Lecture 19 Summary.

Half of the class-time was spent on revisiting and summarizing the results obtained so far. This part of the discussion ended on the topic of how to construct classical Lagrangian given the degree of freedom and associated symmetries.

In the rest of the lecture we applied this procedure for spin 0, & spin 1/2 system.

Spin 0

In order to allow any non-trivial internal symmetry, we must have more than one real scalar field (or at least one complex scalar field.)

for N Complex scalar ϕ's. symmetry group $U(1) \times SU(N)$.

invariants include.

functions of $\phi^* \phi$ or $\epsilon^{i_1 \dots i_n} \phi_1 \dots \phi_n$.

Spin 1/2

at least two $(1/2, 0)$ spinors are needed in order to have a massive system with internal symmetries (continuous)

Consider a $\psi \equiv \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} \rightarrow$ massless case $\rightarrow$ symmetry $U(1)_A \times SU(2)$

mass. $m_{ij} \psi_i \psi_j \rightarrow$ m_{ij} $\rightarrow$ arbitrary $\rightarrow$ no symmetry

only m_{11} nonzero $\rightarrow U(1)_B$

$m_{ij} = 0 \;\forall\; i = j \rightarrow U(1)_C$

$\Rightarrow$ a mass term completely destroys global $SU(2)$.

Note: $U(1)_A$, $U(1)_B$, $U(1)_C$ are different $U(1)$ symmetries.