

Hot Stuff

Sourendu Gupta

Wednesday Colloquium
TIFR Mumbai
October 3, 2012

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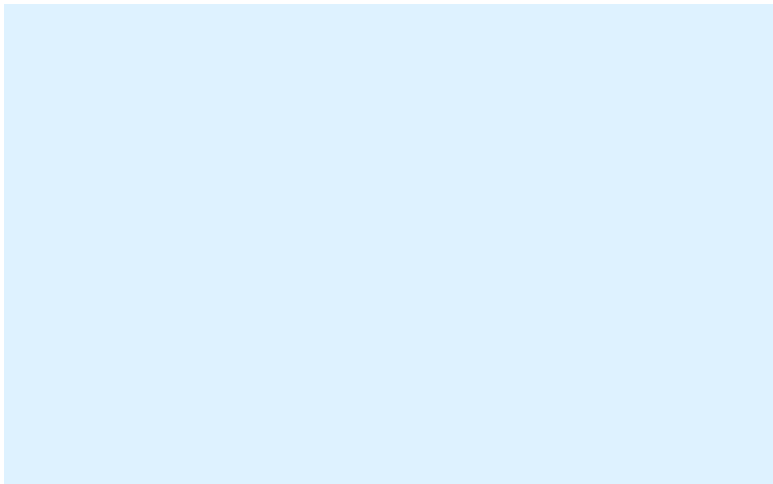
Outline

- 1 Why study hot matter?
- 2 The theoretical puzzle
- 3 Looking for a critical point in a collider
- 4 The three revolutions in science

A hot big bang

The baby universe was very nearly in thermal equilibrium

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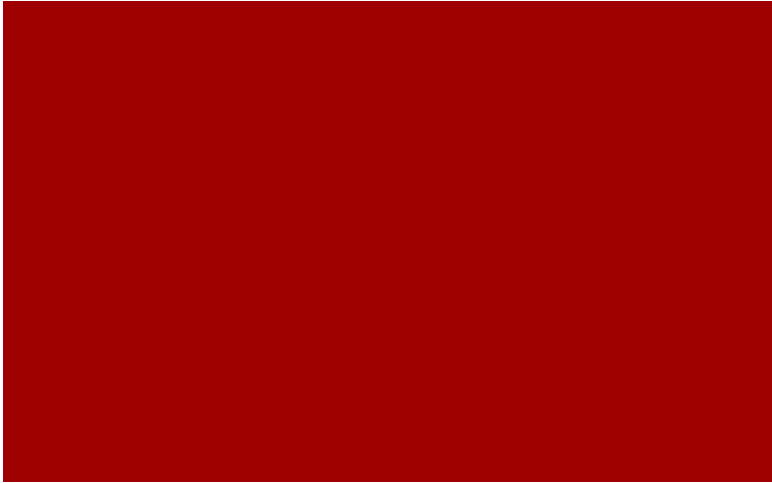
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A hot big bang



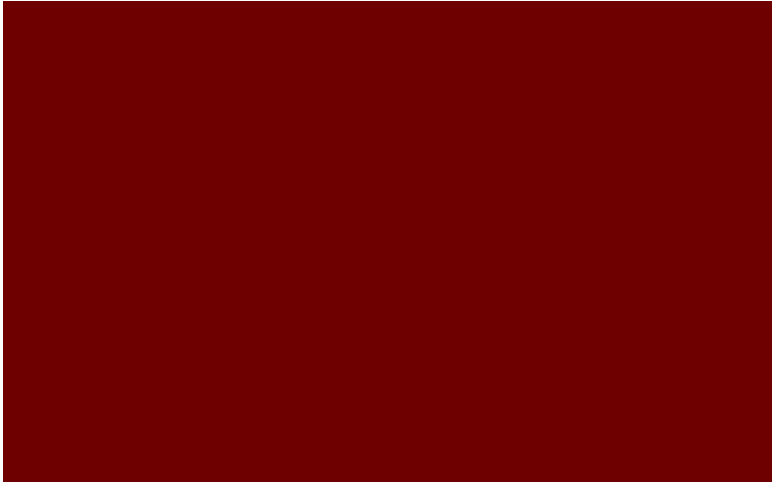
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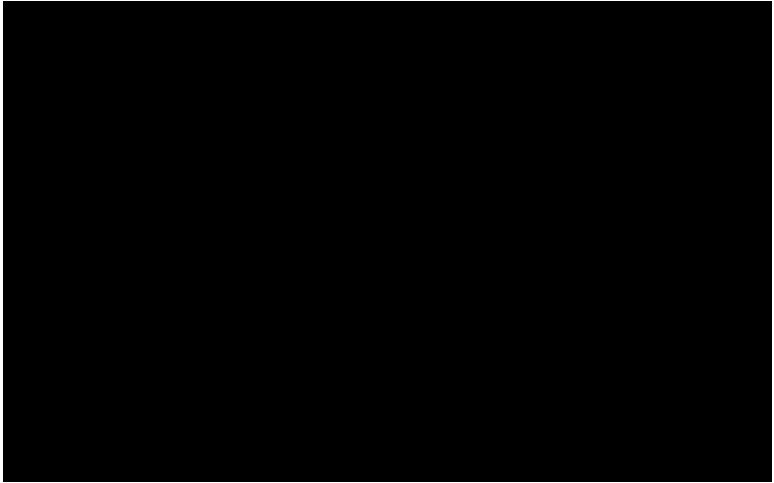
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Choice of units

$k = 1$ temperature is energy

$c = 1$ length is time,
mass is energy

$\hbar = 1$ energy is frequency

Energy is the only dimensionful quantity

Relativity and particle production

In thermal equilibrium particles have kinetic energy typically equal to their temperature: T . If two particles collide, their total kinetic energy is of order T .

If the mass of the particles is M , and the kinetic energy in a collision is much larger ($T \gg M$), then particles can be produced.

So the Lorentz factor $\gamma \equiv E/M \simeq T/M$ is much larger than 1 when particles are easily produced in a thermal medium.

Maxwell, Boltzmann, Einstein

Quantum mechanics and field theory

Quantum mechanics perfectly fine for problems with fixed number of particles. Say, spectrum of acetylcholine, information transfer through entangled states, transport in nanowires ...

Quantum mechanics fails when particle number changes:
 $H^* \rightarrow H + \gamma$. Then need quantum field theory.

Thermal matter with $M \ll T$ requires relativistic quantum field theory. All matter that we know of obeys the standard model. So **standard model at finite temperature**.

Pauli, Dirac, Bethe (1930s) ... Weinberg (1972)

The Discovery of the Strong Interactions

The atomic nucleus discovered in the scattering of α and β particles by matter. Positively charged nucleus unstable unless there is a new force to keep it together: the strong interactions.

Rutherford (1911)

Half a century of discoveries of mesons and baryons. All attempts to understand strong interactions failed. Realization that the true constituents of matter were quarks and gluons.

Nambu (1960); Gell-Mann, Ne'eman (1961)

Forces between quarks and gluons hundred times stronger than electrodynamics; forces between nucleons is a shadow of these.

The Discovery of the Theory of Strong Interactions

A relativistic quantum field theory of quarks developed. Initially faced technical difficulties similar to electrodynamics, but problems resolved. Theory has an intrinsic momentum scale, $\Lambda \simeq 200 \text{ MeV}$. Well tested for $\log(p/\Lambda) \gg 1$: perturbative QCD.

Gross, Wilczek, Politzer (1973)

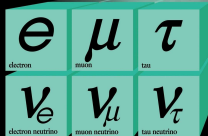
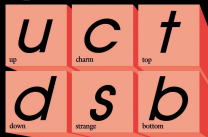
Three “flavours” of quarks: light flavours up, down ($m_u, m_d \ll \Lambda$), strange ($m_s \simeq \Lambda$). Later three heavy flavours discovered ($m \gg \Lambda$). Quarks and gluons **confined**, seen together only in combinations of mesons and baryons. Remained a theoretical mystery; until a radically new approach developed.

Strong interactions: 30 Nobel prizes to about 50 people

Particle content

Fermions: spin = 1/2 particles

Quarks

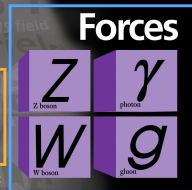


Leptons



Higgs Boson:
spin = 0
fundamental
scalar particle

Vector Bosons: spin = 1 particles



Little bangs

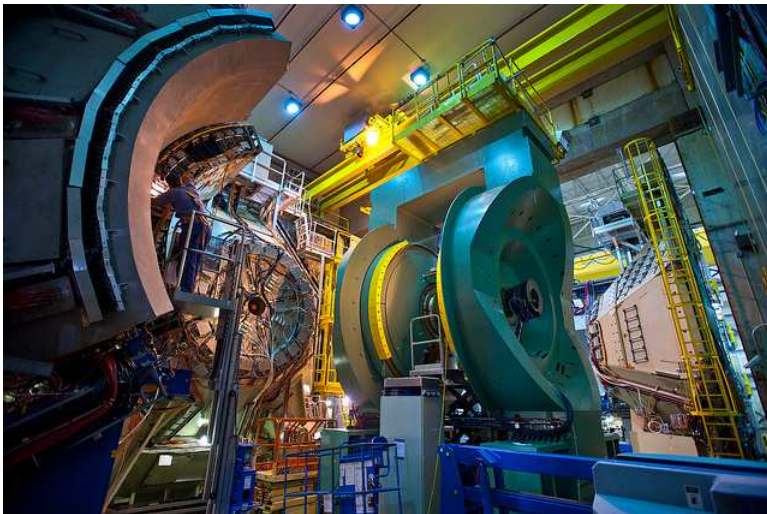
Recreate the conditions of the big bang in controlled experiments in a lab: relativistic collisions of heavy ions. Create a fireball which thermalizes at high temperature, then expands and cools.

Size of the fireball $\simeq 10$ femto meters. Detectors placed 10 meters away. See only the late stages of the bang.

Similar to today's telescopes looking back for traces of the big bang. Main difference: small bangs can be repeated. Statistical accuracy as high as you want.

CERN SPS 1980s, BNL RHIC 2000s, CERN LHC 2010s, GSI FAIR 2020s ...

A typical experiment



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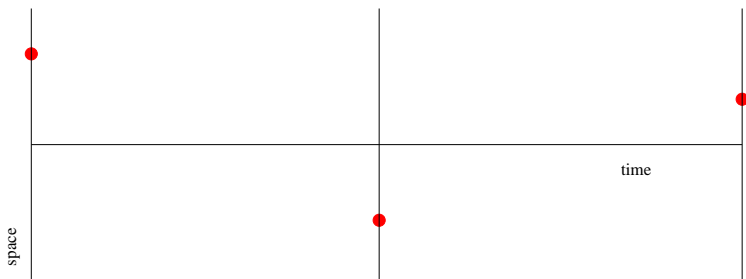
Post-colonial quantum field theory

$$\langle x_i, t_i | x_f, t_f \rangle$$



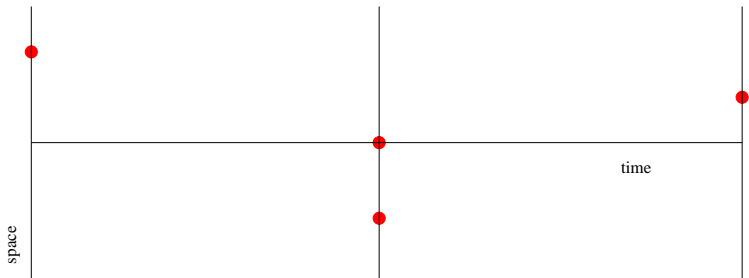
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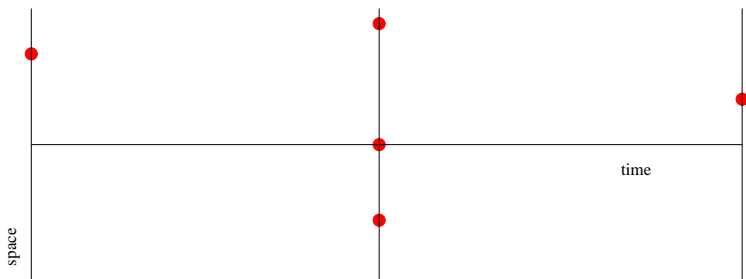
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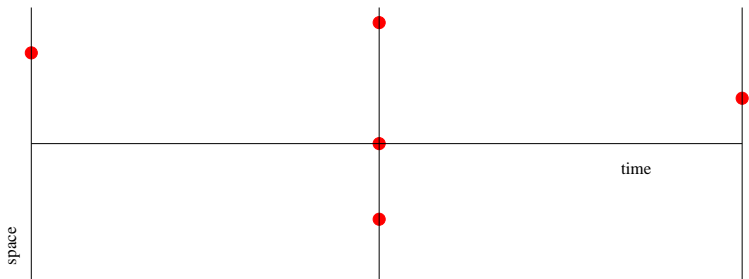
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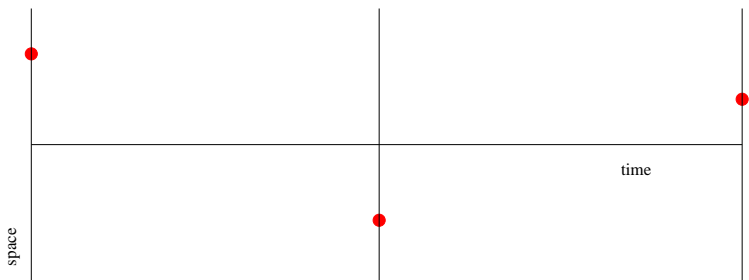
Post-colonial quantum field theory

$$\langle x_i, t_i | x_f, t_f \rangle = \sum_{x_m} \langle x_i, t_i | x_m, t_m \rangle \langle x_m, t_m | x_f, t_f \rangle$$



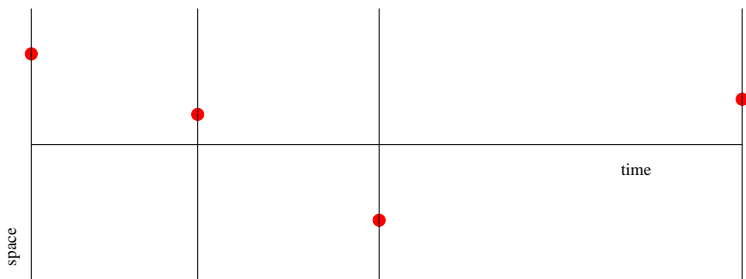
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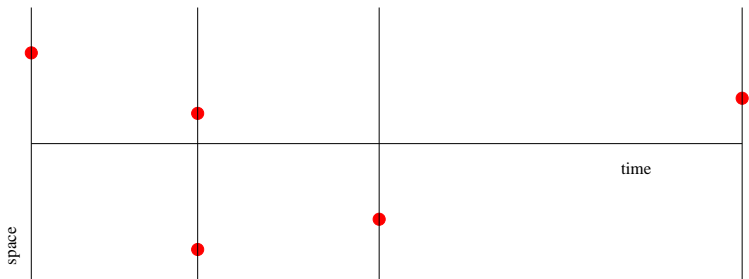
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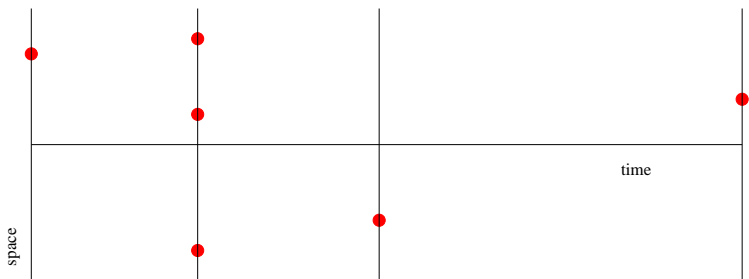
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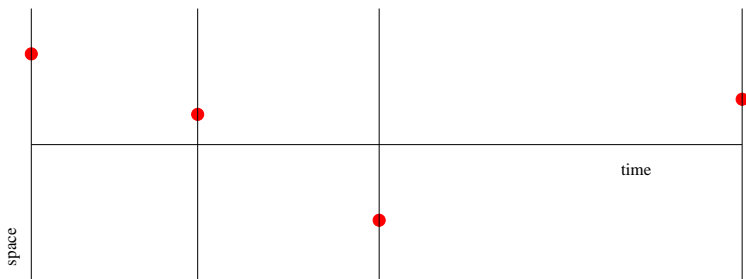
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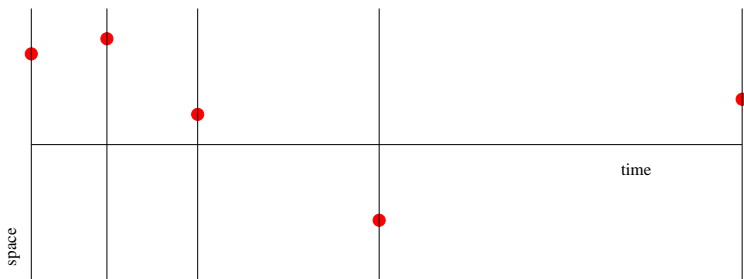
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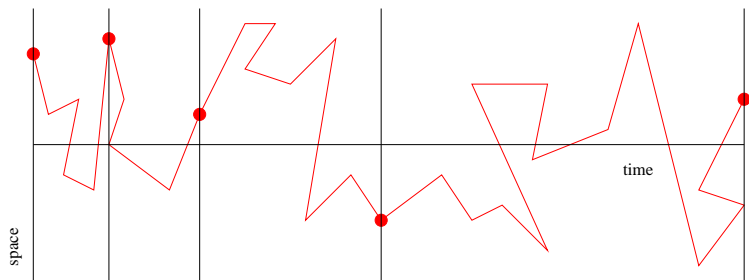
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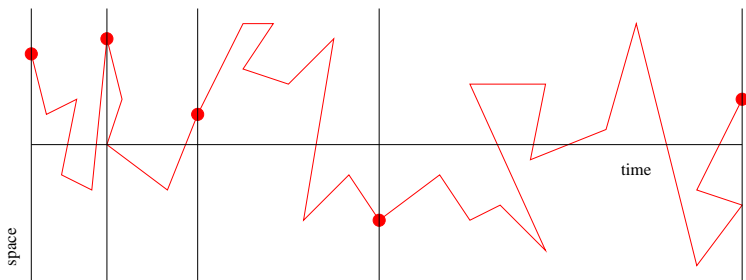
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$$\langle x_i, t_i | x_f, t_f \rangle = \sum_{x_m} \langle x_i, t_i | x_m, t_m \rangle \langle x_m, t_m | x_f, t_f \rangle$$


Dirac (1933), Feynman (1948)

The grand synthesis

Notice a relation between quantum evolution operator (transition matrix) and the thermal density operator

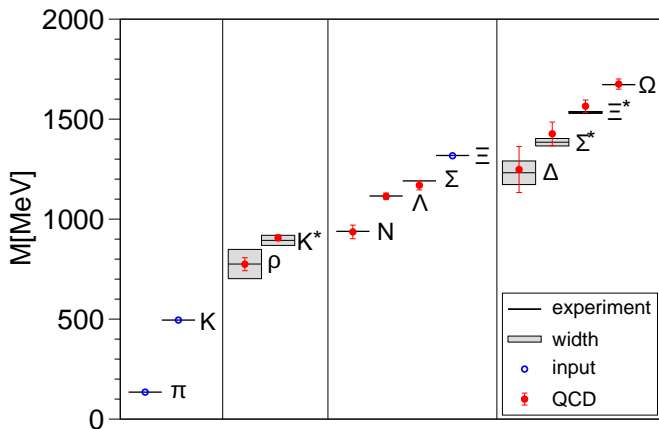
$$\exp(iHt) \quad \text{and} \quad \exp(-H/T),$$

Wick rotation $t \rightarrow it$. Makes path integral real; then use Monte Carlo to do the integral.

Fisher, Kadanoff; Wilson (1974), Creutz, Jacobs, Rebbi (1979)

Opens the door to the study of almost any quantity in a field theory. Applied to the study of hadron masses and widths, hadron form factors, decay constants, weak matrix elements, muon $g - 2$, rare decays of hadrons, exotics and glueballs, nucleon-nucleon scattering, nuclear structure, equation of state of neutron stars, phase transitions at finite temperature ...

A sample result



Durr, Fodor, Katz, et al (2009)

The sign problem

When there are Fermions in the external states, then the integrand of the path integral is complex. Monte Carlo method fails.

Examples: QCD at non-zero particle density, high temperature superconductors, ...

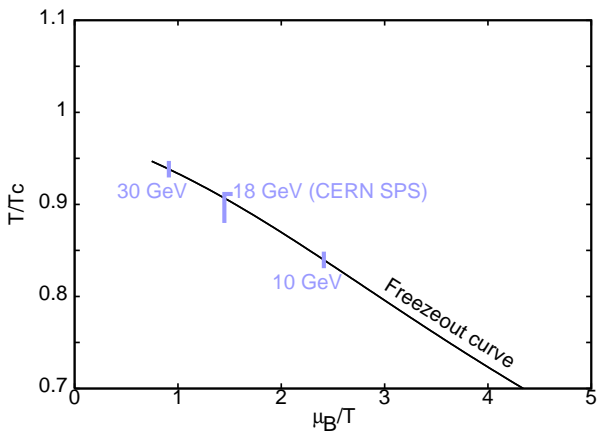
Proposed workaround, use the Maclaurin expansion:

$$P(\mu) = P(0) + \sum_n \chi_n \frac{\mu^n}{n!}$$

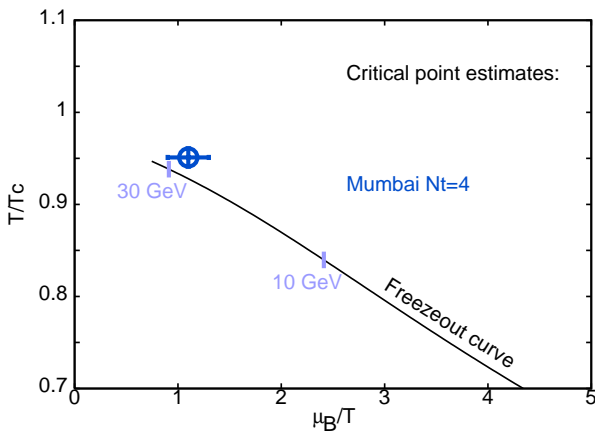
χ_1 is mean particle number, χ_2 is a particle number susceptibility, χ_3, \dots are non-linear susceptibilities. Developed methods to compute the coefficients χ_n .

Gavai, SG (2002)

The QCD critical point



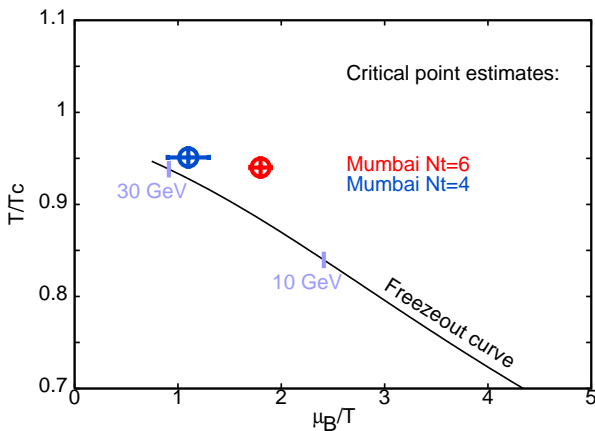
The QCD critical point



Critical point for the onset of confinement and chiral symmetry breaking; note unspecified scale T_c

Gavai, SG (2005, 2008), Datta, Gavai, SG (2012)

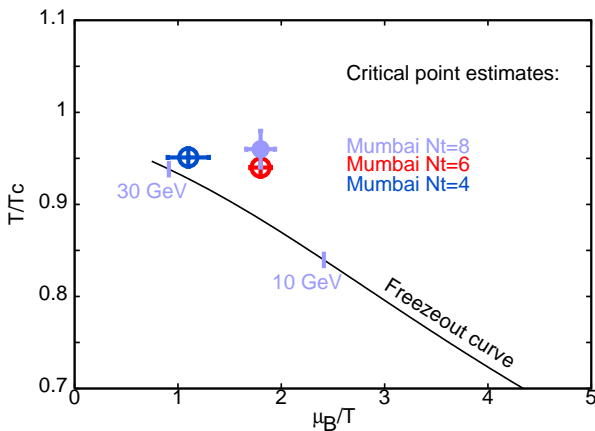
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Relativistic Collisions of Nuclei

At relativistic energies if two heavy nuclei (Au, Pb, etc) collide, then produced particles interact and form a dense hot fluid, which cools as it expands.

The particles in the fluid are strongly interacting, matter is opaque: no knowledge of the early stages of the collision. When fluid becomes dilute then particles freeze out, and observations can be made.

Fluctuations of conserved quantities possible, between one event and another.

Asakawa, Heinz, Muller — Jeon, Koch (2000)

Thermodynamic Fluctuations

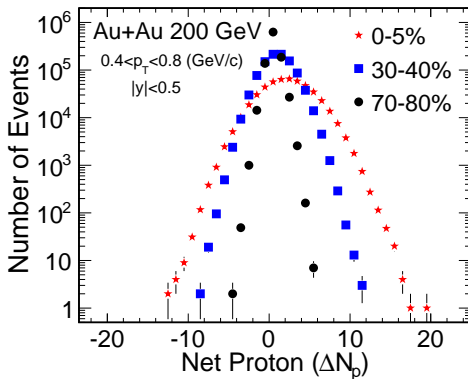
Since there are 10^{28} molecules of gas in this room, the pressure, entropy, heat content, etc can be accurately determined. Limits on our knowledge are due to instrumental limitations.

If the number of molecules was 10^6 then there would be inherent limits on the accuracy. Repeated accurate measurements would not give the same value but would reveal a distribution of values.

Fluctuations give physical information. Gaussian distribution of energy; width give specific heat. Specific heat can be computed from molecular properties.

Carnot (1824), ... Einstein (1905)

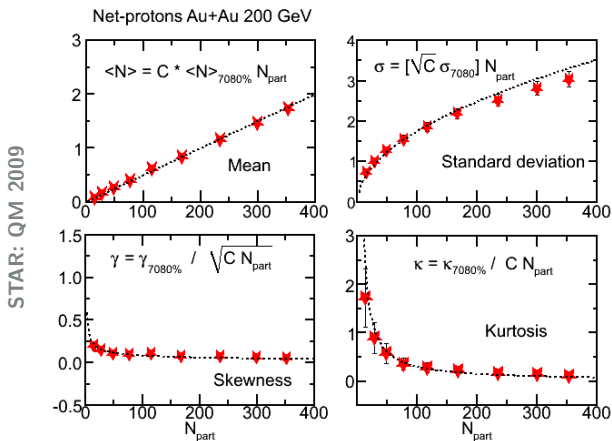
Observed fluctuations



STAR arxiv:1004.4959

Central rapidity slice taken. Protons accepted with p_T of 400–800 MeV.

Shape of distribution



Shape of distribution captured in cumulants $[B^n]$. Cumulants change with volume (proxy: N_{part}), by central limit theorem.

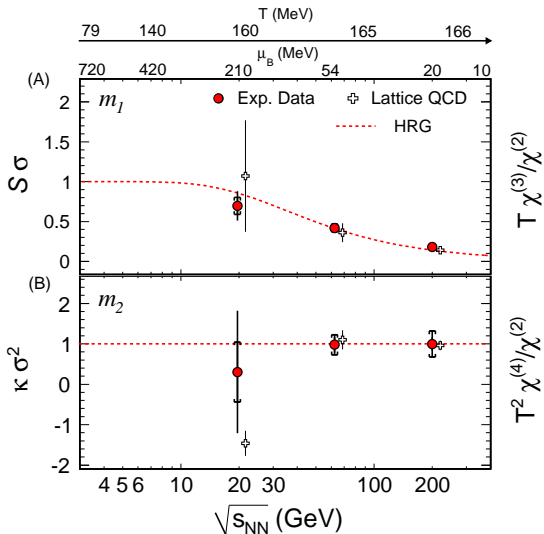
QCD predictions needed at finite μ_B

Shape variables: $[B^n] = (VT^3) T^{n-4} \chi_n(T, \mu)$. Ratios of cumulants are thermodynamic state variables:

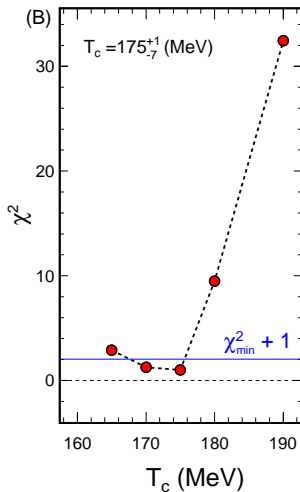
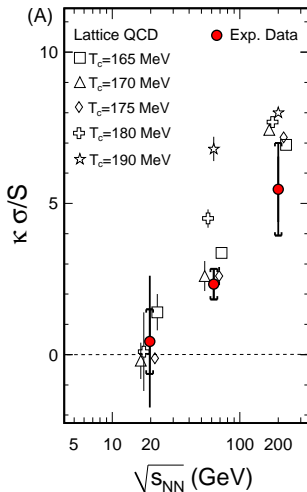
$$\begin{aligned} m_0 : \quad & \frac{[B^2]}{[B]} = \frac{T\chi_2}{\chi_1} \\ m_1 : \quad & \frac{[B^3]}{[B^2]} = \frac{T\chi_3}{\chi_2} \\ m_2 : \quad & \frac{[B^4]}{[B^2]} = \frac{T^2\chi_4}{\chi_2} \\ m_3 : \quad & \frac{[B^4]}{[B^3]} = \frac{T\chi_4}{\chi_3} \end{aligned}$$

SG, 2009; Athanasiou, Rajagopal, Stephanov, 2010

Checking the match



Tuning lattice scale to match data



GLMRX (2011)

Conclusions

Thermalization

After 1 parameter tuning agreement of thermodynamic predictions with data for 2 ratios at 3 energies. Indicates thermalization of the fireball at freezeout.

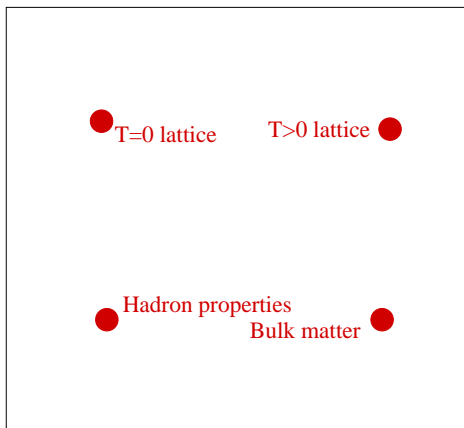
T_c

Comparison of lattice and data along the freezeout curve gives

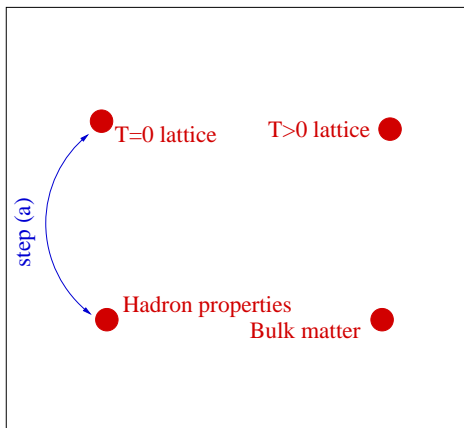
$$T_c = 175^{+1}_{-7} \text{ MeV},$$

in agreement with other scale settings on the lattice. Indicates that non-perturbative phenomena in single hadron physics and strong interaction thermodynamics are mutually consistent through QCD.

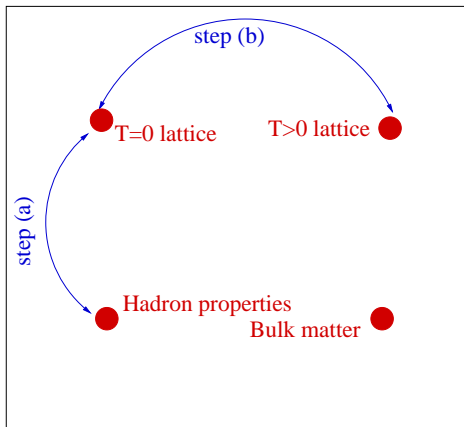
Implications for QCD



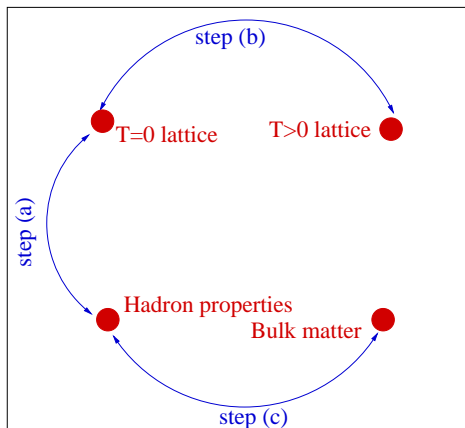
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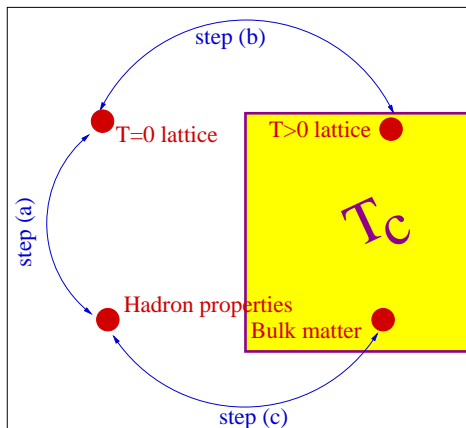
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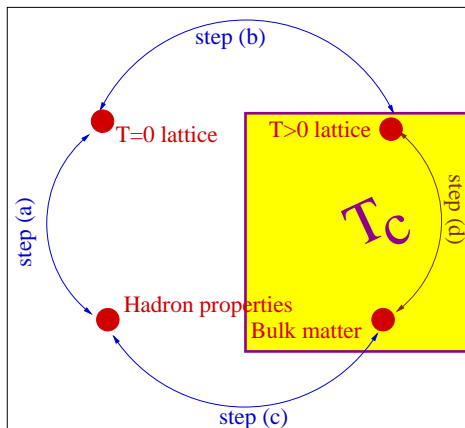
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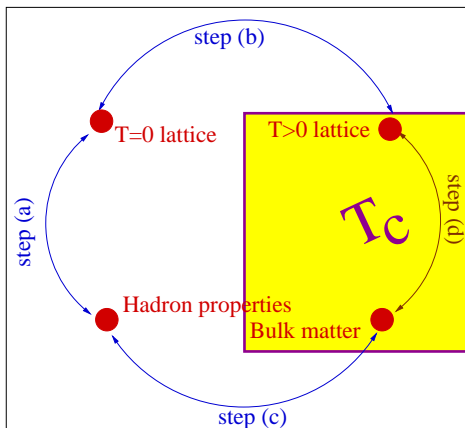
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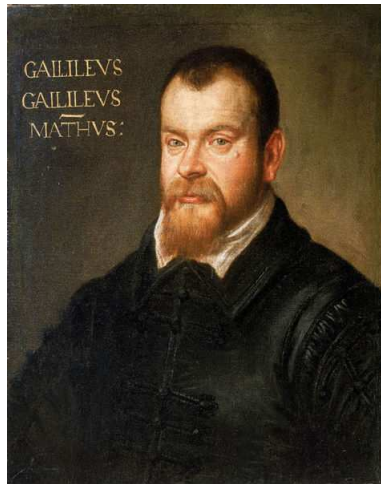


Beginning of quantitative theory for hot relativistic matter. Extend this method to the search for the critical point of QCD.

Outline

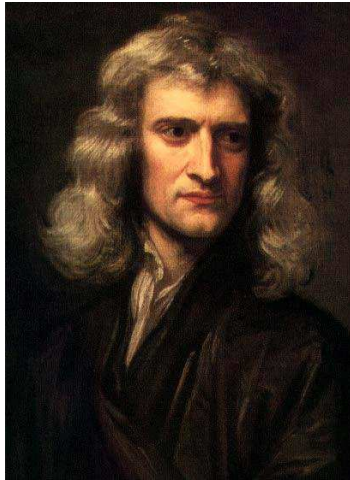
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The beginning of experimental sciences



Usually attributed to Galileo, often dated to 1609, but could be a little earlier.

The start of mathematical sciences



Usually attributed to Newton and dated to the establishment of the inverse square law of gravity in 1686.

The founding of computational sciences



Often attributed to Alan Turing, and traced to his 1937 proof that the behaviour of computer programs is observable but not mathematically predictable.

The computational mode is a conceptual revolution

Quantum mechanics was the first revolution of modern science. Conceptual unification of physically totally different fields: atomic spectroscopy, chemistry, solid state physics, etc.

Today the computational aspects of very large scale problems creates a methodological unification of lattice gauge theory with fermions, Google search and data mining, extraction of sky maps from noisy radio telescopes, atomic spectroscopy, fluid dynamics of nanorobots, ... Conceptual advances in one can cross fertilize other problems.

Similar methodological unification also occurring elsewhere within the computational mode of doing science.

A pedagogical shortcoming

Work in every area of science today involves an interplay of these three modes of science.

The science training programs developed in the 1920s and 30s still persists in our country today with updated content but little structural change. Not a problem earlier since the founding generations of the computational mode of doing science grew up with the subject.

This is a problem because a new generation is entering the sciences now. We need to train them in the computational mode of doing science: new course work, new text books. Computatiion must be embedded into course work.