Physics across the phase diagram of QCD

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Strong and Electroweak Matter
Prifysgol Abertawe
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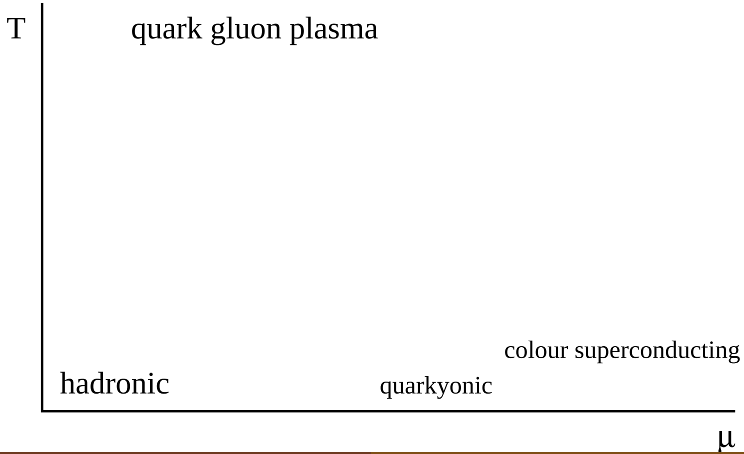
Outline

Heavy-ion collisions

Fireballs produced in heavy-ion collisions thermalize.



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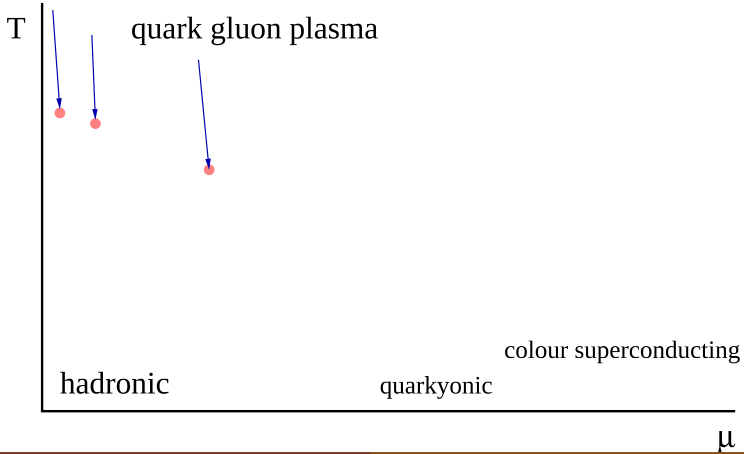

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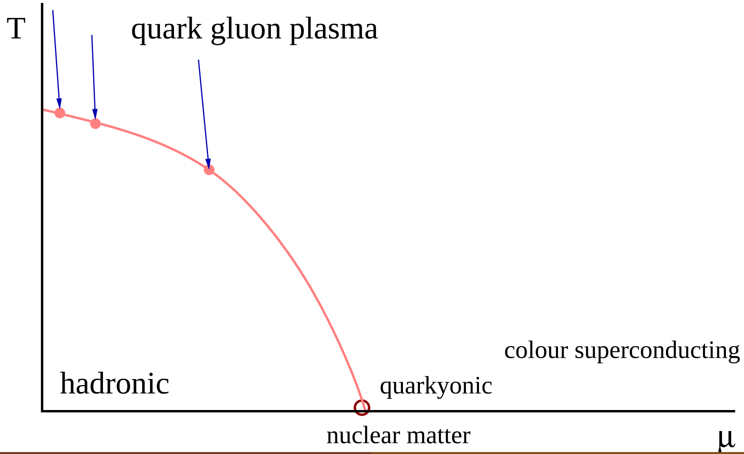

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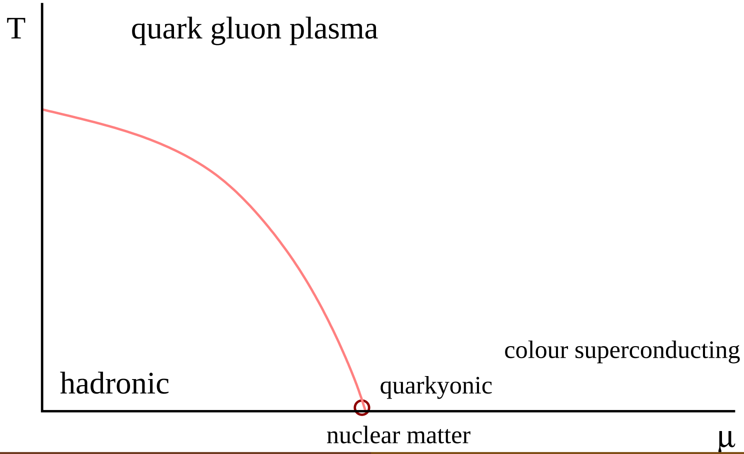

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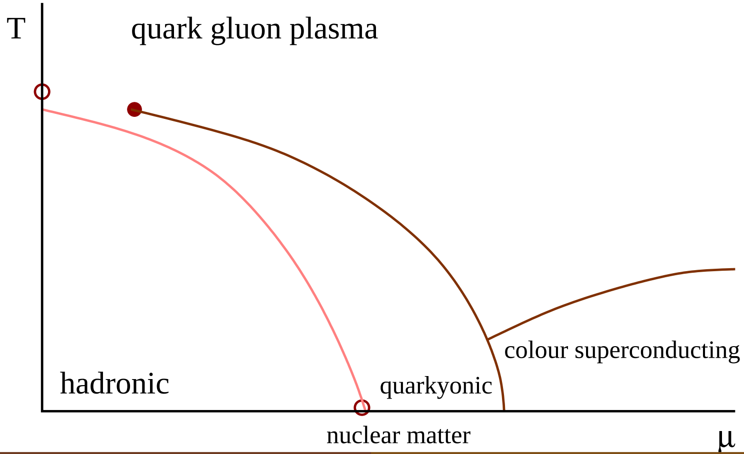


The analysis paradigm

Freeze out parameters

The hadronic final state is the easiest to study. This gives information on the freezeout hypersurface. Hence freezeout parameters could be the best determined quantities, and provide good checks of thermalization. Lattice can help.

Hydrodynamic evolution

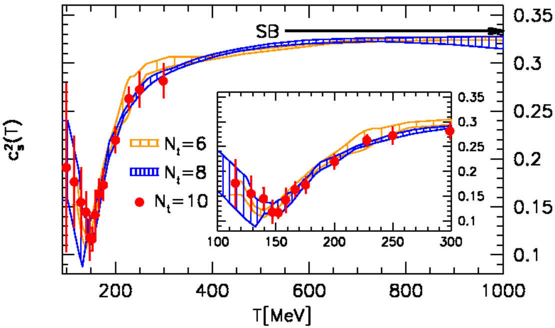
Some kind of transport theory before freezeout; maybe hydrodynamics. Methods for lattice predictions of transport coefficients still under development. If hydro, then lattice can constrain this with precision computations of the equation of state.

Initial conditions

Currently very active field of phenomenology. Seems to be presently outside the scope of lattice computations.

The QCD thermal cross-over

There is no phase transition in QCD at $\mu = 0$: gradual change from hadrons to quarks. Physically important: how fast does the fireball cool?



Endrodi et al, arxiv:1007.2580

Crucial question: what are the dof from 130 MeV$\leq T \leq$ 200 MeV?

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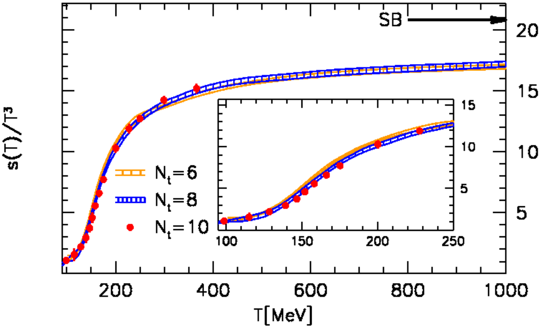

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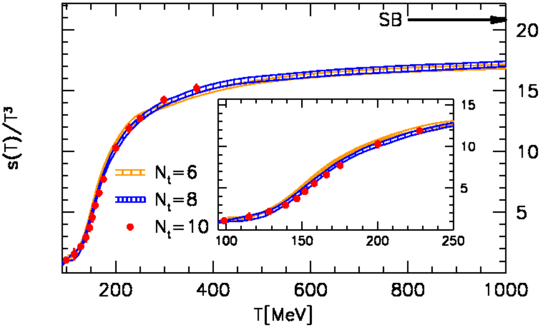

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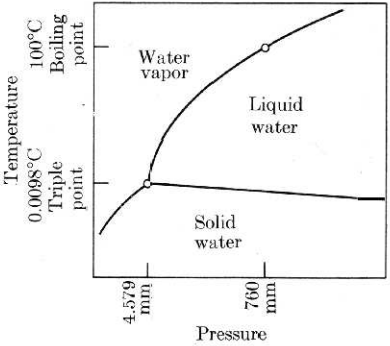
Outline

The nature of a cross over



Phase diagram: map of the singularities of the free energy

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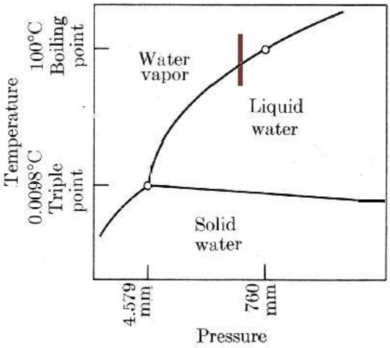

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1st order: latent heat
2nd order: divergent χ, c_V
cross over: neither.

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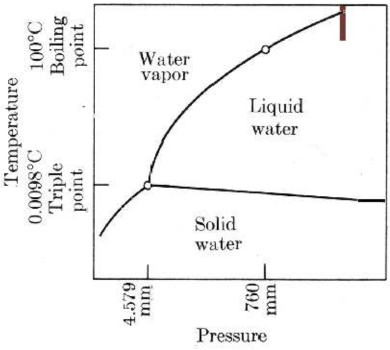

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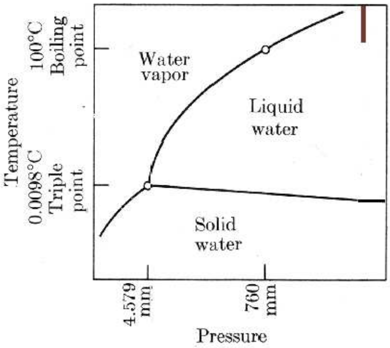

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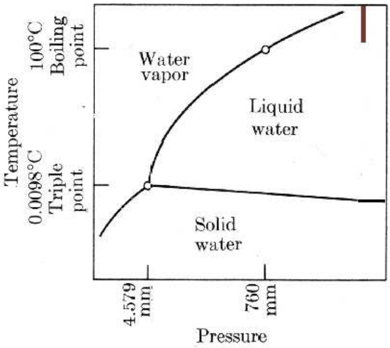

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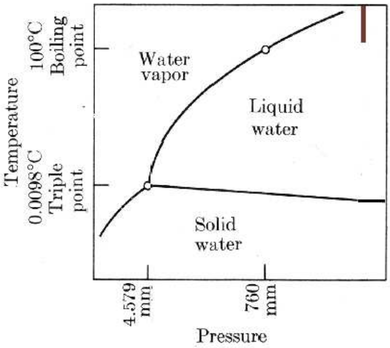

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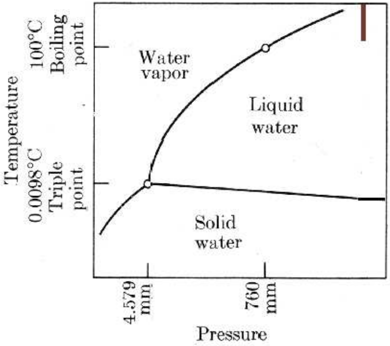

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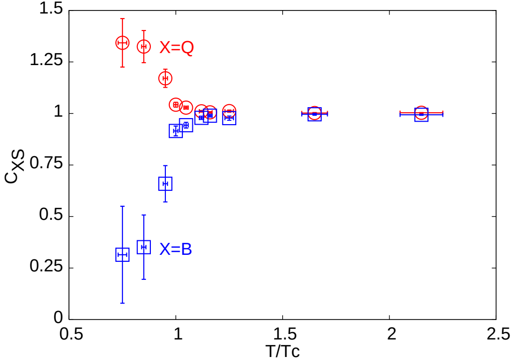
Peak in chiral susceptibility is hard!
Use the peak of the Polyakov loop susceptibility as a (non-unique but definite) measure of the cross over

$$T_c \simeq 175 MeV$$

Is there deconfinement in QCD?



A Fermi liquid of quarks at deconfinement **Gavai, SG, 2005**

$$C_{BS} = -3\frac{\chi_{BS}}{\chi_{SS}}$$

Koch, Majumder, Randrup, 2005

Is the continuum infinitely far away?

Typical argument for continuum limit of lattice computations: compute with $N_t = 4, 6, 8, 10, 12, \cdots$ Last three values of N_t agree; so we reached the continuum.

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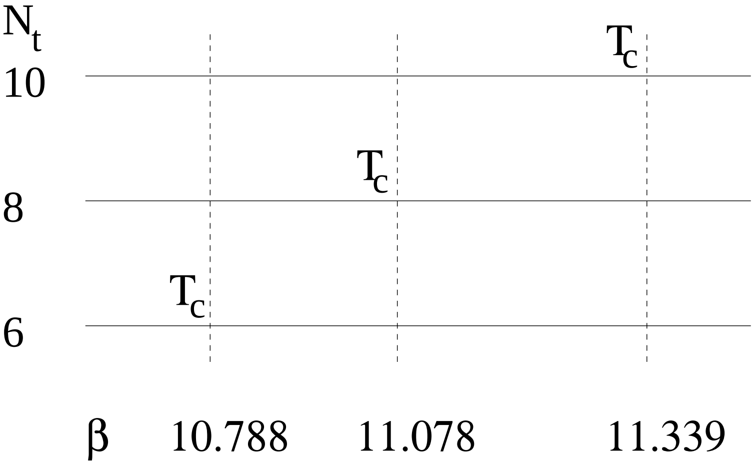
What if corrections are large but slow?

Correct answer must involve the RG. Inverse of lattice spacing is the momentum cutoff, and universal RG coefficients govern the approach to continuum. Can this be used?

SG, 2000; Datta and SG, 2010

Precision test of RG: SU(4) pure gauge theory



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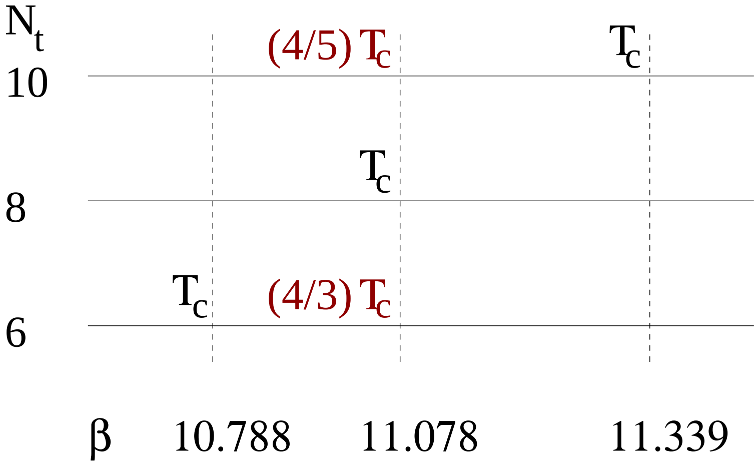

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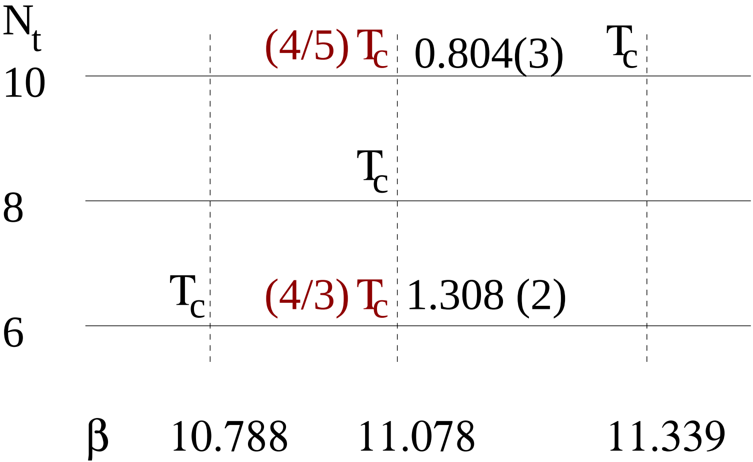
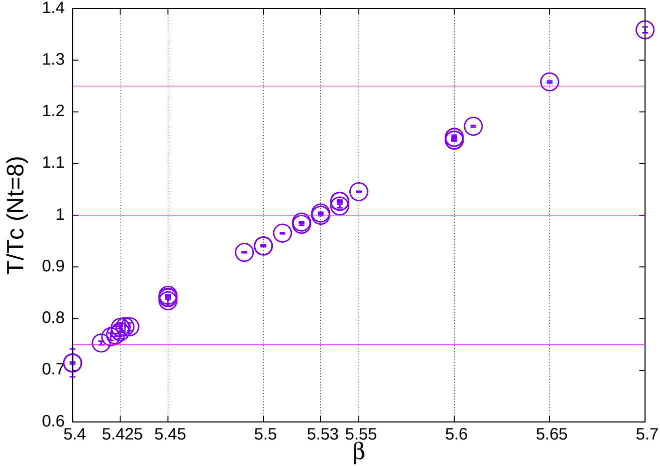


Approach to continuum: $N_f = 2$ staggered QCD



Scale set by T_c for $N_t = 8$ also good for $N_t = 10$ and 6.

Datta, Gavai, SG, 2012

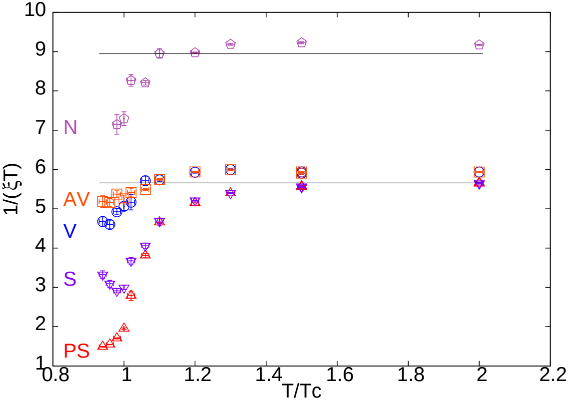
Outline

1. Heavy-ion collisions: a setting for QCD thermodynamics
2. QCD at $\mu = 0$: setting a temperature scale
3. QCD at $\mu = 0$: finding the scales of size
4. The theory of fluctuations: QCD at $\mu \neq 0$
5. Summary

Initial thermalized state

- Little direct access; too few penetrating probes.
- Recent attempts to analyze direct photons and dileptons in order to extract thermal parameters from them.
  Renk, 2005; Frodermann, Heinz, 2009; Mohanty, Alam, Mohanty, 2011; Csanad, Majer, 2011
- Fitted temperature, T, of about 500 MeV at early time of 0.5 fm in RHIC. Cross checks not yet possible because of paucity of data. Is it possible to have thermalization so early? Theory still in flux.
- Spatial size, L, is bounded by time. Hence $LT \simeq 1.25$. Is this large enough for thermodynamics? Need to know correlation lengths.

Hot screening masses



Nikhil Karthik and SG, 2012: QCD $N_f = 2$, $a = 1/(4T)$

Implies that $L/\xi \simeq 8$ for gauge invariant probes; volumes large enough for thermodynamics if dynamics is slow enough.

Technical improvements

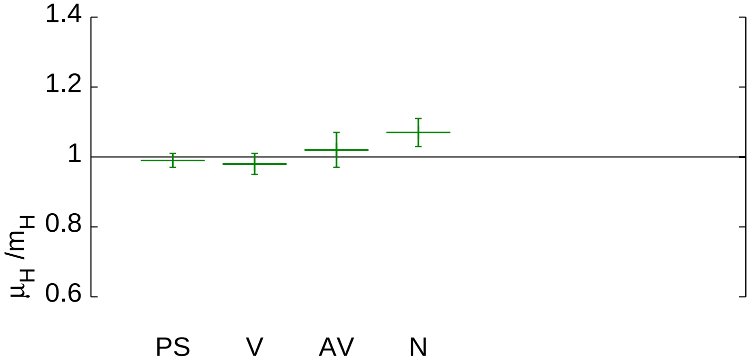
Previous recent: **HotQCD 2011; Banerjee, Gavai, SG, 2011**

- Staggered quarks with fat links. Results shown for HYP improvement ($\epsilon = 0.6$). Similar results with Ape smearing ($\epsilon = 0.6$) or stout ($\epsilon = 0.1$) or HEX ($\epsilon = 0.15$).
- High temperature results insensitive to bare quark mass. Varied sea quark mass to change m_π by 15% without affecting screening masses at high temperature.
- Screening masses insensitive to spatial volume (L^3) at all temperatures, provided $m_\pi L > 4$. Varied L by 50% to check.
- Noise controlled by using wall sources after Coulomb gauge fixing.

Freeze out conditions

- Chemical freeze out occurs when reaction rates fall below expansion rate: either densities of reactants diluted or thermal activation rates inadequate. For diffusion dominated reactions, this happens at the Peclet scale.
- Stochastic freeze out conditions approximated by space-like surfaces. Thermal conditions on this surface estimated by modelling yields by an ideal gas of hadrons. Best fits at RHIC top energy give $T \simeq 165\ \mathrm{MeV} \simeq 0.95\,T_c$ and $L = 15$–20 fm.
- Cross checks possible because of large amount of data.
- Model also fits bulk thermal quantities from lattice (but this is a stronger statement). Directly look for the eigenvalues of the transfer matrix. Are there temperature dependences?

Cold screening masses



Padmanath, Datta, SG, Mathur, 2012: quenched QCD

Improved Wilson quarks with heavy pion ($m_\pi \simeq 2T_c$).

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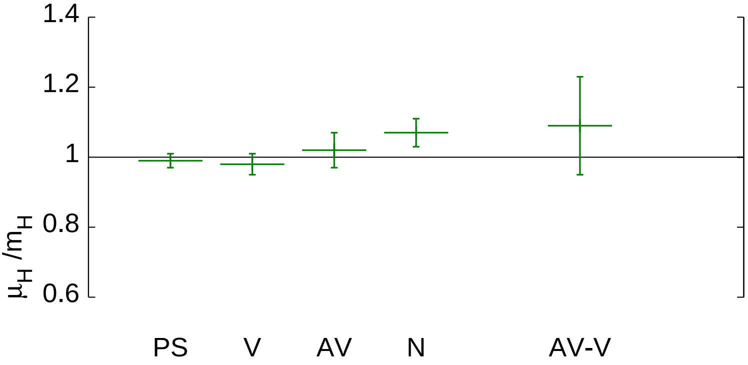

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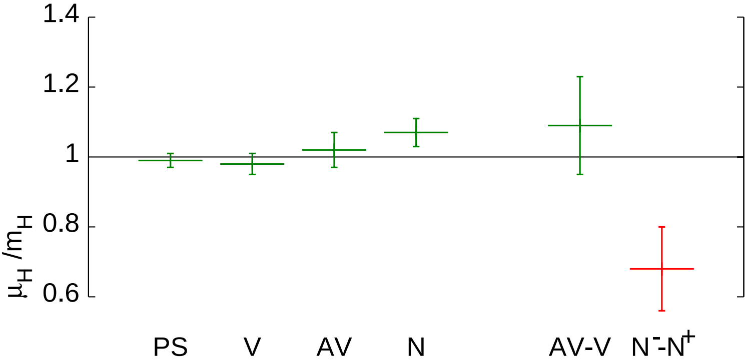

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Improved Wilson quarks with heavy pion ($m_\pi \simeq 2T_c$). Shift in nucleon sector mass splitting persists when pion mass is shifted

Details and conjectures

Previous: **Laermann, Schederin, Wissel, 2007**

- Clover improved Wilson quarks in quenched QCD with lattice spacing $a = 1.05/T_c$ at $T = 0$ and $T = 0.95T_c$. Zero temperature pion masses ranging from $2.4T_c$ to $1.5T_c$.
- Correlation functions measured with point sources turn out to be rather noisy. Wall sources preferred.
- Deconfining 1st order transition in pure gauge theory but tendency for precursor parity symmetry restoration. Could it become more dramatic if quarks are unquenched?
- No study of nucleon parity partner between 130 and 175 MeV. Unquenched improved Wilson quarks would be very interesting. Could the region between peaks of chiral and deconfinement susceptibilities be parity restored hadrons?

Thermodynamics and fluctuations

Since $Lm_\pi \simeq 10$–14 for fireball at freezeout in top RHIC energy, thermodynamics is not bad approximation, but corrections good to have.

However, detectors observe a central wedge defined by $\Delta\eta = 1$. So the volume becomes correspondingly smaller, and corrections more important.

Extend the theory of fluctuations beyond the usual second term if needed. Lattice can be used to compute these terms with precision.
SG 2009; Gavai, SG, 2010

Outline

A grand-canonical ensemble

With full acceptance detector, all conserved charges equal to initial value of charge. Any fluctuations only initial state fluctuations: one way to characterize initial state.

Partial acceptance in rapidity implies only part of the initial volume accepted. When this is a small part of the full volume then grand canonical ensemble.

Grand canonical fluctuations identifiable with eventwise fluctuations of conserved quantities. Finite volume fluctuations of conserved quantities in equilibrium requires higher order susceptibilities.

Madhava-Maclaurin series method

Series expansion of pressure ($t = T/T_c$ and $z = \mu_B/T$):

$$\frac{1}{T}^4 P(t,z) = \frac{P(T)}{T^4} + \frac{\chi^{(2)}(T)}{T^2}\frac{z^2}{2!} + \chi^{(4)}(T)\frac{z^4}{4!} + T^2\chi^{(6)}(T)\frac{z^6}{6!} + \cdots ,$$

Gavai, SG (2003)

Derivatives give the successive "susceptibilities":

$$\frac{\chi^{(1)}}{T^3}(t,z) = \frac{\chi^{(2)}}{T^2} z + \chi^{(4)}\frac{z^3}{3!} + T^2\chi^{(6)}\frac{z^5}{5!} + \cdots ,$$

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$$\frac{\chi^{(3)}}{T}(t,z) = \chi^{(4)} z + T^2\chi^{(6)}\frac{z^3}{3!} + T^4\chi^{(8)}\frac{z^5}{5!} + \cdots ,$$

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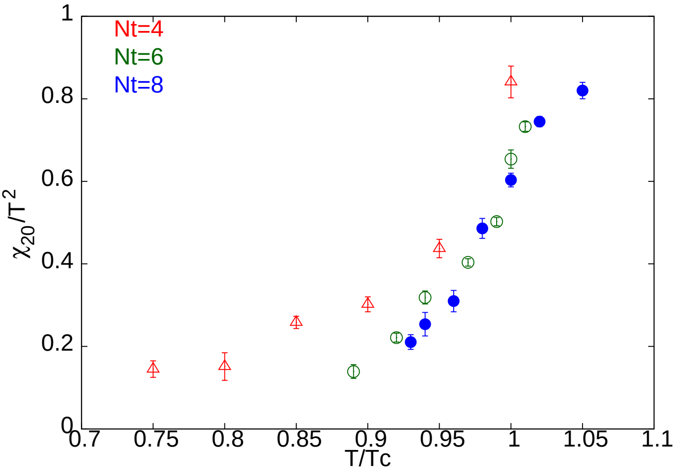
Series diverge at the critical point: can be used to estimate the position of the critical point:

$$z_* = 1.8 \pm 0.1 \qquad \text{lattice cutoff 1.2 GeV}$$

Gavai, SG, 2008

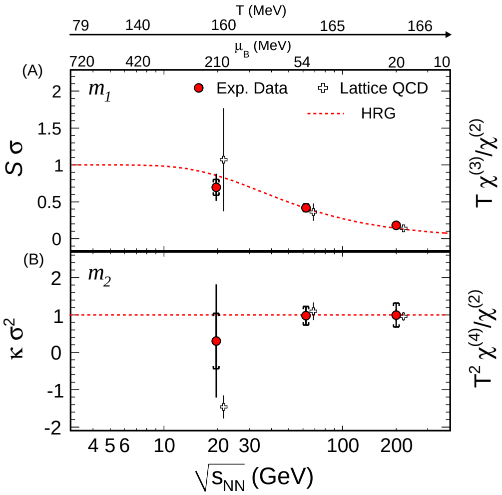
$$z_* = ?? \qquad \text{lattice cutoff 1.6, 2 GeV}$$

Closing in on continuum results



Datta, Gavai, SG, 2012

Making contact with experiments



Gavai, SG (2010); STAR (2010); GLMRX, Science (2011)

Two earlier suggestions

If the critical point is far from the freezeout curve over a certain range of energy, then m_1 decreases with increasing $\sqrt{S_{NN}}$ (since z decreases) and m_3 increases. Using these two measurements and comparing with lattice predictions, it is possible to estimate the freezeout conditions: T/T_c and μ_B/T. This method is independent of the usual one in which hadron yields are interpreted through a resonance gas picture [15]. Comparison of the two methods then allows us to estimate T_c by inverting the argument of the previous paragraph. Mutual agreement of the values of T_c

so derived at different $\sqrt{S_{NN}}$ would constitute the first firm experimental proof of thermalization. If this proof holds then one also obtains the simplest and most direct measurement of T_c found till now. Since such a thermometric measurement can be made reliably with data at large $\sqrt{S_{NN}}$, where μ_B is small, it would remain a valid measurement whether or not a critical point is found in the low energy scan at RHIC.

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The first strategy

Use the chemical freezeout curve and the agreement of data and prediction along it to measure

$$T_c = 175^{+1}_{-7}\ \mathrm{MeV}.$$

GLMRX, 2011

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The second strategy

Because of the critical divergence of $\chi^{(2)}(t,z)$, near the critical point the ratios of shape variables have poles as a function of $z = \mu/T$.

$$m_0 = \frac{[B^2]}{[B]} = \frac{\chi^{(2)}(t,z)/T^2}{\chi^{(1)}(t,z)/T^3} = \frac{1}{z\left[1-3\left(\frac{z}{z_*}\right)\right]}$$

$$m_3 = \frac{[B^4]}{[B^3]} = \frac{\chi^{(4)}(t,z)}{\chi^{(3)}(t,z)/T} = \frac{1}{z\left[1-10\left(\frac{z}{z_*}\right)\right]}$$

Match lattice predictions and data (including statistical and systematic errors) assuming knowledge of z_* to get estimates of freeze-in conditions.

Follow up: **Karsch 2011**

A third strategy

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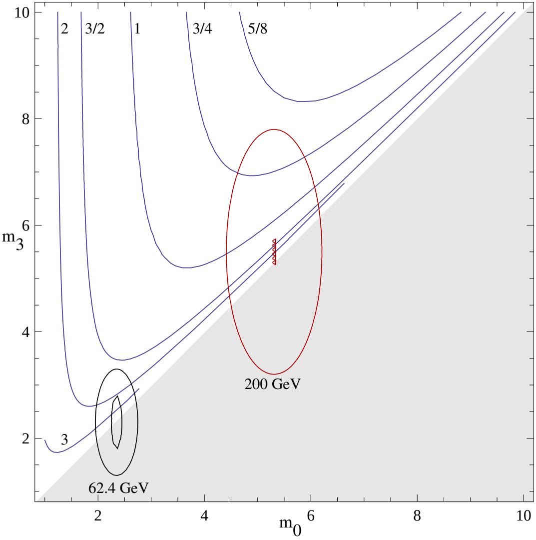
Fit m_0 and m_3 simultaneously to get both z and z_*. Since z_* is the position of the critical point: high energy data already gives information on the critical point without lattice input!
Analysis: **SG, 2012**

Critical point from the top RHIC energy



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Questions for investigation

1. Deconfinement and chiral symmetry cross overs in QCD are displaced from each other. Good evidence that deconfinement cross over point determined from the Polyakov loop corresponds to the formation of a Fermi liquid of quarks. Is there parity doubling of baryons at the chiral symmetry cross over?
2. Correlation lengths of colour singlet operators very small at high temperatures. No bar to thermalization at short times, although finite volume corrections may be important.
3. Finite volume corrections easily seen in the shape of the eventwise distribution of baryon number; in good agreement with lattice results. Opens door to much more physics.