phase: chiral symmetry spontaneously broken, colour confined. Excitations are the normal hadrons.
- The quark-gluon plasma phase: approximate chiral symmetry restored, colour deconfined. Excitations are quasi-quarks. **Cabibbo and Parisi, Phys. Lett. B 59 (1975) 67**
- Quarkyonic phases: conjectured approximate chiral symmetry restored but confined phases. Excitations could be abnormal hadrons. **McLerran and Pisarski, Nucl. Phys. A 796 (2007) 83**
- Colour superconducting phases: quasi-quarks bound into coloured Cooper pairs, colour gauge symmetry broken. Possibility of many different kinds of superconducting phases. Excitations are coloured hadron-like and coloured Higgs bosons. **Alford, Rajagopal and Wilczek, Phys. Lett. B 422 (1998) 247**

The phase diagram of QCD

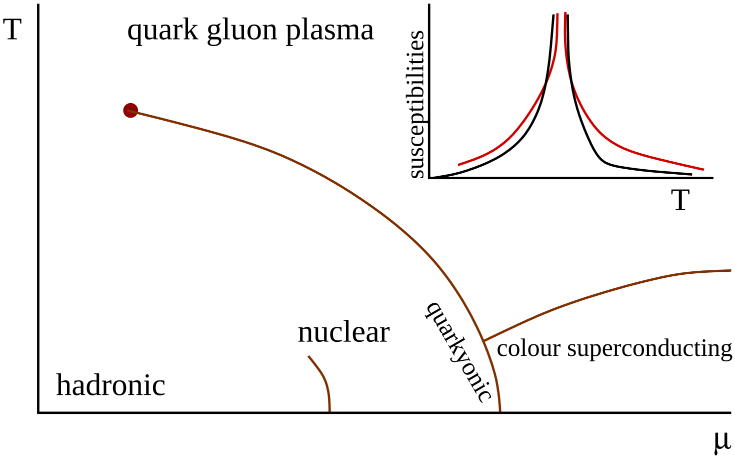


The phase diagram of QCD



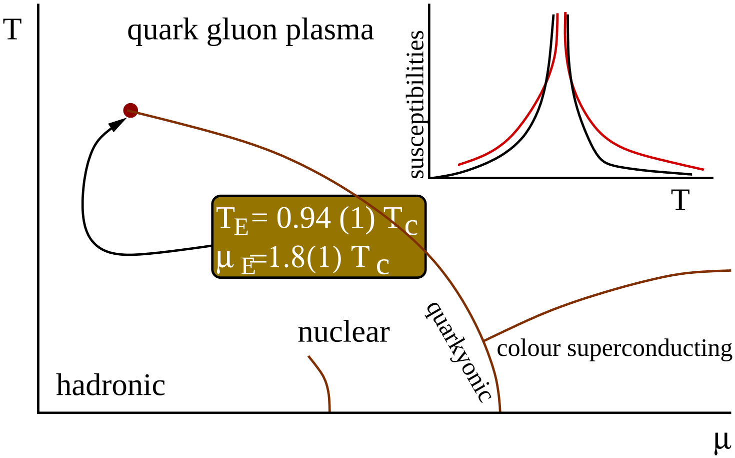
Gavai and Gupta, Phys. Rev. D 71 (2005) 110414, D 78 (2008) 114503

The phase diagram of QCD



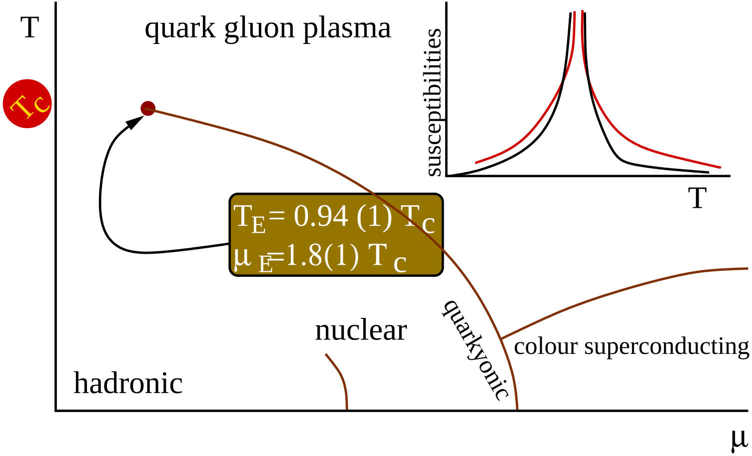
Gavai and Gupta, Phys. Rev. D 71 (2005) 110414, D 78 (2008) 114503

The phase diagram of QCD



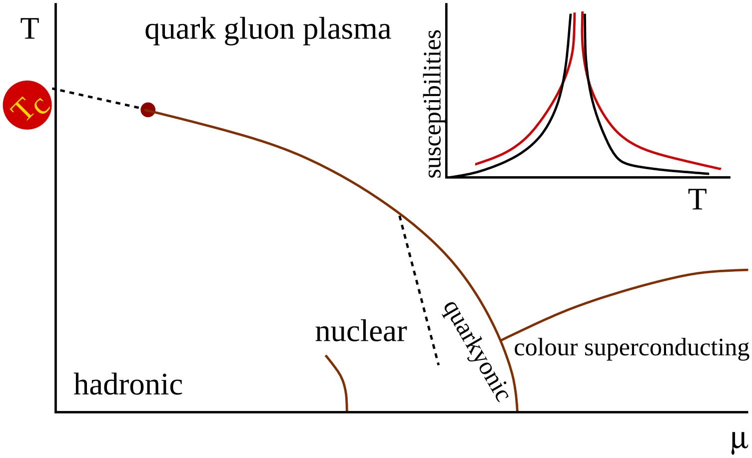
Gavai and Gupta, Phys. Rev. D 71 (2005) 110414, D 78 (2008) 114503

The phase diagram of QCD

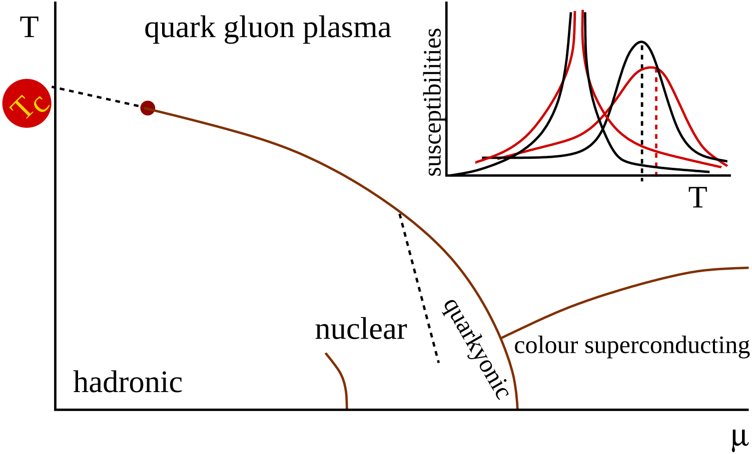


Gavai and Gupta, Phys. Rev. D 71 (2005) 110414, D 78 (2008) 114503

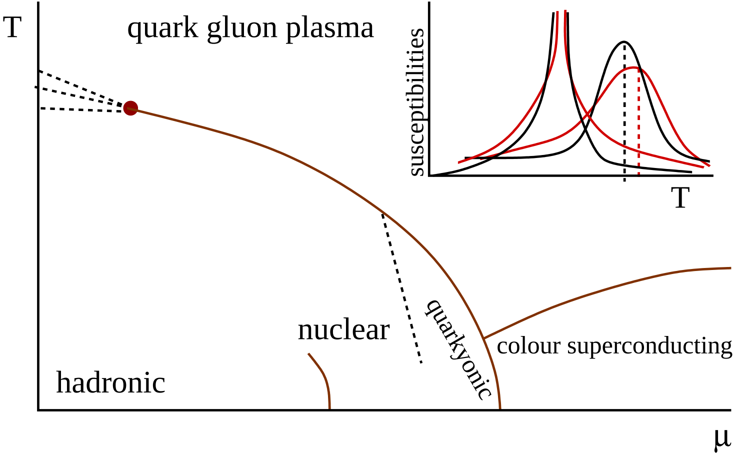
The phase diagram of QCD



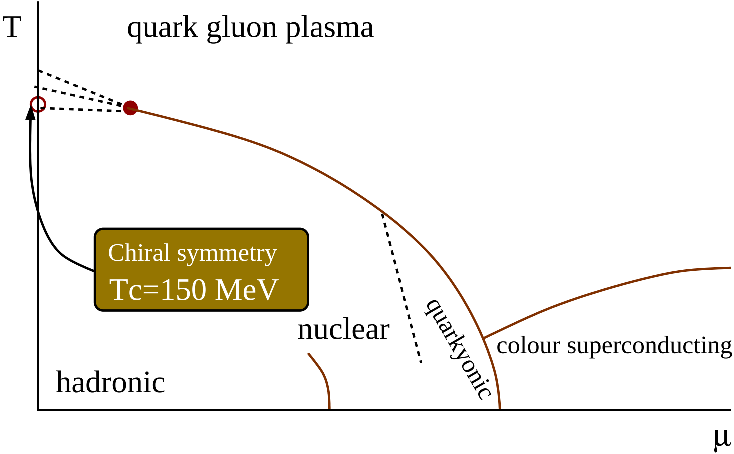
The phase diagram of QCD



The phase diagram of QCD

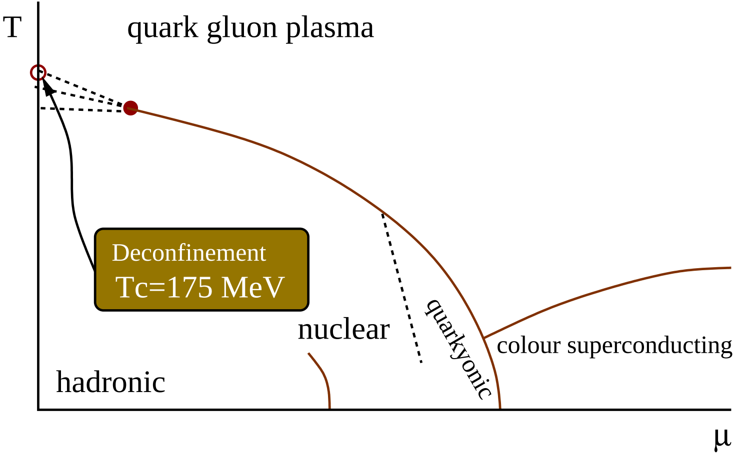


The phase diagram of QCD



Y. Aoki *et al.*, Phys. Lett. B 643 (2006) 46

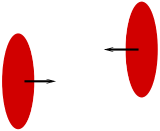
The phase diagram of QCD

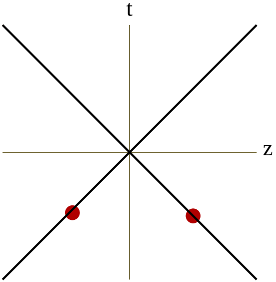


Y. Aoki *et al.*, Phys. Lett. B 643 (2006) 46

Relativistic Heavy-ion Collisions

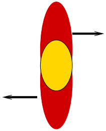
Beating a path through the phase diagram of QCD

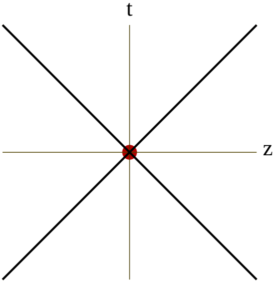




Anishetty, Koehler, McLerran, Phys. Rev. D22 (1980) 2793

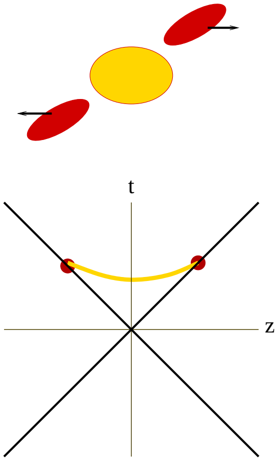
Beating a path through the phase diagram of QCD





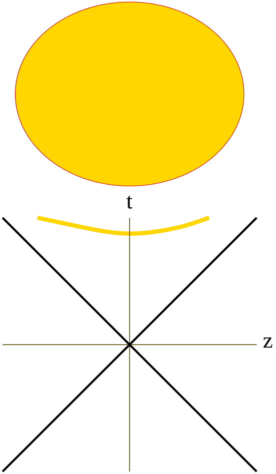
Anishetty, Koehler, McLerran, Phys. Rev. D22 (1980) 2793

Beating a path through the phase diagram of QCD



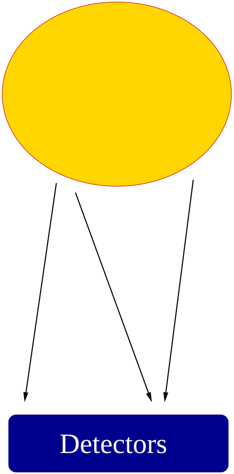
Anishetty, Koehler, McLerran, Phys. Rev. D22 (1980) 2793

Beating a path through the phase diagram of QCD



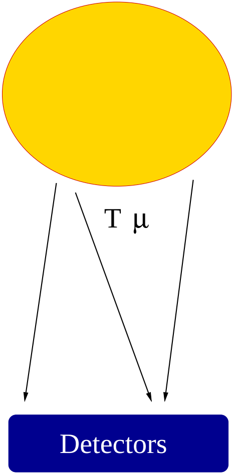
Anishetty, Koehler, McLerran, Phys. Rev. D22 (1980) 2793

Beating a path through the phase diagram of QCD



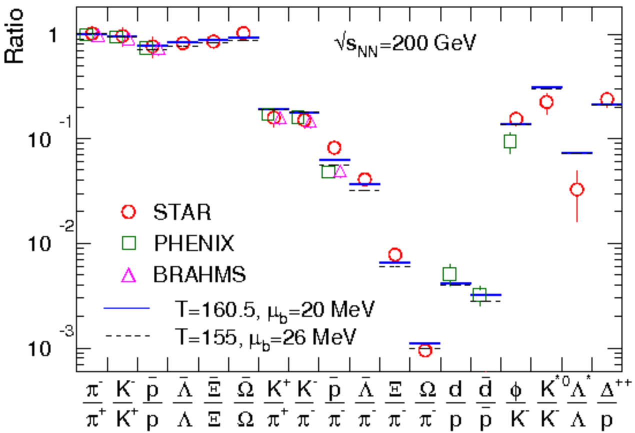
Anishetty, Koehler, McLerran, Phys. Rev. D22 (1980) 2793

Beating a path through the phase diagram of QCD



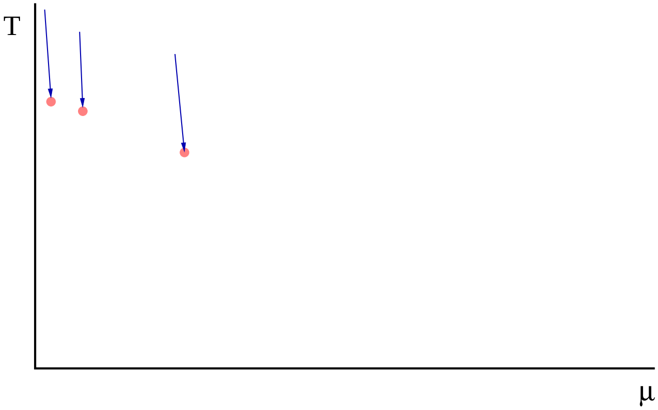
Anishetty, Koehler, McLerran, Phys. Rev. D22 (1980) 2793

Inferring freeze out conditions



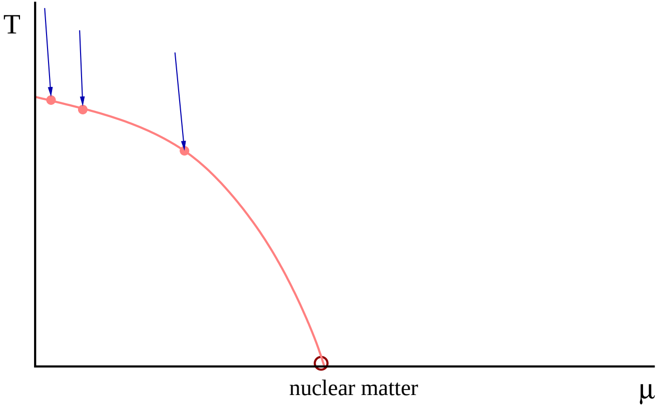
Andronic et al, nucl-th/051107

The freeze-out curve: the final state



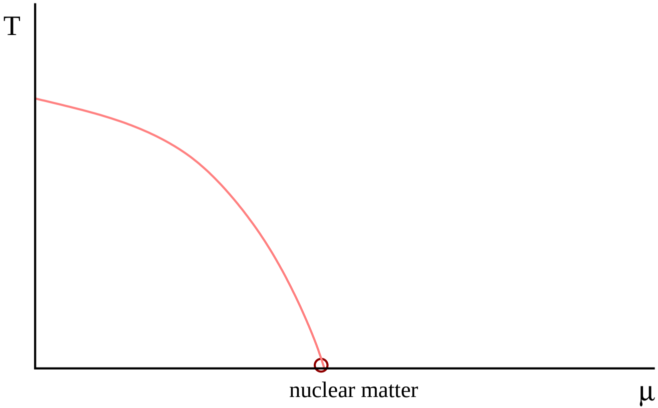
Braun-Munzinger, Cleymans, Redlich, Stachel, Xu

The freeze-out curve: the final state



Braun-Munzinger, Cleymans, Redlich, Stachel, Xu

The freeze-out curve: the final state



Braun-Munzinger, Cleymans, Redlich, Stachel, Xu

Heavy-ion physics today

Experimental observations

Many interesting new puzzles: jet quenching, elliptic flow, strange chemistry, fluctuations of conserved quantities ...

Heavy-ion physics today

Experimental observations

Many interesting new puzzles: jet quenching, elliptic flow, strange chemistry, fluctuations of conserved quantities ...

Systematic understanding

Matter formed: characterized by T and μ. History of fireball described by hydrodynamics and diffusion. Small mean free paths.

Heavy-ion physics today

Experimental observations

Many interesting new puzzles: jet quenching, elliptic flow, strange chemistry, fluctuations of conserved quantities ...

Systematic understanding

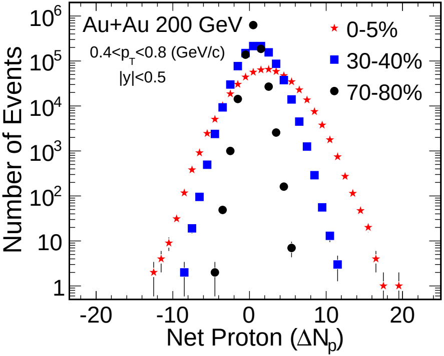
Matter formed: characterized by T and μ. History of fireball described by hydrodynamics and diffusion. Small mean free paths.

Theoretical underpinning

Does QCD describe this matter? Is there a new nonperturbative test of QCD?

Fluctuations of Conserved Quantities

Event-by-Event fluctuations of baryons



Central rapidity slice. Experiments blind to neutrons; but isospin fluctuations small. Assumption tested in event generators.

STAR 2010, Hatta and Stephanov 2003, Asakawa and Kitazawa 2011

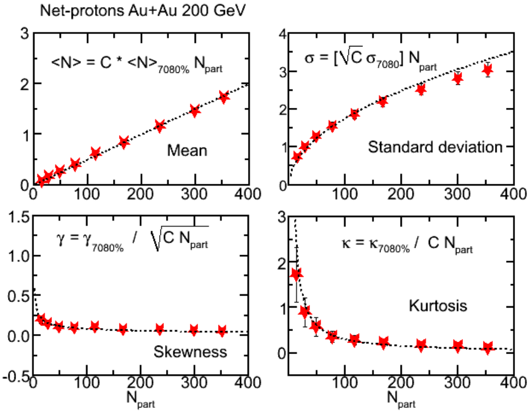
Shapes of distributions

1. Experiments simulate a GCE when cuts are chosen appropriately.
2. Cumulants of the distribution, $[B^n]$, measure the shape; then connected to measurables in GCE:

$$[B^n] = \left(VT^3\right) T^{n-4} \chi_B^{(n)}(t, z).$$

3. Shape variables $\langle B \rangle$, $\sigma^2 = [B^2]$, skewness $S = [B^3]/\sigma^3$ and Kurtosis $\kappa = [B^4]/\sigma^4$ scale as expected with change in V (proxy measure: $V \propto N_{part}$). Central limit theorem.
4. Study of finite-volume effects gives more information about the theory than thermodynamic analysis. In the $V \to \infty$ limit the distribution is Gaussian; S, κ *etc.* vanish.

At a normal point fluctuations are Gaussian



Mohanty, QM 2009

QCD predictions at finite μ_B

Make a MacLaurin expansion of the (dimensionless) pressure:

$$\frac{1}{T^4}P(t,z) = \sum_{n=0}^{\infty} T^{n-4}\chi_B^{(n)}(t,0)\frac{z^n}{n!}, \qquad \text{where} \quad t = \frac{T}{T_c}, z = \frac{\mu_B}{T}.$$

and measure each NLS at $z = 0$. **Gavai and SG 2003**

By resumming the series, construct the lattice predictions for:

$$T^{n-4}\chi_B^{(n)}(t,z) = \frac{1}{T^4}\frac{\partial^n P(t,z)}{\partial z^n}, \qquad \text{where} \quad t = \frac{T}{T_c}, z = \frac{\mu_B}{T}.$$

Series resummation needed since the series can diverge near a critical point: ie, any term of the series is as important as any other, and neglect of an infinite number of terms is not justified.

Gavai and SG 2003

Ratios of cumulants

Cumulants depend on volume: large uncertainties. But, ratios of cumulants are state variables independent of the volume: well-determined functions on the phase diagram. Direct measures of finite size effects.

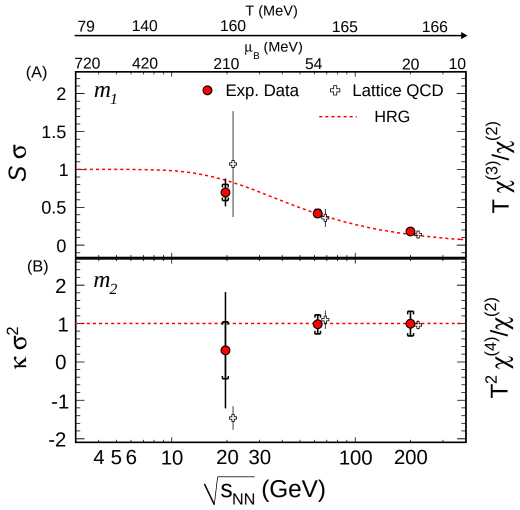
$$m_1: \quad \frac{[B^3]}{[B^4]} = \frac{T\chi_B^{(3)}}{\chi_B^{(2)}} = S\sigma$$

$$m_2: \quad \frac{[B^4]}{[B^2]} = \frac{T\chi_B^{(4)}}{\chi_B^{(2)}} = \kappa\sigma^2$$

$$m_3: \quad \frac{[B^4]}{[B^3]} = \frac{T\chi_B^{(4)}}{\chi_B^{(3)}} = \frac{\kappa\sigma}{S}$$

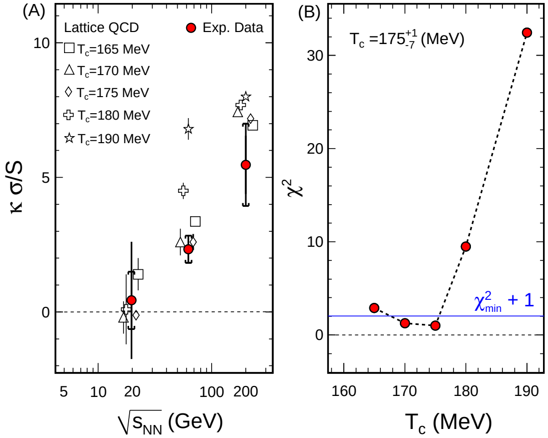
SG, PoS CPOD2009 (2009) 025

Checking the match



GLMRX, Science, 332 (2011) 1525

Tuning lattice scale to match data



GLMRX, Science, 332 (2011) 1525

Results

Thermalization in bulk matter

1 parameter tuning makes thermodynamic predictions agree with data for 2 ratios at 3 energies. Indicates thermalization of the fireball at chemical freezeout. Simultaneously, test of central limit theorem shows that correlation lengths are small. Thermodynamics may be applied.

T_c

First direct test of lattice against data for bulk matter needs

$$T_c = 175^{+1}_{-7}\ \mathrm{MeV}.$$

In agreement with other scale settings on the lattice. Indicates that non-perturbative phenomena in single hadron physics and strong interaction thermodynamics are mutually consistent through QCD.

Conclusions

The future

Next step: the critical point

At the critical point correlations and relaxation times diverge: system falls out of equilibrium. Agreement of shape variables with QCD predictions implies normal points. Lack of agreement due to failure of CLT implies critical point. Second step in exploring the phase diagram of QCD.

Technical issues remain

Do fluctuations freeze out at the same time as yields? How significant are isospin fluctuations beyond chemical freezeout? How much does the freezeout point for fluctuations change as the acceptance is changed? There are small remaining cutoff and quark mass effects in the lattice predictions: how do they affect physics?

The history of the universe

The symmetric universe

Many of the broken symmetries of the standard model of particle physics which are visible to experiments today are expected not to have been broken in the early universe. The standard picture of cosmology involves multiple phase transitions in relativistic quantum field theories.

Symmetrizing a part of the universe

Not a single phase transition in any part of the standard model has been observed yet. Now there is the first possibility of creating and observing such a phase transition in the laboratory.