Outline

Lecture 1

Review of QM:

(a) A physical state is represented by the family of elements $e^{i\alpha}|\psi\rangle$, (α real), and $\langle\psi|\psi\rangle = 1$; $|\psi\rangle \in \mathcal{H}$ (a Hilbert space).

(b) Observables are represented by Hermitian operators

(c) Probability of finding a system originally in $|\psi_0\rangle$ in a state $|\psi_n\rangle$ is

$$P(\psi_0 \to \psi_n) = |\langle\psi_n|\psi_0\rangle|^2 \; ; \quad \sum P_n = 1 \quad \text{if } |\psi_n\rangle \text{ form a complete set.}$$

Symmetry transformation:

A symmetry transformation is a change of point of view that doesn't change result of possible experiment.

A necessary condition

$$P(\psi_i \to \psi_f) = P(\psi_i' \to \psi_f')$$

Wigner proved for these transformations $|\psi'\rangle = \Theta|\psi\rangle$

where (a) Θ is unitary and linear

or (b) Θ is antiunitary and antilinear

Example:

A system of 3 states is found to be invariant under $(1 \to 2 \to 3)$ $\equiv a$ and $(1 \leftrightarrow 2)$ $\equiv b$

we showed that a^2, ba and ba^2 are also symmetries

in fact $\{\mathbb{1}, a, a^2, b, ba, ba^2\}$ → form a group S_3

we found a three dimensional representation of S_3.