Lecture 6

Outline.

A Lie algebra is characterized by ℓ commuting generators

$$[H_i, H_j] = 0 \qquad H_i^\dagger = H_i$$

and $(N_a - 1)$ generators labeled by ℓ dimensional vectors $\vec{\alpha} \equiv \{\alpha_1, \dots, \alpha_\ell\}$

$$[H_i, E_{\vec{\alpha}}] = \underbrace{\alpha_i}_{\to \text{ root vectors}} E_{\vec{\alpha}}$$

for every $\vec{\alpha}$, there is a $(-\alpha)$ a root vector.

$$[E_{\bar{\alpha}}, E_{-\bar{\alpha}}] = \alpha_i H_i$$

$$[E_{\bar{\alpha}}, E_{\bar{\beta}}] = \begin{cases} 0 & \text{if } \bar{\alpha}+\bar{\beta} \text{ isn't a root} \\ N_{\alpha\beta} E_{\bar{\alpha}+\bar{\beta}} & \text{if a root} \end{cases}$$

Positivity: sign of first non-zero component of $\vec{\alpha}$

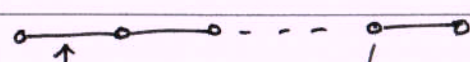
Simple: if it can't be expressed a positive combination of other simple roots.

if $\bar{\alpha}_1 = \underbrace{\bar{\alpha}_2 + \bar{\alpha}_3}_{\hookrightarrow \text{ simple.}}$ $\quad \alpha_1, \alpha_2, \alpha_3 \to$ positive

basis independent measure: $A_{ij} = 2\dfrac{\bar{\alpha}_i \cdot \bar{\alpha}_j}{(\alpha_i \cdot \alpha_i)}$ $\quad$ Cartan Matrix

$\to$ simple roots.

SU(N) characterize by simple roots at an angle $120^\circ \to$



120° representation

simple root

Dynkin diagram.

Tensor method

under symmetry transformation

$$T_a |_i\rangle = (X_a)_{ji} |_i\rangle \qquad \text{and} \qquad |^i\rangle = \epsilon^{ijk} |_{jk}\rangle \to (-X_a)^i_j |^j\rangle$$

a general tensor built using (n, m)

m anti fundamental indices and n fundamental indices

(upper) $\qquad$ (lower)

$$T_a \left|{}^{i_1 \dots i_m}_{j_1 \dots j_n}\right\rangle = \sum_{\ell=1}^{n} (X_a)^{j_k}_{j_\ell} \left|{}^{i_1 \dots i_m}_{j_1 \dots j_{\ell-1} j_k j_{\ell+1} \dots j_n}\right\rangle - \sum_{\ell=1}^{m} (X_a)^{i_\ell}_{i_k} \left|{}^{i_1 \dots i_{\ell-1} i_k i_{\ell+1} \dots i_m}_{j_1 \dots j_n}\right\rangle$$

$\parallel$

(n, m)

Invariants: $\left\{ \begin{matrix} \epsilon^{ijk} |_{ijk}\rangle \\ \epsilon_{ijk} |^{ijk}\rangle \end{matrix} \right.$ and $\delta^i_j |^j_i\rangle$ $\Big\}$ for SU(3)

&

We also learned: (a) How to construct Higher dimensional irreducible repn using tensor method

(b) More about Young Tableaux

Young Tableaux