Lecture 18 — Summary

We discussed the construction of classical Lagrangian for ~~mass~~ spin 1 degrees of freedom both in the massless as well as massive case.

We showed that a Lagrangian

$$\mathcal{L} = \frac{a}{2} A^\mu \Box A_\mu + \frac{b}{2} A^\mu \partial_\mu \partial^\nu A_\nu + \frac{1}{2} m^2 A^\mu A_\mu$$

describes the right d.o.f for spin 1 case if $a+b=0$

We constructed solution from the eom in terms of basis vectors (ϵ^i_μ) that automatically forces $A_\mu(x)$ to satisfy $\partial_\mu A^\mu = 0$

$$p_\mu = (E,0,0,p_z) \rightarrow \begin{matrix} \epsilon^1_\mu \\ \epsilon^2_\mu \end{matrix} \left.\begin{matrix} = (0,1,0,0) \\ = (0,0,1,0) \end{matrix}\right. \text{ or } \left\{\begin{matrix} \frac{1}{\sqrt{2}}(0,1,i,0) \\ \frac{1}{\sqrt{2}}(0,1,-i,0) \end{matrix}\right\} \equiv \begin{matrix} \epsilon^R_\mu \\ \epsilon^L_\mu \end{matrix}$$

and $\epsilon^L_\mu = \left(\frac{p_z}{m}, 0, 0, \frac{E}{m}\right) \xrightarrow[E \gg m]{} \frac{E}{m}(1,0,0,1)$

$\Rightarrow$ Scattering of ϵ^L_μ modes $\rightarrow$ $d\sigma \sim g^2 (\epsilon^L)^2 \sim \frac{g^2 E^2}{m^2}$

blows up at $E \gg \frac{m}{g}$

$\rightarrow$ representation of the theory is sick.

Massless case:

An extra redundancy of the theory $A_\mu \rightarrow A_\mu + \partial_\mu \alpha(x)$

$\rightarrow$ gauge transformation.

only ϵ^1_μ and ϵ^2_μ (or $\epsilon^{R,L}_\mu$) are physical $\rightarrow$ gauge fixing to get the right # d.o.f.

$\epsilon^L_\mu \rightarrow$ pure gauge

for spin 0 – spin 1 (say ϕ) interactions to be present $\rightarrow$ need ϕ also to transform.

gauge invariance (not real) $\xrightarrow{\text{implies}}$ existence of global symmetry (physical) $\rightarrow$ has an associated current and charge

$\hookrightarrow$ only physical is spectrum.