The QCD critical point

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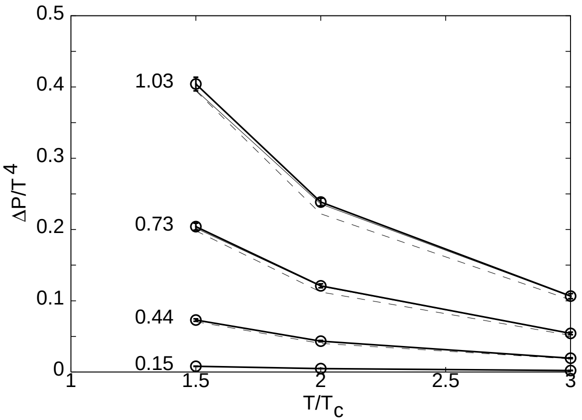
XQCD 2013, Bern, Swaziland

EOS at $\mu \neq 0$



Gavai, SG: Phys.Rev. D68 (2003) 034506

$$\Delta P = P(\mu, T) - P(0, T).$$

The mathematical problem

Perform a series expansion of the pressure in powers of chemical potential

$$\Delta P(\mu_u, \mu_d, T) = \sum_{m,n} \chi_{m,n}(T) \frac{\mu_u^m \mu_d^n}{m!n!}.$$

Does this converge? Can one reconstruct the function? Well studied classical problem. Special complications: few coefficients known, with errors.

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Also, expansion in $z = \mu_B / T$

$$\chi_B(\mu_B, T) = \frac{\partial^2 \Delta P}{\partial \mu_B^2} = \chi_B^0(T) + \frac{T^2}{2!}\chi_B^2(T) z^2 + \frac{T^4}{4!}\chi_B^4(T) z^4 + \cdots$$

Our data

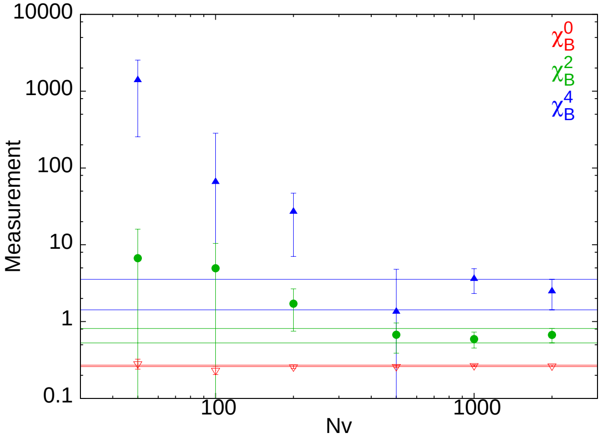
Lattice simulations with $N_f = 2$ staggered quarks and Wilson action. Used $N_t = 8$, 6 and 4; $m_\pi \simeq 0.3 m_\rho$ MeV; spatial size $L = 4/T$.

Temperature scale, T_c, found by the point at which χ_L peaks. If $T_c \simeq 170$ MeV, then $1/a = 1.36$ GeV.

Configurations: 50K+ at each coupling; large number of fermion sources used for determination of fermion traces.

Partial statistics reported in: **Datta, Gavai, SG: arXiv:1210.6784**

Numerical errors



Errors depend on number of fermion sources for evaluation of propagator as well as number of gauge configurations. Multiple fermion loops are source hungry.

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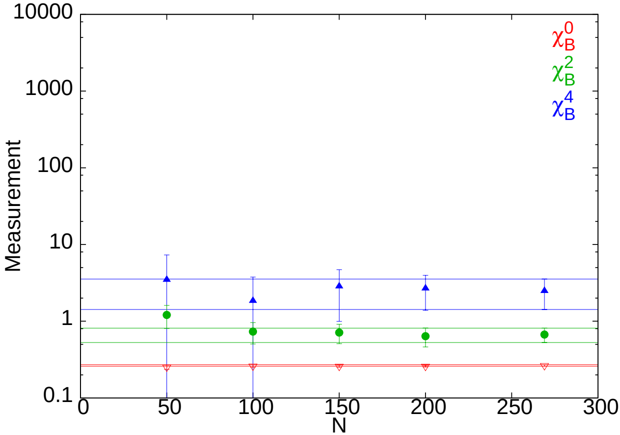

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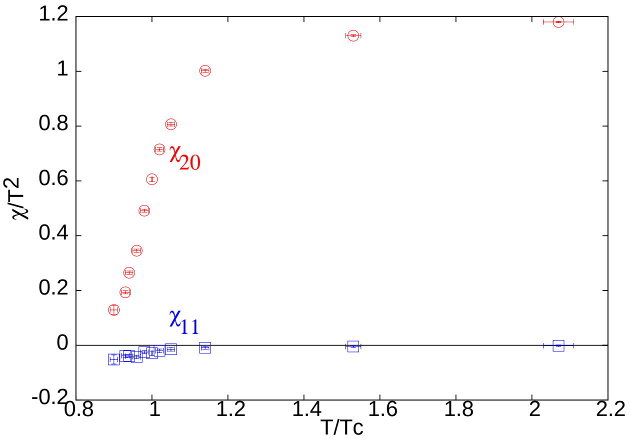
Susceptibilities at $\mu = 0$



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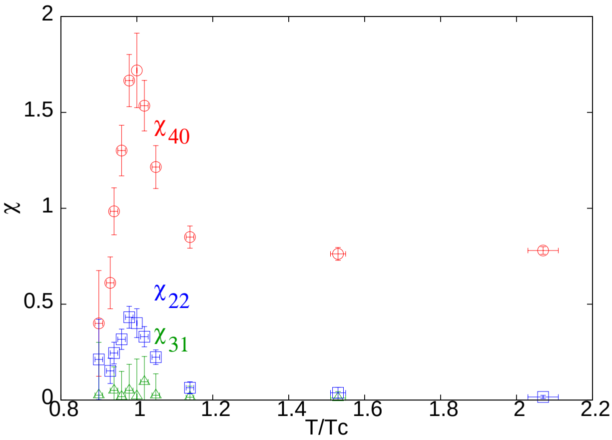
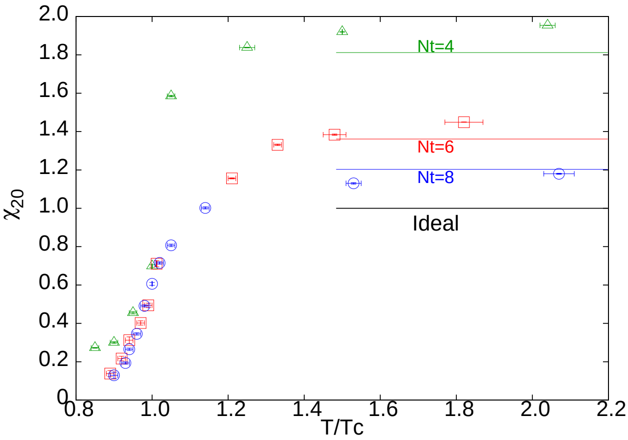
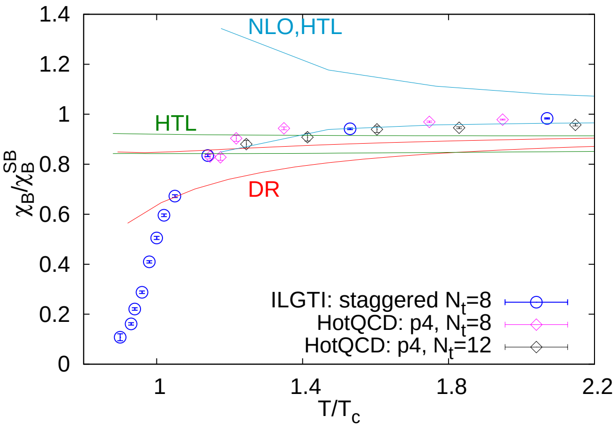


Lattice spacing effects



Nearing continuum physics



Continuum: $T_c = 170$ MeV; p4 with $N_t = 8$: $T_c = 180$ MeV.

HTL, DR: Andersen etal, 1307.8098; NLO: Haque etal, 1302.3228; HotQCD: Petreczky, Lattice 2013

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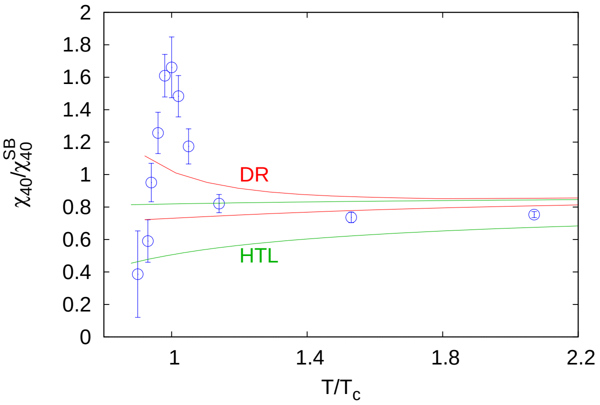

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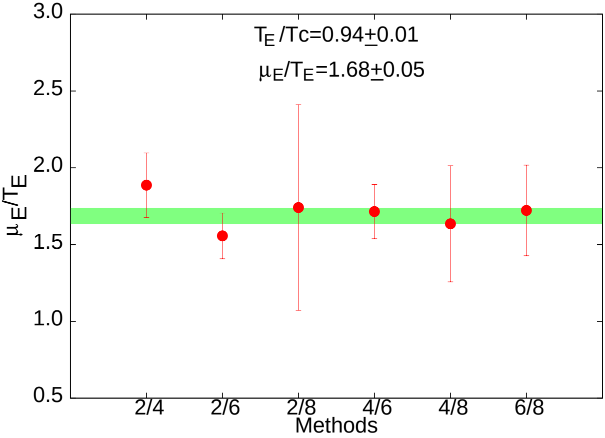
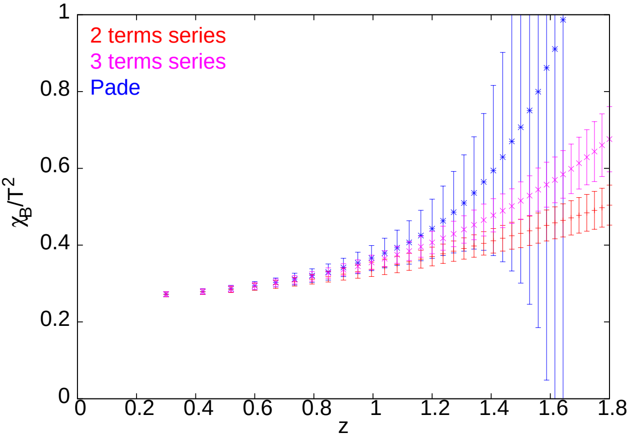
The radius of convergence



For $N_t = 6$, $\mu_E / T_E = 1.7 \pm 0.1$ **Gavai, SG: 2008**

Must resum a series expansion



Truncated series sum is regular even at the radius of convergence, so is missing something important.

Critical behaviour of m_1

If $\chi_B(z) \simeq (z_* - z)^{-\psi}$, then $m_1 = d\log\chi_B/dz$ has a pole. Series expansion of χ_B gives series for m_1. Resum series into a Padé approximant:

$$[0,1]: \qquad m_1(z) = \frac{c}{z_* - z}$$

Width of the critical region? If we define it by

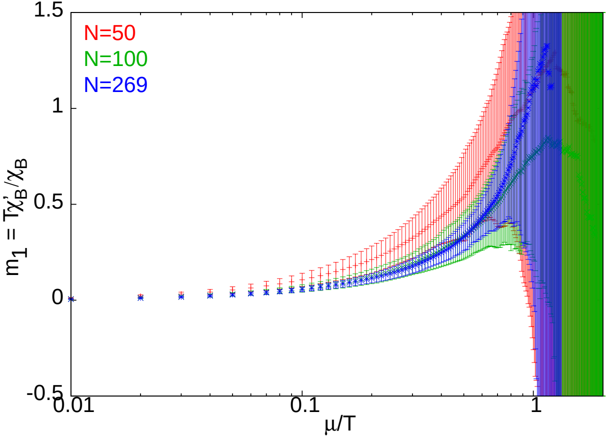
$$\left|\frac{m_1(z)}{m_1(0)}\right| > \Lambda,$$

then $|z - z_*| \leq z_*/\Lambda$.
Errors in extrapolation? We have

$$\left|\frac{\Delta m_1}{m_1}\right| > \frac{1}{1 - \Lambda\delta},$$

where δ is fractional error in z_*.

Critical slowing down



The DLOG Pade

At a critical point

$$\chi_B = \frac{\partial^2 (P/T^4)}{\partial z^2} \simeq (z_*^2 - z^2)^{-\psi}.$$

Continuity and finiteness of P at the CEP forces $\psi \leq 1$.

Since

$$m_1(z) = \frac{d \log \chi_B}{dz} \simeq \frac{2\psi z}{z_*^2 - z^2},$$

use the series to estimate the critical exponent. Series for m_1 has one term less than series for χ_B.

Accurate results require fine statistical control of at least 3 series coefficients of χ_B: 2 of m_1.

Widom scaling

Widom scaling for the order parameter gives

$$|\Delta\mu| = |\Delta n|^\delta J\left(\frac{|\Delta T|}{|\Delta n|^{1/\beta}}\right),$$

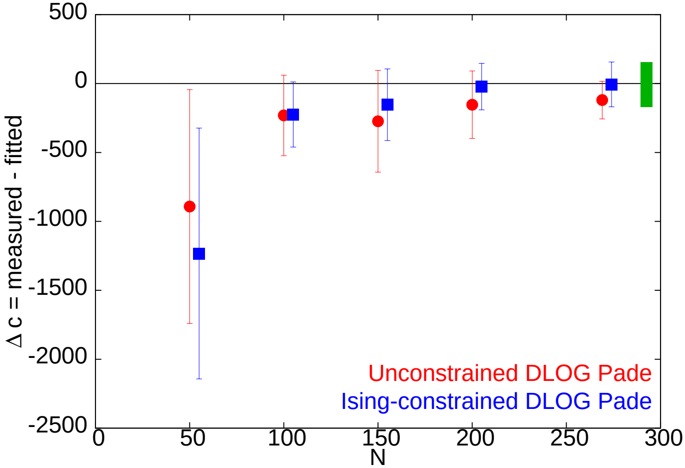
where $\Delta T = T - T_E$ and $\Delta\mu = \mu - \mu_E$. For $\Delta T = 0$ one finds $\Delta n \propto |\Delta\mu|^{1/\delta}$ in the high density phase. Then clearly one has

$$\psi = 1 - \frac{1}{\delta}.$$

For the 3d Ising model, $\delta = 1.49$, so $\psi = 0.79$. Since the identification of the two scaling directions is arbitrary, one can vary these. This gives $0.79 \leq \psi \leq 1$.

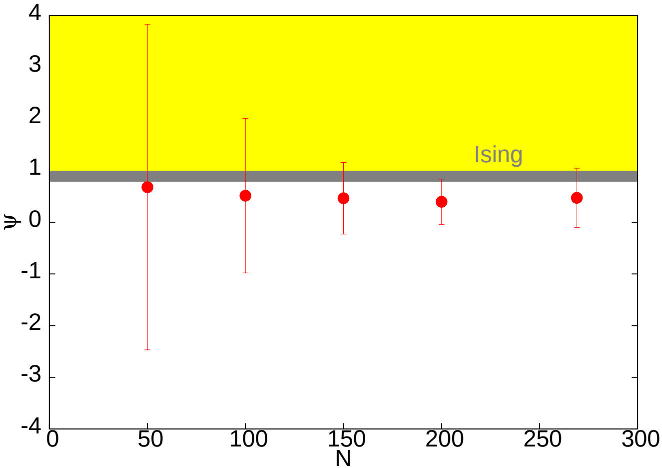
In mean field theory one has $\delta = 3$, so $0.66 \leq \psi \leq 1$. The data cannot yet distinguish between these cases.

Testing the DLOG Pade



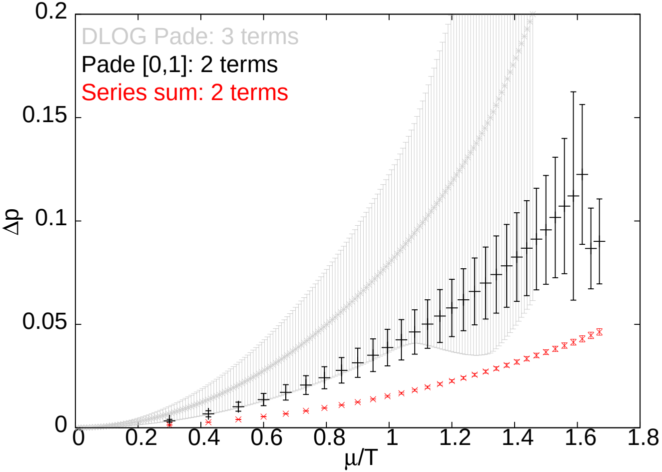
Test resummation by using 3rd term of m_1.

Critical exponent

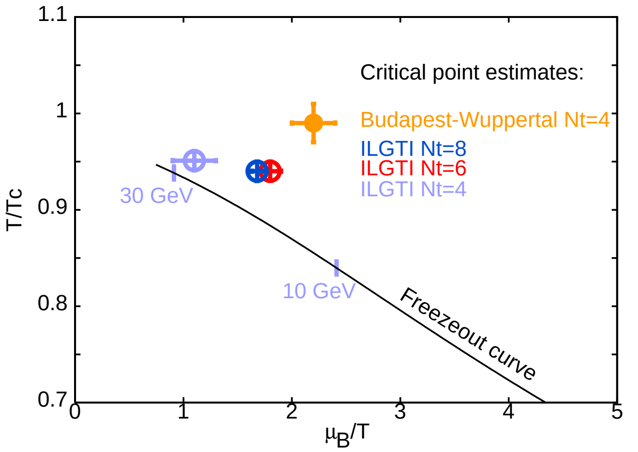


Large errors in ψ, but $\psi < 1$ as expected from continuity of pressure. Ising prediction: $\psi \geq 0.79$.

The pressure



Critical point and critical region



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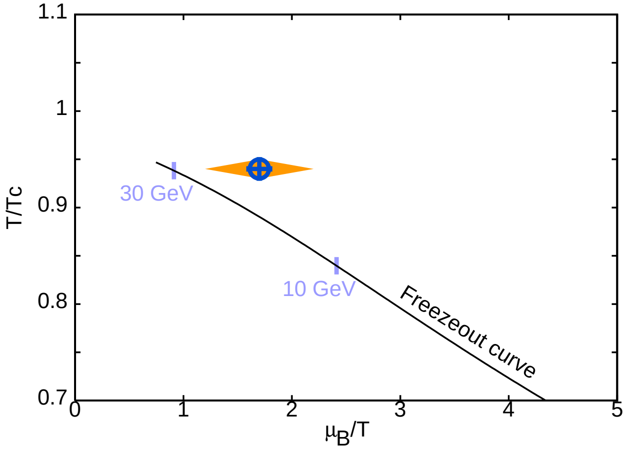

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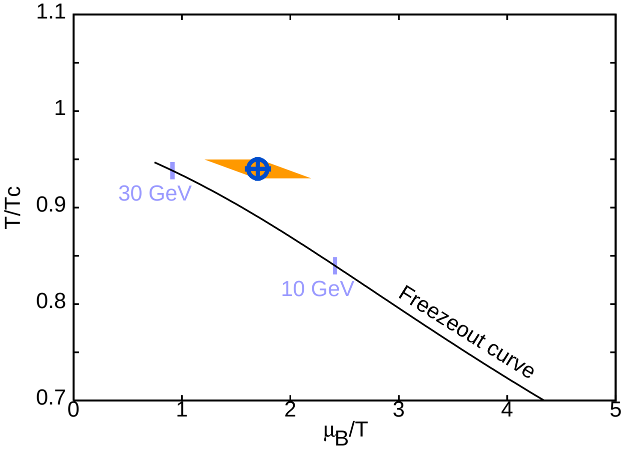

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