Pressure in QCD at finite μ

Sourendu Gupta

TIFR, Mumbai

19 January, 2014
STAR Regional Meeting
IOP, Bhubaneshwar

ILGTI computations with Saumen Datta and Rajiv Gavai

The mathematical problem

Series expansion of the pressure in powers of chemical potential

$$\begin{aligned} \Delta P(\mu_u, \mu_d, T) &= P(\mu_u, \mu_d, T) - P(0, 0, T) \\ &= \sum_{m,n} \chi_{m,n}(T) \frac{\mu_u^m \mu_d^n}{m! n!}. \end{aligned}$$

Well studied classical problem. Special complications in QCD: few coefficients known, with errors. Questions:

- **Does this converge?** Estimate whether the series is summable, radius of convergence and location of nearest singularity.
- **Can the function be reconstructed?** More complicated: estimating value of the function, nature of divergence.

Our simulations

Lattice simulations with $N_f = 2$ staggered quarks and Wilson gauge action. Used $m_\pi \simeq 0.3 m_\rho$; spatial size $L = 4/T$. Temperature scale, T_c, found by the point at which χ_L peaks. If $T_c \simeq 170$ MeV, then $1/a = 0.7$ GeV, 1 GeV, 1.4 GeV for $N_t = 4$, 6 and 8.

Partial statistics reported in: QM 2012, Lattice 2013 **Datta, Gavai, SG: arXiv:1210.6784** Now doubling the statistics reported in Lattice 2013.

Our simulations

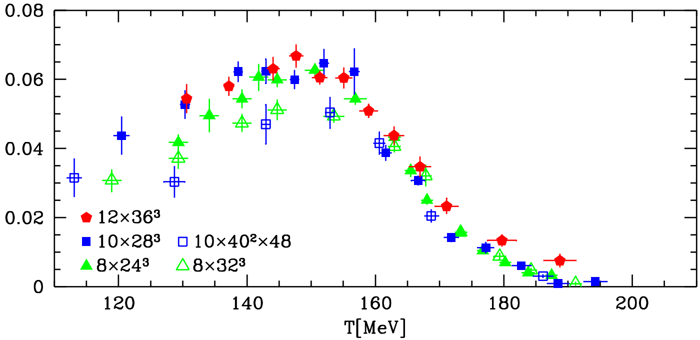
Lattice simulations with $N_f = 2$ staggered quarks and Wilson gauge action. Used $m_\pi \simeq 0.3 m_\rho$; spatial size $L = 4/T$. Temperature scale, T_c, found by the point at which χ_L peaks. If $T_c \simeq 170$ MeV, then $1/a = 0.7$ GeV, 1 GeV, 1.4 GeV for $N_t = 4$, 6 and 8.

Partial statistics reported in: QM 2012, Lattice 2013 **Datta, Gavai, SG: arXiv:1210.6784** Now doubling the statistics reported in Lattice 2013.

Notation: $z = \mu_B/T$ and

$$\chi_{20}(\mu_B, T) = \frac{\partial^2 \Delta P}{\partial \mu_B^2} = \chi^0(T) + \frac{T^2}{2!}\chi^2(T) z^2 + \frac{T^4}{4!}\chi^4(T) z^4 + \cdots$$

On T_c

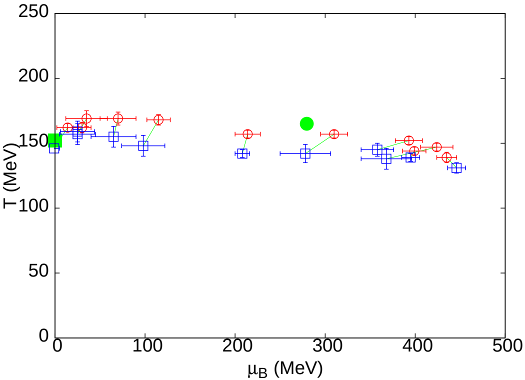


Broad crossover: even with one single measure (figure: chiral susceptibility) T_c uncertain by 20 MeV. Reflected in quoted values.

Aoki, Borsanyi, Dürr, Fodor, Katz, Krieg, Szabo: JHEP 0906 (2009) 088

Select any definition and stick with it: we use Polyakov loop susceptibility.

Freezeout: not unique

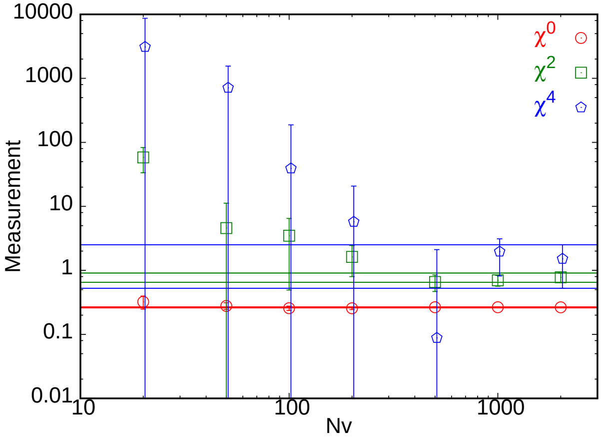


Sandeep Chatterjee, Rohini Godbole and SG

Also see argument by Jajati Nayak and Sandeep Chatterjee

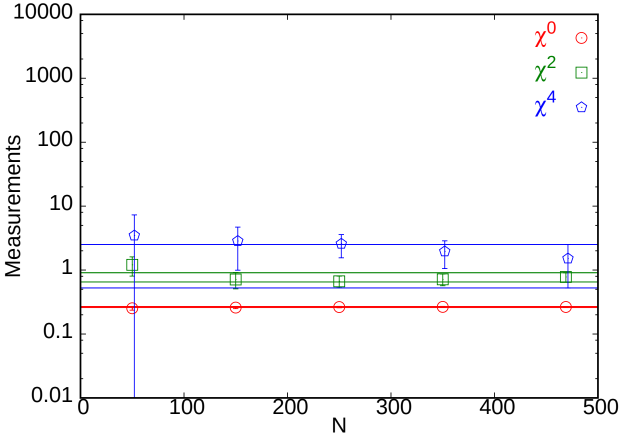
Upsilon freezes out at $T \simeq 250$ MeV at LHC **Rishi Sharma and SG**.
Fluctuations freeze out when Peclet number reaches about 1.

Numerical errors



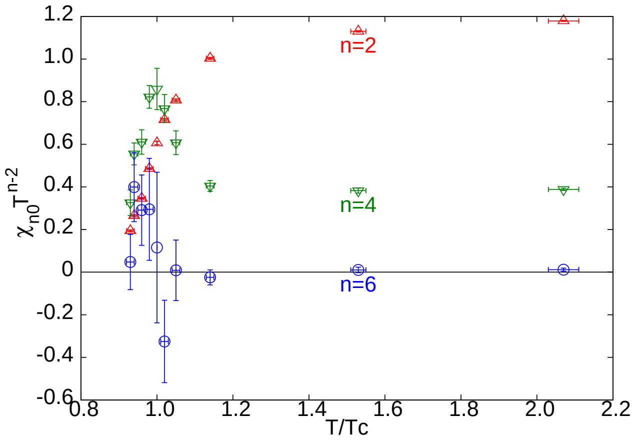
Errors depend on number of fermion sources for evaluation of propagator as well as number of gauge configurations. Multiple fermion loops are source hungry.

Numerical errors



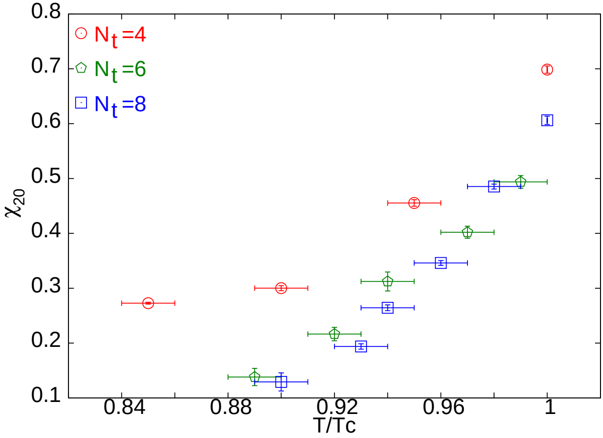
Errors depend on number of fermion sources for evaluation of propagator as well as number of gauge configurations. Multiple fermion loops are source hungry.

Susceptibilities at $\mu = 0$



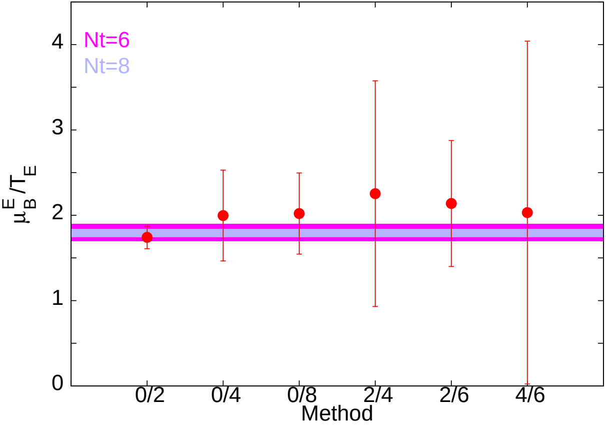
ILGTI result

Nearing continuum physics



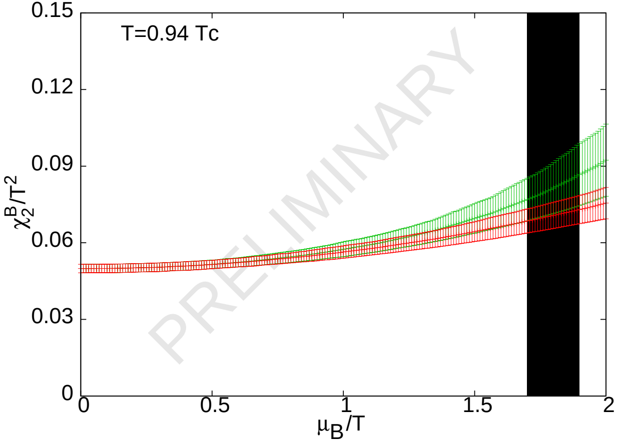
ILGTI result

The radius of convergence



ILGTI result

Must resum a series expansion



Truncated series sum is regular even at the radius of convergence, so is missing something important.

Critical behaviour and the pressure

At a critical point

$$\chi_B = (z_*^2 - z^2)^{-\psi}.$$

Continuity and finiteness of P at the CEP forces $\psi \leq 1$. Also

$$m_1(z) = \frac{d \log \chi_B}{dz} \simeq \frac{2\psi z}{z_*^2 - z^2}.$$

Convert series for χ_B to series for m_1, and use it to estimate the critical exponent. Use 2 terms to fix 2 parameters, remaining terms serve as checks of critical behaviour.

From the Padé approximant to $m_1(z)$, integrate to find χ_B and again twice to find ΔP.

Critical region and critical slowing down

If $\chi_B(z) \simeq (z_* - z)^{-\psi}$, then $m_1 = d\log\chi_B/dz$ has a pole. Series expansion of χ_B gives series for m_1. Resum series into a Padé approximant:

$$[0,1]: \qquad m_1(z) = \frac{c}{z_* - z}$$

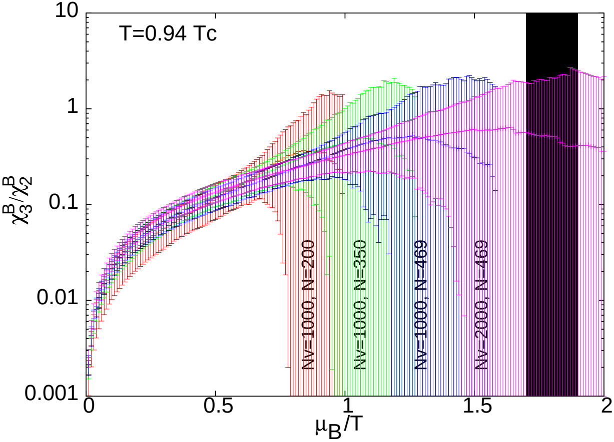
Width of the critical region? If we define it by

$$\left|\frac{m_1(z)}{m_1(0)}\right| > \Lambda,$$

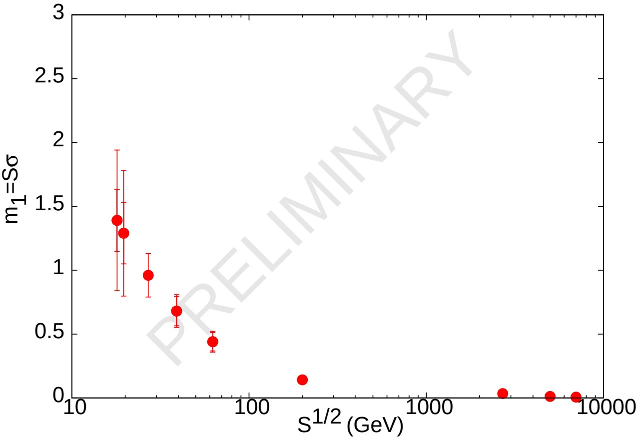
then $|z - z_*| \le z_*/\Lambda$.
If δ is fractional error in measurement of z_*, then error in Padé? Easy to check

$$\left|\frac{\Delta m_1}{m_1}\right| > \frac{1}{1-\Lambda\delta}.$$

Critical slowing down

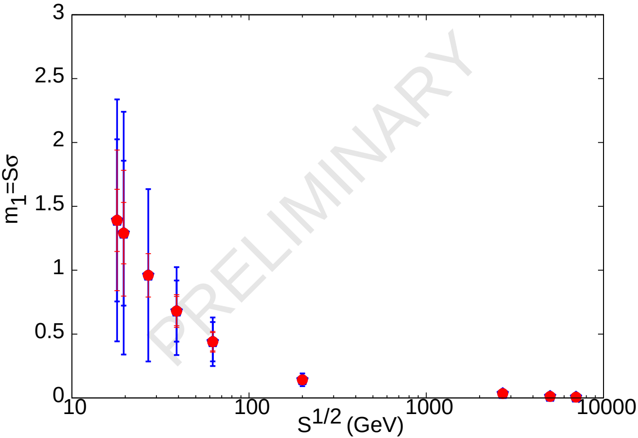


Predictions along the freezeout curve



Interesting region inside the critical region: hence large errors.

Predictions along the freezeout curve



Interesting region inside the critical region: hence large errors.

Critical exponents: Widom scaling

Widom scaling for the order parameter gives

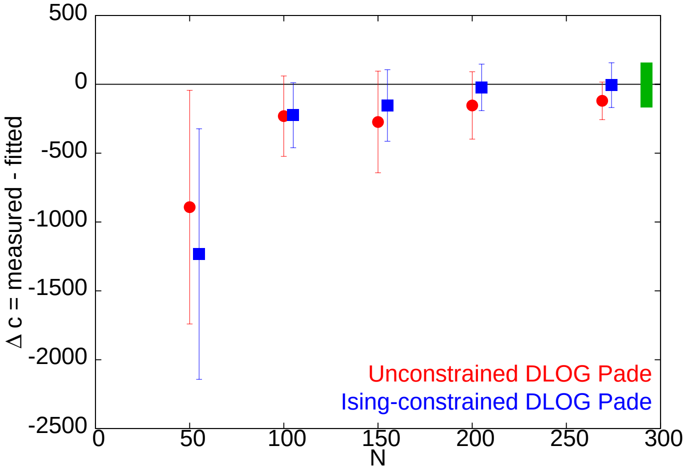
$$|\Delta\mu| = |\Delta n|^{\delta} J\left(\frac{|\Delta T|}{|\Delta n|^{1/\beta}}\right),$$

where $\Delta T = T - T_E$ and $\Delta\mu = \mu - \mu_E$. For $\Delta T = 0$ one finds $\Delta n \propto |\Delta\mu|^{1/\delta}$ in the high density phase. Then clearly one has

$$\psi = 1 - \frac{1}{\delta}.$$

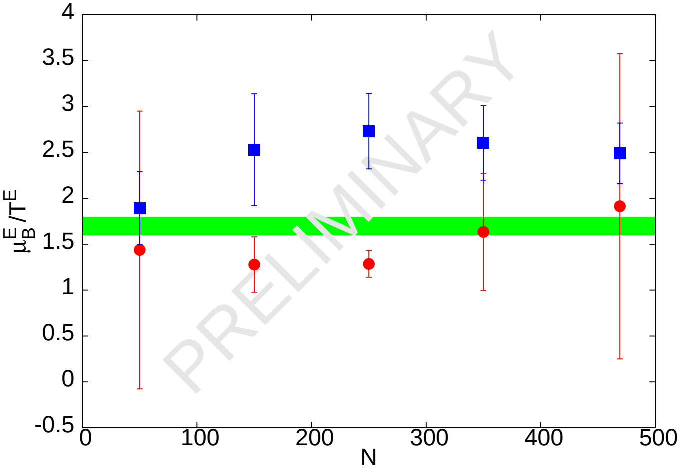
For the 3d Ising model, $\delta = 1.49$, so $\psi = 0.79$. In mean field theory one has $\delta = 3$, so $\psi = 0.66$. Our computations consistent with both: cannot distinguish between them yet.

Testing the DLOG Pade



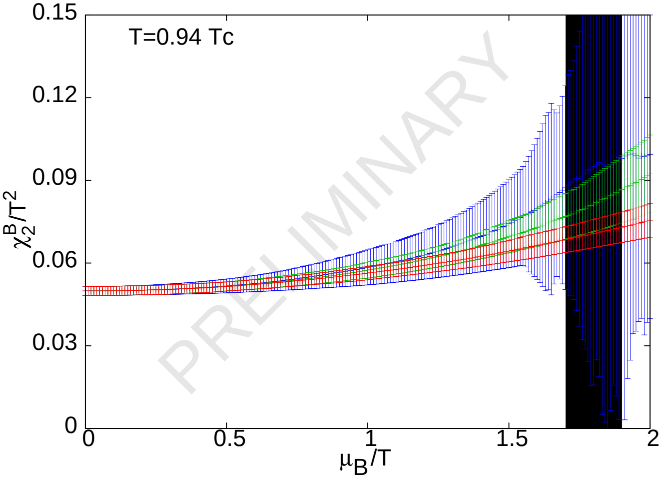
Padé uses 2 terms of the series for m_1. Using these it predicts the 3rd: critical behaviour.

Radius of convergence is critical point



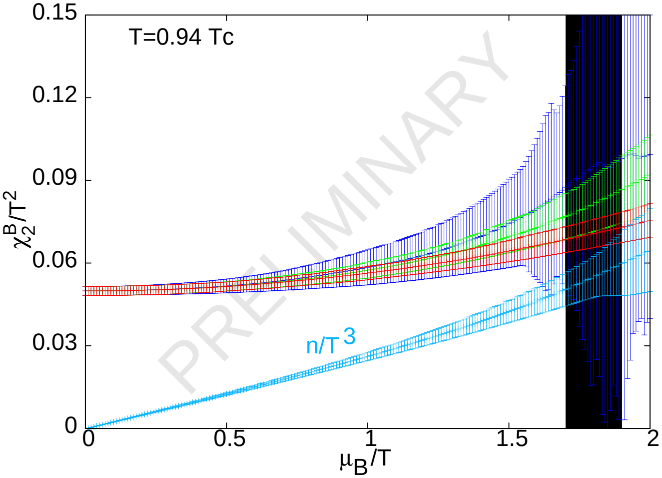
Position of pole agrees with radius of convergence.

The pressure



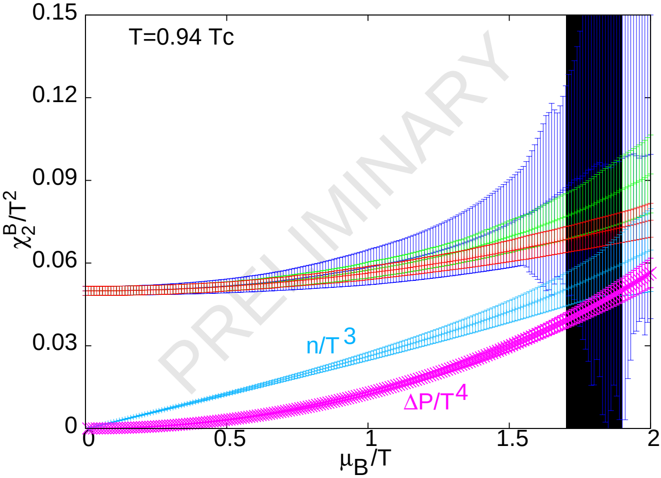
Integrated m_1 to find χ_2.

The pressure



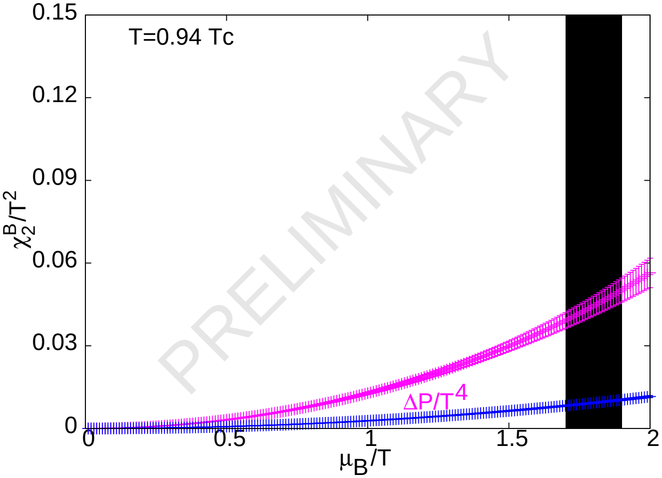
Integrated m_1 to find χ_2.

The pressure



Integrated m_1 to find χ_2.

The pressure



Integrated m_1 to find χ_2.

Critical point and the pressure

- QNS require huge CPU expenses; we have up to the 8th order. Momentum cutoff of 0.7 GeV, 1 GeV and 1.4 GeV. Able to see the approach to the renormalized values: $T^E \simeq 0.94\, T_c$, $\mu_B^E / T^E \simeq 1.7$.
- When the series diverges then ΔP at finite μ_B cannot be obtained from a partial resummation of the series.
- Since $\chi_B \simeq |\mu_B - \mu_B^E|^{-\psi}$, the ratio $m_1 = \chi_B' / \chi_B$ has a simple pole. Resum the series expansion into a simple pole. Integrate this to find χ_B and ΔP. First results for pressure at finite μ_B are reported.
- Lattice uses m_1 along a path of constant T and varying μ_B. Event-to-event fluctuations of baryon number can measure m_1 along the freezeout curve. At high energies, this can be used to estimate the critical point or the critical index ψ.