P-4811 Autumn 2015-2016

Physics of Standard Model: Part I

Homework # 4 **Due Date: 06/11/2015**

4.1 In the class I claimed that gauge symmetry automatically guarantees a global symmetry and a corresponding conserved charge associated with it. Let us demonstrate it.

(a) Write down the gauge invariant Lagrangian density for a massless spin 1 field and a massive complex scalar field that interact with it. Identify the global symmetry (use α to denote the parameter of transformation) and find the Noether current associated with it.

(b) You should notice at this point that the term in the Lagrangian density linear in A^μ is just $A^\mu J_\mu$. This is a general feature. Can you make sense out of it?

4.2 Let us try to understand the massless limit of spin 1 particle in more detail. The massless spin 1 field is quantized using two polarizations. These are the transverse polarizations and are physical. A sample basis (for p^μ in the Z-direction) is

$$\epsilon_1^\mu(p) = (0,1,0,0) \qquad \text{and } \epsilon_2^\mu(p) = (0,0,1,0) \tag{48}$$

these satisfy $\epsilon^*_{i\mu}\epsilon^\mu_j = -\delta_{ij}$, and $\epsilon_i \cdot p = 0$. Two orthogonal polarizations are

$$\epsilon_f^\mu(p) = (1,0,0,1) \qquad \text{and } \epsilon_b^\mu(p) = (1,0,0,-1) \tag{49}$$

You can see that these are not normalizable modes and hence are unphysical. Now, you can't really ignore these polarizations though! Let's see why.

We have shown in the class that the little group goes to $ISO(2)$ for massless case. Find an example, where a member of this group mixes the physical polarizations to the unphysical one. In particular find a Lorentz transformation Λ, such that:

$$\begin{aligned} \Lambda^\mu_\nu p^\nu &= p^\mu \qquad \text{for} \qquad p^\mu = (E,0,0,E) \\ \Lambda^\mu_\nu \epsilon_1^\mu(p) &= c_{11}\epsilon_1^\mu(p) + c_{12}\epsilon_2^\mu(p) + c_{13}\epsilon_f^\mu(p)\ , \end{aligned} \tag{50}$$

The first line confirms that Λ is a member of the little group. c_{ij} are numbers, and you only need to find an example where $c_{13} \neq 0$

Note: This is a disaster. If you scatter a photon using the field A^μ, the matrix element must depend on the polarization, *i.e.*, $\mathcal{M} = \epsilon_\mu M^\mu$, where you have carefully prepared the polarization ϵ to be a linear combination of physical modes. However, under Lorentz transformation

$$\mathcal{M} \to \mathcal{M}' = \epsilon'_\mu M'^\mu = \cdots + \epsilon_{f\mu} M'^\mu \tag{51}$$

Since ϵ_f is not in our Hilbert space, there is no physical polarization for which the matrix element is same in the new frame as in the old frame. There is only one solution, since $\epsilon_f \propto p^\mu$, we must have $p_\mu M^\mu = 0$, for a Lorentz invariant matrix element. This is **Ward Identity**.

4.3 In order to calculate anything with a photon, we are going to need to know its propagators $\Pi^{\mu\nu}$, defined by

$$\langle 0|T\Big\{A^\mu(x)A^\nu(y)\Big\}|0\rangle \;=\; i\int \frac{d^4p}{(2\pi)^4}\; e^{ip(x-y)}\;\Pi^{\mu\nu} \tag{52}$$

evaluated in the free theory. Derive it.