Lecture 16 — Outline

We introduced a toy model in the last class, which we continue using it

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi')^2 - \frac{\mu^2}{2}\phi'^2 + \partial_\mu \psi^{*\prime} \partial^\mu \psi' - m^2 \psi'^* \psi' - g\, \psi^{*\prime}\psi^{*}\phi + \mathcal{L}_{CT}$$

where

$$\mathcal{L}_{CT} = A\phi' + B/_2\, (\partial_\mu \phi')^2 - \frac{C}{2}\phi'^2 + D\, \partial_\mu \psi^{*\prime}\partial^\mu \psi' - E\, \psi^{*\prime}\psi' - F\, \psi^{*\prime}\psi'\phi' + \text{Const.}$$

Six new parameters and six renormalization conditions

1. $\langle 0 | \phi' | 0 \rangle = 1$
2. $\langle q | \phi'(0) | 0 \rangle = 1$
3. $\langle \psi | \psi'(0) | 0 \rangle = 1$
4. ϕ mass $= \mu$
5. ψ mass $= m$
6. g is defined sensibly

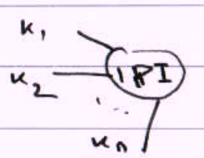
Need to express conditions (2-6) in terms of Green's function.

$$\tilde{G}^2(k_0, k') = \overset{k\rightarrow}{-}\!\!\!\bigcirc\!\!\!\overset{\leftarrow k'}{-} = (2\pi)^4\, \delta^4(k+k')\, \underbrace{D'(k^2)}_{\text{"renormalized propagator"}}$$

ϕ mass $= \mu \iff D'$ has a pole in μ.

$\langle q | \phi(0) | 0 \rangle = 1 \iff$ residue at the pole $= i$

Introduced



$\equiv$ Sum of all connected graphs that cannot be disconnected by cutting a single line

$$\Rightarrow \quad D'(k^2) = \frac{i}{k^2 - \mu^2 - \Pi'(k^2) + i\epsilon} \quad \text{where} \quad -(\text{1PI})- \;\equiv\; i\,\Pi'(k^2)$$

$\Pi'(k^2)$: self energy

$D'(k^2)$ has a pole $\iff \Pi'(\mu^2) = 0$

residue at pole $= i \iff \left.\frac{d\Pi'}{dk^2}\right|_{\mu^2} = 0$

We determine B and C parameters order by order in perturbation.

Show example: $\Pi'(k^2)$ at $\mathcal{O}(g^2)$. $\Rightarrow -i\Pi'(k^2) = -(\text{1PI})- = -\bigcirc- + \overset{(2)}{-\times-}$

$$= i\Pi_f(k^2) + iB_2 k^2 - iC_2$$

$$\Pi_f(k^2) = \frac{g^2}{16\pi^2}\int_0^1 dx\, \ln\left(-k^2 x(1-x) + m^2 - i\epsilon\right)$$

+ terms that vanish in convergent conditions.

$$\Pi'(k^2) = \frac{g^2}{16\pi^2}\int_0^1 dx \left[\ln \frac{-k^2 x(1-x) + m^2 - i\epsilon}{-\mu^2 x(1-x) + m^2 - i\epsilon} + \frac{(k^2-\mu^2)(x(1-x))}{-\mu^2 x(1-x) + m^2} \right\}$$