Lecture 4 outline

We started with the review of earlier classes — restating the problem we are trying to understand (namely understanding the symmetry structure of a system of 3 physical states, where observables are invariant under arbitrary mixture of physical states.

(a) We found the Cartan generators of the groups.

(b) we learned that states can be represented by weights: i.e. eigenvalues of Cartan generators, namely $\vec{H}|e_i\rangle = \vec{\mu}|e_i\rangle$

(c) what remains is to label /understand /construct the remaining generators.

Can Operators be designated by weights of states?

we constructed example operators such as $E_{1\to 2}$ defined via

$\langle 2|E_{1\to 2}|1\rangle = 1$ and $=0$ for all other elements

$\Downarrow$

in matrix $\begin{pmatrix} 0&0&0\\ 1&0&0\\ 0&0&0 \end{pmatrix}$ where states are represented by $|1\rangle = \begin{pmatrix}1\\0\\0\end{pmatrix}$, $2 = \begin{pmatrix}0\\1\\0\end{pmatrix}$, $3 = \begin{pmatrix}0\\0\\1\end{pmatrix}$.

Then, we showed that.

$$[H_n, E_{i_0\to j_0}] = (\mu^n_{j_0} - \mu^n_{i_0})\, E^{+}_{i_0\to j_0}$$

$\Rightarrow$ these operators can be represented by weight vectors $(\vec{\mu}_{j_0} - \vec{\mu}_{i_0})$.

However there is no guarantee that these operators furnishes Lie Algebra.

(d) We constructed a complex valued vector space $\mathcal{H}_A$ such that if

$$X_a \in \mathcal{H}_A \Rightarrow \sum C_a X_a \in \mathcal{H}_A \quad \text{for } C_A = \text{Complex } \#$$

$\Rightarrow X_a|X_b\rangle = |[X_a, X_b]\rangle$

$\Rightarrow H_i|H_j\rangle = 0 \Rightarrow [H_i, H_j] = 0$

for eigenvectors of H_n

$H_n|X_{\alpha_n}\rangle = \alpha_n|X_{\alpha_n}\rangle \Rightarrow [H_n, X_{\vec{\alpha}}] = \alpha_n X_{\vec{\alpha}}$ → we can use these as basis vectors.

we also need scalar product defined for vector space. A convenient one is

$\langle X_a|X_b\rangle = k\,\mathrm{Tr}[X_a^\dagger X_b]$