Theory and Experiments for the phase diagram of QCD

Sourendu Gupta

ILGTI: TIFR

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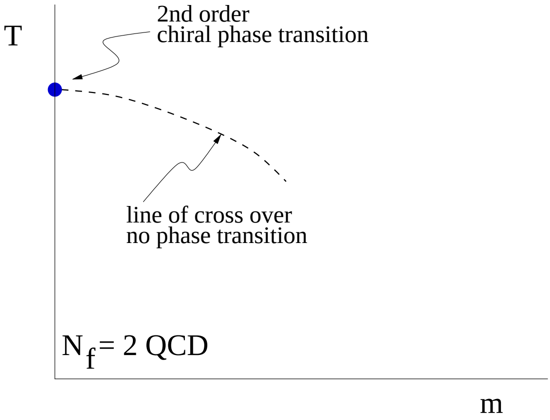
Outline

Symmetries of QCD

- If quarks massless then different chiralities do not mix: flavour symmetries act on each chirality independently. Quark mass breaks chiral symmetry explicitly to flavour symmetry. Limit $m_\pi = 0$: chiral symmetry.
- If some $m \gg \Lambda_{QCD}$ then that quark is not even approximately chiral. Recall $m_\pi = 0.2 m_\rho$ but $m_K = 0.7 m_\rho$. In QCD two flavours are light ($m_{u,d} \ll \Lambda_{QCD}$) and one is medium heavy ($m_s \simeq \Lambda_{QCD}$).
- Flavour symmetries not exact: difference in masses of different flavours breaks symmetry. Since $m_{\pi^0} \simeq m_{\pi^\pm}$, flavour SU(2) is a good approximate symmetry of the hadron world. Flavour SU(3) is not useful without strong symmetry breaking terms (Gell-Mann and Nishijima).
- Phase diagram of QCD close to 2-flavour phase diagram.
  Endrodi etal, 0710.0988 (2007)

The two flavour phase diagram



Pisarski and Wilczek, PR D 29, 338 (1984)

Phase diagram records positions of singularities of free energy:

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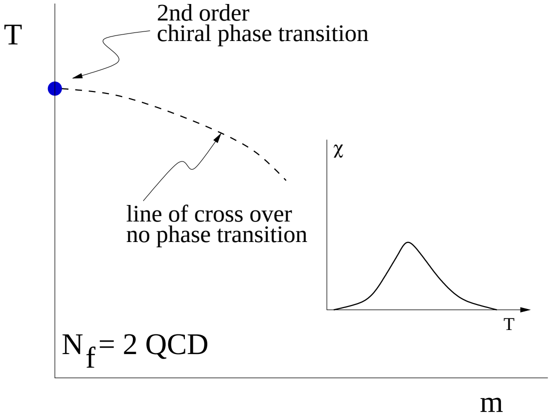

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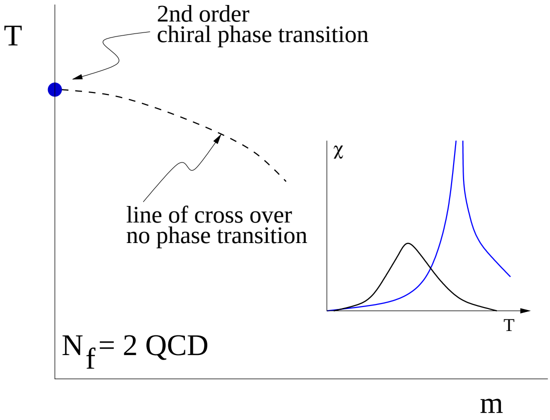

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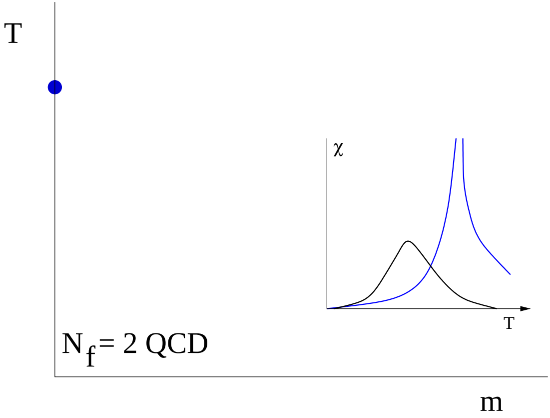

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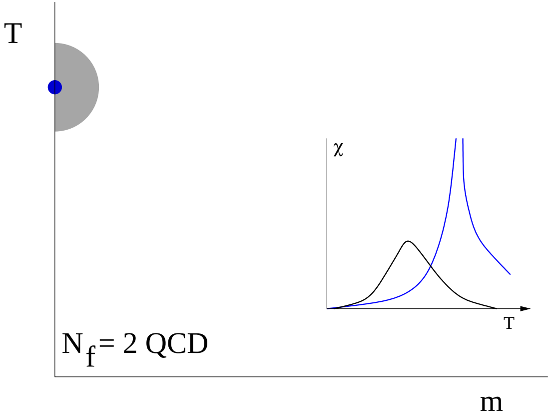

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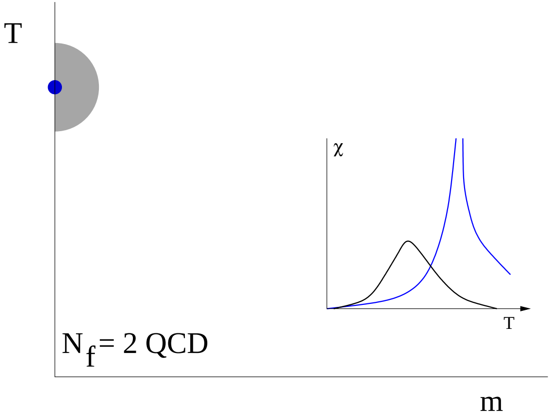

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Phase diagram records positions of singularities of free energy:

$$F(T,m) = [F_r(T,m) + g(T,m)] + [F_s(T,m) - g(T,m)]$$

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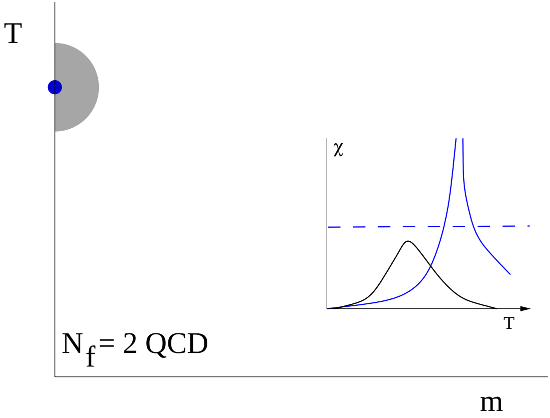

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Universality

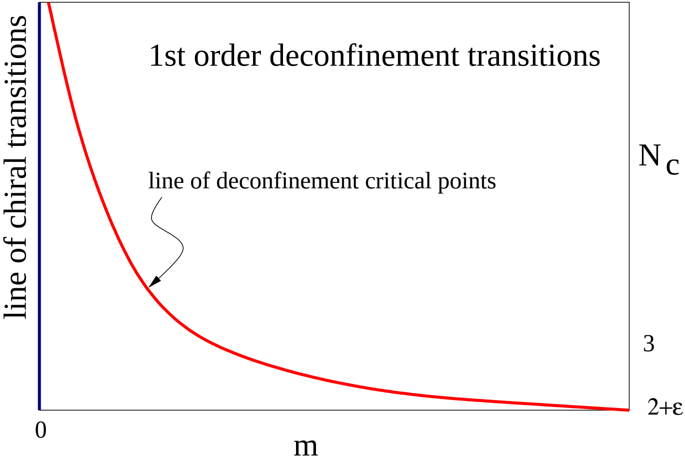
- Susceptibility closely related to a correlation function:

$$\chi = \int d^3x C(x), \quad C(x) \simeq \exp(-x/\xi).$$

  When correlation length, ξ, finite then χ, always finite. Screening mass: $\overline{m} = 1/\xi$.

- Critical point: $\xi \to \infty$; so integral diverges, and $\chi \to \infty$.
- Universality: $F_s^{QCD}(T, m)$ related to $F_s^{O(4)}(\overline{T}, H)$. Generally $\overline{T}(T, m)$ and $H(T, m)$. But if $F^{QCD} - F^{O(4)} = g$? Since g non-singular, go close enough to critical point and its effects are small.
- Useful definition: if $\chi(T_c + \epsilon, 0) = A\epsilon^{-\gamma}$ then $\gamma_{QCD} = \gamma_{O(4)}$. Also if $\chi(T_c - \epsilon, 0) = B\epsilon^{-\gamma'}$, then $\gamma'_{QCD} = \gamma'_{O(4)}$, and $(A/B)_{QCD} = (A/B)_{O(4)}$. Experimental relevance?

Guidance from large N_c QCD



No small parameter for non-perturbative QCD: expand in $1/N_c$.

Datta, SG, 1006.0938

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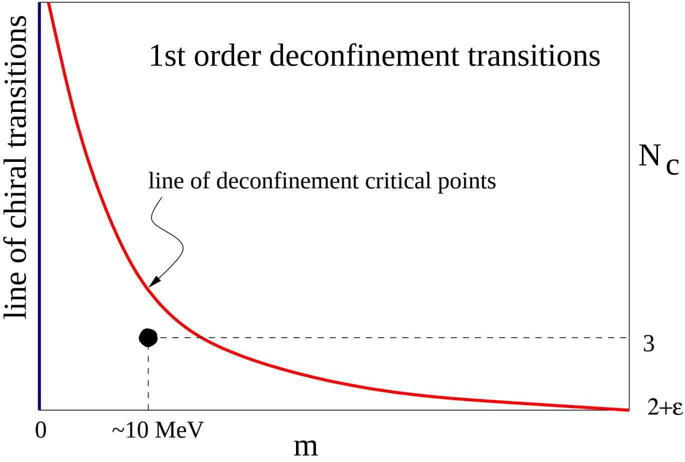

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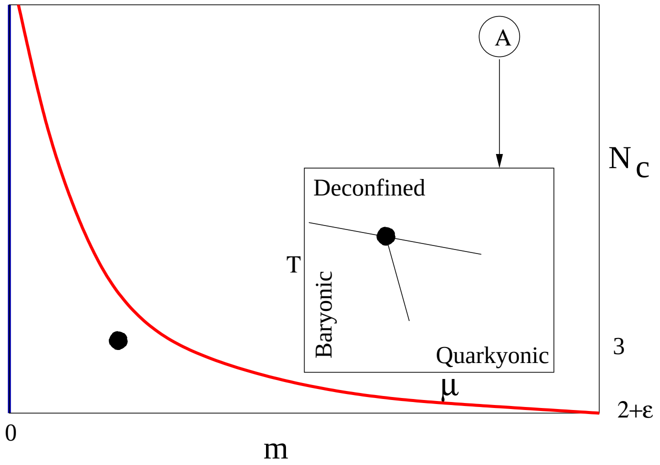

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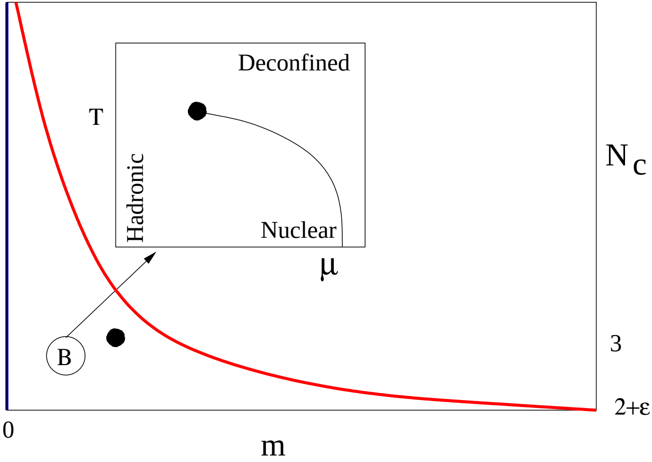

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QCD cross over: chiral or deconfinement?

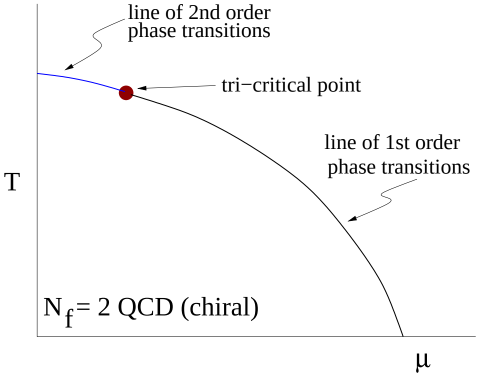
1. QCD with realistic quark masses at $\mu = 0$ has no finite T phase transition: only a cross over. Large N_c argument indicates that the topology of the QCD phase diagram is controlled by chiral symmetry.
2. Lattice shows that cross over temperature defined by peaks of susceptibilities differ:

$$T_c^{\mathrm{deconf}} \simeq 175 \text{ MeV}, \qquad \text{and} \qquad T_c^{\mathrm{chiral}} \simeq 155 \text{ MeV}.$$

3. Which of these influences RHIC/LHC fireball evolution? Whatever controls hydrodynamics, *i.e.*, c_s^2. Region of rapid crossover in energy density close to T_c^{deconf}.

   Borsanyi ***et al.*****, 1109.5032**
4. Both effects important: topology of phase diagram controlled by chiral cross over, evolution of fireball controlled by deconfinment cross over!

The QCD phase diagram



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Rajagopal, Stephanov, Shuryak hep-ph/9806219 and hep-ph/9903292

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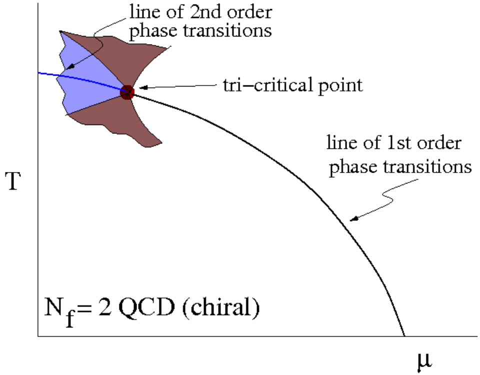

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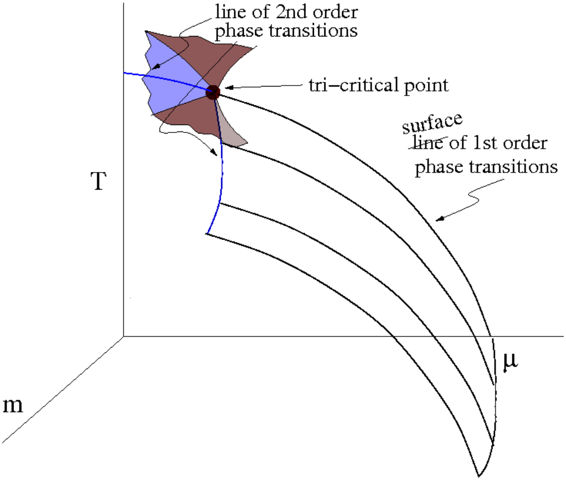

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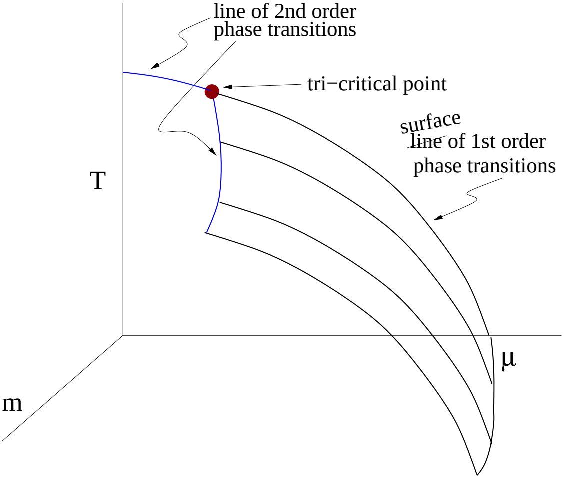

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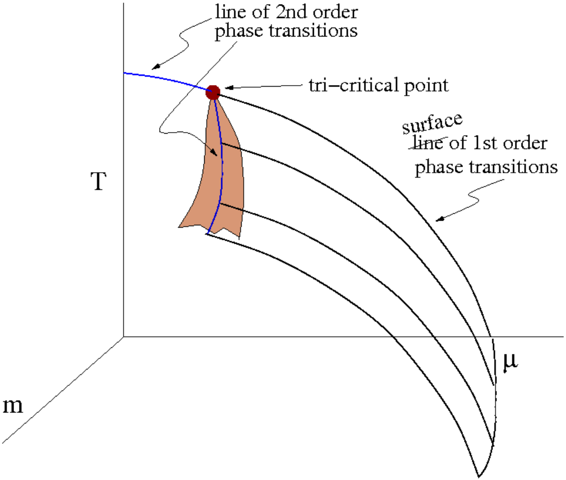

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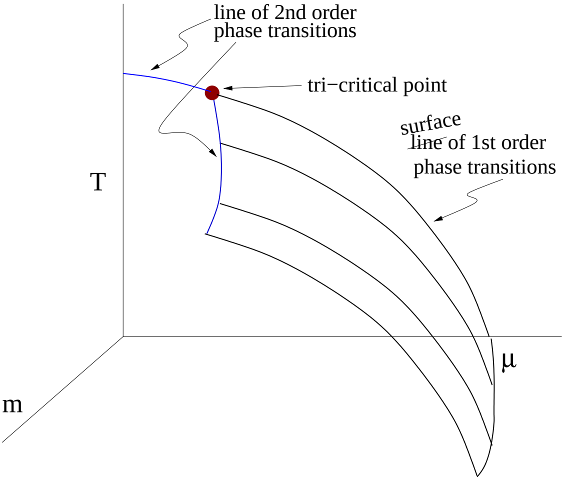

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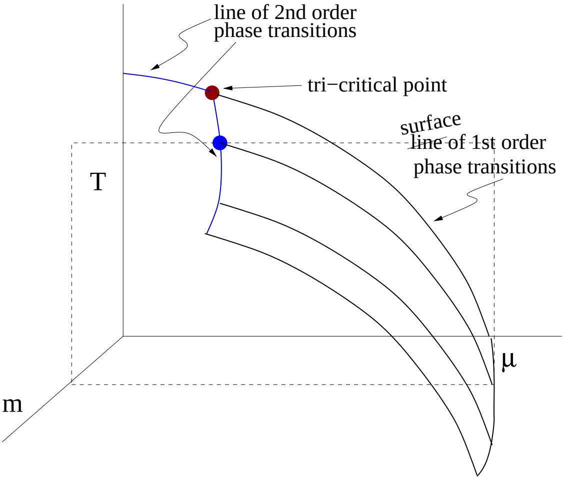

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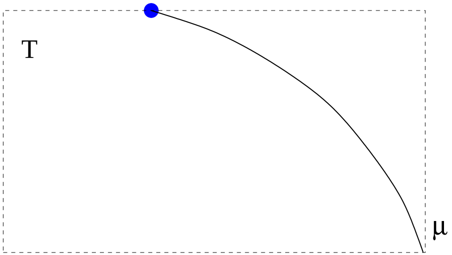

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Outline

Strategy of lattice QCD

- Lagrangian has free parameters: cutoff a, quark masses $m_u \simeq m_d \ll \Lambda_{QCD}$, $m_s \simeq \Lambda_{QCD}$, $\cdots$
- Compute enough quantities from QCD: $m_\pi(a, m_{ud}, m_s, \cdots)$, $m_K(a, m_{ud}, m_s, \cdots)$, $f_K(a, m_{ud}, m_s, \cdots)$, $f_\pi(a, m_{ud}, m_s, \cdots)$, $m_\rho(a, m_{ud}, m_s, \cdots)$, $m_p(a, m_{ud}, m_s, \cdots)$, $T_c(a, m_{ud}, m_s, \cdots)$, $T_E(a, m_{ud}, m_s, \cdots)$, $\mu_E(a, m_{ud}, m_s, \cdots)$
- Fix the free parameters using some of the predictions. Then the remaining are scale-free predictions.

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- Most robust part of the solution: use Moore's law

Lattice setup

Lattice simulations impossible at finite baryon density: **sign problem**. Basic algorithmic problem in all Monte Carlo simulations: no solution yet.

Bypass the problem; make a Taylor expansion of the pressure:

$$P(T,\mu) = P(T) + \chi_B^{(2)}(T)\frac{\mu^2}{2!} + \chi_B^{(4)}(T)\frac{\mu^4}{4!} + \cdots$$

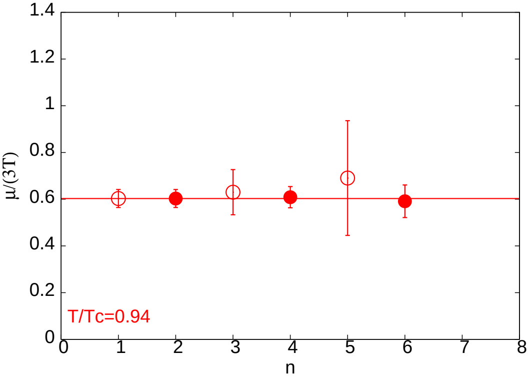
Series expansion coefficients evaluated at $\mu = 0$.
Implies

$$\chi_B^2(T,\mu) = \chi_B^{(2)}(T) + \chi_B^{(4)}(T)\frac{\mu^2}{2!} + \chi_B^{(6)}(T)\frac{\mu^4}{4!} + \cdots$$

Series fails to converge at the critical point.

Gavai, SG, hep-lat/0303013

Series diverges at critical point



Radius of convergence of the series as a function of order ($a^{-1} = 1200$ MeV)

Gavai, SG, 0806.2233 (2008)

Systematic effects

1. Series expansion carried out to 8th order. What happens when order is increased? Intimately related to finite volume effects. Finite size scaling tested; works well

   Gavai, SG 2004, 2008; Moore, York, 1106.2535

2. What happens when strange quark is unquenched (keeping the same action)? Numerical effects on ratios of susceptibility marginal when unquenching light quarks

   Gavai, SG, hep-lat/0510044; see also RBRC 2009; de Forcrand, Philipsen, 2007, 2009

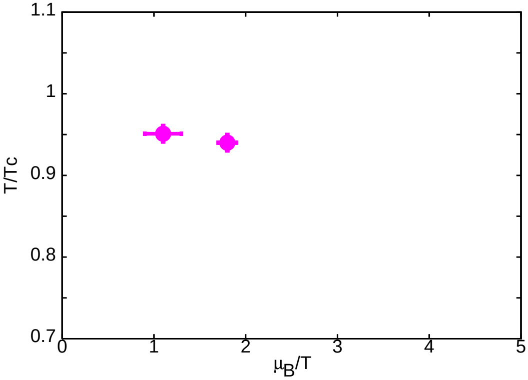
3. What happens when m_π is decreased? Estimate of μ_B^E may decrease somewhat: first estimates in

   Fodor, Katz hep-lat/0106002; Gavai, SG, Ray, nucl-th/0312010

4. What happens in the continuum limit? Estimate of μ_B^E may increase somewhat

   Gavai, SG 2008; SG 2009

The critical point of QCD



$$\frac{\mu^E}{T^E} \simeq \begin{cases} 1.8 \pm 0.1 & N_f = 2,\ 1/a = 1200 \text{ MeV} \\ 1.5 \pm 0.4 & N_f = 2+1,\ 1/a = 800 \text{ MeV} \end{cases}$$

Gavai, SG, 0806.2233 (2008); **RBRC, unpublished, 2010**

comparable m_π; normalized to same estimator.

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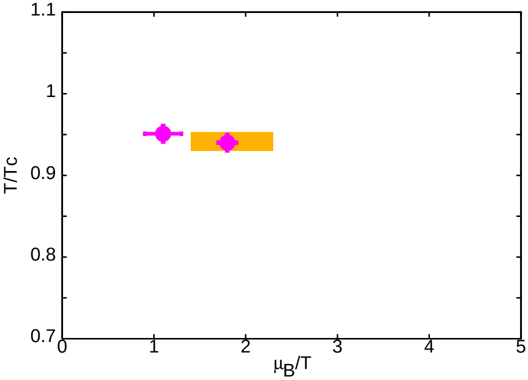

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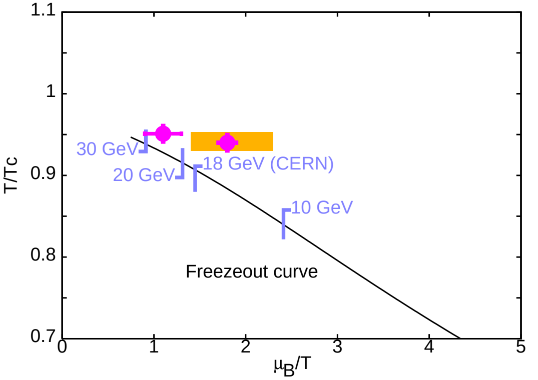

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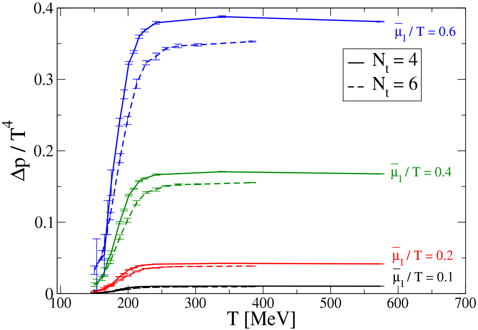
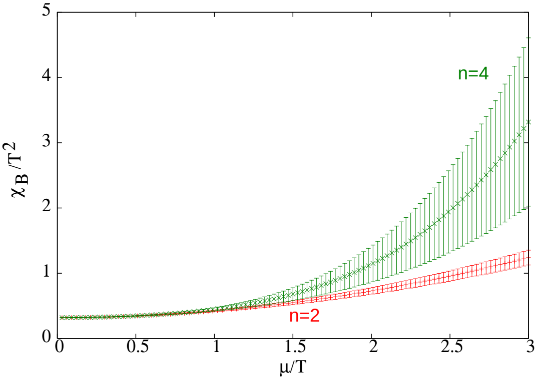
Extrapolating physical quantities



$$\Delta p = \chi_B^{(2)} \frac{\mu^2}{2!} + \chi_B^{(4)} \frac{\mu^4}{4!} + \cdots - \chi_S^{(2)} \frac{\mu_S^2}{2!} - \cdots$$

MILC Collaboration, 1003.5682 (2010)

Critical divergence: summation bad, resummation good



Infinite series diverges, but truncated series finite and smooth: sum is bad. Resummations needed to reproduce critical divergence. Padé resummation useful **Gavai, SG, 0806.2233 (2008)**.

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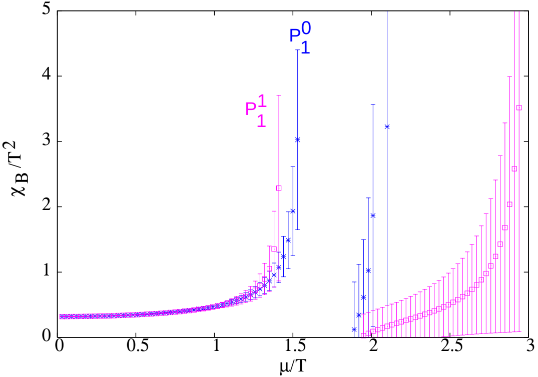

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Outline

Locating the critical end point in experiment

Measure the (divergent) width of momentum distributions

Stephanov, Rajagopal, Shuryak, hep-ph/9903292

Better idea, use conserved charges, because at any normal (non-critical) point in the phase diagram:

$$P(\Delta B) = \exp\left(-\frac{(\Delta B)^2}{2VT\chi_B}\right). \qquad \Delta B = B - \langle B \rangle.$$

At any non-critical point the appropriate correlation length (ξ) is finite. If the number of independently fluctuating volumes ($N = V/\xi^3$) is large enough, then net B has Gaussian distribution: central limit theorem

Landau and Lifschitz

Bias-free measurement possible

Asakawa, Heinz, Muller, hep-ph/0003169 (2000); Jeon, Koch, hep-ph/0003168.

Three length scales

$V_{\rm obs} \ll V_{\rm fireball}$: conserved charges can fluctuate.

1. The longest correlation length, ξ, controls the scale of microscopic physics. When $\xi \ll \sqrt[3]{V_{\rm obs}}$, CLT works: many independently fluctuating volumes.
2. The Peclet length scale, $\lambda = \xi/M$, where M is the Mach number in the fireball controls the freezeout of fluctuations. At critical point c_s vanishes, and fluctuations never reach equilibrium. **Bhalerao, SG: 2009**
3. The typical distance between baryons: $\zeta = \sqrt[3]{V_{\rm obs}/B_+}$ where B_+ is the net number of baryons. This controls whether the sample of events is "typical". Very non-typical events may take different times to thermalize or freezeout; very high cumulant orders, $[B^6]$ *etc.*, may not be thermal. May give information on transport.

Is the top RHIC energy non-critical?

Check whether CLT holds. Then $V_{\mathrm{fireball}} \gg V_{\mathrm{obs}} \gg \xi^3$.

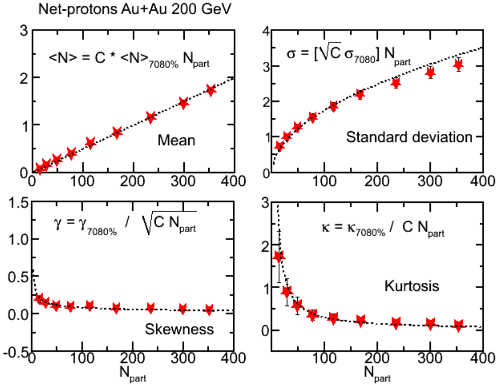
Recall the scalings of extensive quantity such as B and its variance σ^2, skewness, $\mathcal{S}$, and Kurtosis, $\mathcal{K}$, given by

$$B(V) \propto V, \quad \sigma^2(V) \propto V, \quad \mathcal{S}(V) \propto \frac{1}{\sqrt{V}}, \quad \mathcal{K}(V) \propto \frac{1}{V}.$$

Coefficients depend on T and μ. So make sure that the nature of the physical system does not change while changing the volume.

This is a check that microscopic physics is forgotten (except two particle correlations).

STAR measurements



Perfect CLT scaling: remember only $VT\chi_B$? or some other physics?

Can we recover microscopic physics? Try finite size scaling

Can we test QCD?

STAR Collaboration: QM 2009, Knoxville

What to compare with QCD

The cumulants of the distribution are related to Taylor coefficients—

$$[B^n] = T^3 V \left(\frac{\chi^{(n)}}{T^3} \right).$$

V is hard to measures, so remove it by taking ratios. Define number, $n = [B]$, variance $\sigma^2 = [B^2]$, skew $\mathcal{S} = [B^3]/\sigma^3$ and Kurtosis, $\mathcal{K} = [B^4]/\sigma^4$. Construct the ratios

$$m_0 = \frac{\sigma}{n} = \frac{[B^2]}{[B]}, \qquad m_1 = \mathcal{S}\sigma = \frac{[B^3]}{[B^2]},$$

$$m_2 = \mathcal{K}\sigma^2 = \frac{[B^4]}{[B^2]}, \qquad m_3 = \frac{\mathcal{K}\sigma}{\mathcal{S}} = \frac{[B^4]}{[B^3]}.$$

These are comparable with QCD provided all other fluctuations removed.

SG, 0909.4630 (2009), (2010)

How to compare with QCD

Strategy 1

Check whether CLT holds. If yes, then input T and $z = \mu/T$ from hadron resonance gas model. Extract T_c by comparing lattice predictions and data. Check whether compatible with other results.

Gavai, SG 1001.3796; STAR 1004.4959; GLMRX, Science (2011).

Strategy 2

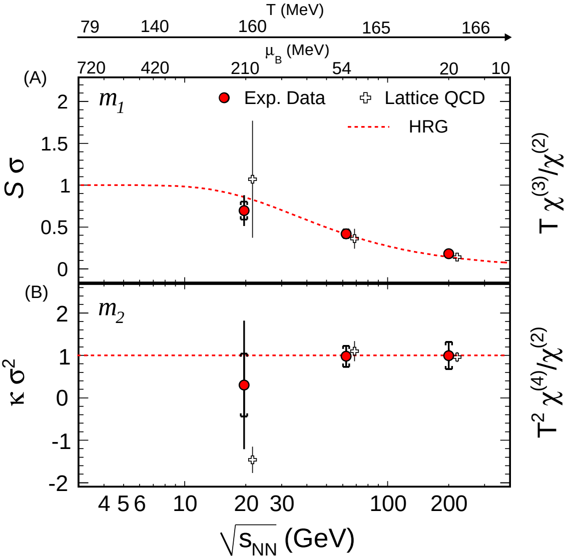
Check whether CLT holds. If yes, then extract $t = T/T_c$ and $z = \mu/T$ by comparing lattice predictions and data. Check whether t and z compatible with other extractions.

Gavai, SG 1001.3796, Karsch unpublished 2011

Near CP system drops out of equilibrium: finite lifetime and finite size. Lack of agreement with CLT and QCD is signal of CP!

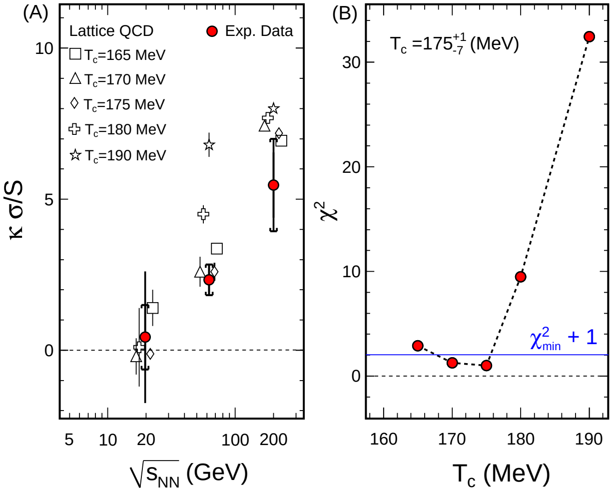
Berdnikov, Rajagopal hep-ph/9912274, Stephanov 0809.3450

Strategy 1: Matching data and prediction



STAR arXiv:1004.4959

Strategy 1: Extracting T_c



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Strategy 2: The fluctuation freezeout curve

The Taylor expansion of the pressure can be written as

$$\frac{P(t,z)}{T^4} = \frac{P(t,0)}{T^4} + \frac{\chi^2(t)}{T^2}\frac{z^2}{2!} + \chi^4(t)\frac{z^4}{4!} + T^2\chi^6(t)\frac{z^6}{6!} + \cdots ,$$

where $t = T/T_c$ and $z = \mu/T$. This gives the expressions

$$\begin{aligned} \frac{1}{m_0(t,z)} &= \frac{z}{1+3(z/z^*)^2+\mathcal{O}(z^4)}, \\ \frac{1}{m_3(t,z)} &= \frac{z}{1+10(z/z^*)^2+\mathcal{O}(z^4)}, \end{aligned}$$

where z^* is the radius of convergence. Using the lattice estimates of z^* one can estimate the freezeout point t and z by comparing the lattice predictions with data either for m_0 or for m_3.

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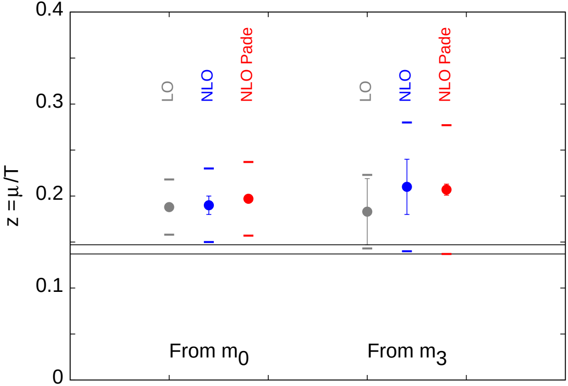
Strategy 2: The freezeout point for fluctuations



Matching lattice predictions and data at top RHIC energy (including statistical and systematic errors) assuming knowledge of the critical point. Beyond LO assumed that $t = 0.94$ (HRG).

Strategy 3: Determining the critical point

Assuming that $t = 0.94$, and using the resummed NLO expressions one can fit m_0 and m_3 simultaneously. Since $t^E = 0.94\,T_c$, this can give simultaneous estimates of the critical end point and the freezeout point.

To NLO with current systematic error estimates, one finds only a lower limit for the critical point, because

$$z = \frac{7}{2m_3 + 5m_0}, \qquad z^* = 2z\sqrt{\frac{1}{1 - z/m_0}},$$

and at the limits of the systematic errors the value of z extends past the singular point. So

$$\frac{\mu^E}{T^E} \geq 1.7$$

This is compatible with current lattice estimates.

Three ways to recognize the critical point

At the critical point $\xi \to \infty$.

1: CLT fails

Scaling $[B^n] \simeq V$ fails: fluctuations remains out of thermal equilibrium. Signals of out-of-equilibrium physics in other signals.

2: Non-monotonic variation

At least some of the cumulant ratios m_0, m_1, m_2 and m_3 will not vary monotonically with $\sqrt{S}$. If no critical point then $m_{0,3} \propto 1/z$ and $m_1 \propto z$.

3: Lack of agreement with QCD

Away from the critical point agreement with QCD observed. In the critical region no agreement.

Outline

Main results

- Chiral cross over determines the shape of the phase diagram; deconfinement cross over determines the evolution of the fireball.
- Lattice determines series expansion of pressure; indicates a critical point in QCD: $\mu^E/T^E \simeq 1.5$–2.5. Physical quantities can be found be resumming the series expansion (*e.g.*, Padé approximants).
- Extrapolation of lattice results to the experimentally known freezeout curve possible. First comparison of predictions and data in good agreement. Both can be improved.
- Opens many new exciting paths: direct comparison of finite temperature data with lattice predictions. T_c compatible with other determinations. Freezeout of fluctuations vs of yields? Self consistent determination of critical point from every energy?