# Lecture 3

Outline.

Review of earlier classes:→ restated the problem we are trying to address; (namely trying to understand the symmetry structure of a system of 3 physical states, where observables are invariant under arbitrary mixture of physical states.

$\boxed{\text{back to lie Algebra:}}$ → which ~~generates~~ gives algebra of of generators.

$$[x_a, x_b] = i f_{abc}\, x_c$$

↓ generators generate Lie group

we learned that

describes the ← symmetry structure of the problem.

① $x_a^+ = x_a$ ; ⇒ $f_{abc}$ real

② $f_{bcd} f_{ade} + f_{cad} f_{bde} + f_{abd} f_{cde} = 0$

② $\det(D(\alpha)) = 1$ ⇒ $Tr(x_a) = 0$

④ $x_a$ form their own vector space where scalar product can be defined as $Tr[x_a, x_b]$

⑤ with a change of basis (linear transformation: we can go to a basis where $Tr[x_a, x_b] = \lambda\, \delta_{ab}$ ⟶ (only in our problem)

we will use $\lambda = \frac{1}{2}$

→ not true if one of eigenvalue is <0

⑥ in this specific basis $f_{abc} = -i \frac{1}{\lambda} Tr([x_a, x_b]\, x_c)$

⇒ $f_{abc}$ is completely antisymmetric ~~tensor~~.

At this point we are ready to represent states in term of symmetry transformation generators:

Since $Tr[x_a] = 0$ ⇒ there can be at most 2 diagonal generators.
$a = 1(1)8$

Call them $H_n \equiv \{H_1, H_2\}$.

then states can be ~~represented~~ ~~as~~ represented by basic vector $|e_1\rangle, |e_2\rangle, |e_3\rangle$ which are eigenvectors of $H_n$

$$H_n |e_i\rangle = \mu_i^n |e_i\rangle \Rightarrow |e_i\rangle \cong |\{\mu_i^1, \mu_i^2\}\rangle$$

↳ weight of states.