Lecture 15 Outline

Scattering theory (Continued).

Next we consider full ~~seat~~ Hamiltonian with

(a) spectrum bounded from below

(b) lowest lying state not part of continuum

(c) physical vacuum denoted by $|0\rangle_p$ satisfy $H|0\rangle_p = 0$, $\vec{P}|0\rangle_p = 0$, ${}_p\langle 0|0\rangle_p = 1$

Introducing $\mathcal{H}_g \equiv \mathcal{H} - \rho(x)\,\Phi(x)$.

$$Z[\rho] \equiv {}_p\langle 0|S|0\rangle_p \quad \rightarrow \quad G^{(n)}(x_1 \ldots x_n) = \frac{1}{i^n}\left.\frac{\delta^n Z[\rho]}{\delta\rho(x_1)\ldots\delta\rho(x_n)}\right|_{\rho=0}$$

(i) We showed that

$$G_1^{(n)}(x_1 \ldots x_n) = G_F^n(x_1 \ldots x_n) \longrightarrow \text{mindless sum of Feynman diagrams with free Hamiltonian}$$

starting with

$$Z_F^n[\rho] = \frac{\langle 0|T\{\exp(-i\int_{-\infty}^{\infty} d^4x(\mathcal{H}_I - \rho\Phi_I)\}|0\rangle}{\langle 0|T\{\exp(-i\int_{-\infty}^{\infty} d^4x\,\mathcal{H}_I)\}|0\rangle}$$

$\rightarrow$ H_0 eigenstate

(ii) LSZ

$$\langle \ell_1 \ldots \ell_r|S-1|k_1 \ldots k_r\rangle = \prod_{a=1\ldots r}\frac{\ell_a^2-\mu^2}{i}\prod_{b=1\ldots s}\frac{k_b^2-m^2}{i}\,\tilde{G}'^{(r+s)}(-\ell_1,\ldots,\ell_r,k_1,\ldots k_s)$$

~~where~~ guaranteed for any Green's fn constructed with a field ϕ', satisfying

$$\langle 0|\Phi'(x)|0\rangle = 0$$
$$\langle k|\Phi'(x)|0\rangle = e^{ik\cdot x}$$

fields getting a vev problematic $\longrightarrow$ $\phi'(x) = \frac{1}{\langle k|\Phi(0)|0\rangle}\{\Phi(x) - \langle 0|\Phi(0)|0\rangle\}$

also normalization

satisfied all

(iii) Toy model

$$\mathcal{L} = \tfrac{1}{2}(\partial_\mu\phi')^2 - \frac{\mu_0^2}{2}\phi'^2 + \partial_\mu\psi'^*\partial^\mu\psi' - m^2\psi'^*\psi' - g_0\psi^*\psi\phi + \mathcal{L}_{CT}$$

$$\mathcal{L}_{CT} = \cancel{A\,\partial_\mu\phi^2}\; A\phi' + \frac{B}{2}(\partial_\mu\phi)^2 - \frac{C}{2}\phi'^2 + D\,\partial_\mu\psi^{*\prime}\psi' - E\psi^{*\prime}\psi - F\psi^{*\prime}\psi'\phi'$$

+ const.

(1) we determined A perturbatively

(2) $+ B, C, D, E \rightarrow$ we derived the mass and normalization conditions from Green's fn

$\rightarrow$ found spectral decomposition