Lecture 8 Outline

We began with the discussion of Lorentz group, after which we started talking about Poincare group.

Complete space-time symmetry

$$x^\mu \to x'^\mu = \Lambda^\mu{}_\nu x^\nu + a^\mu$$

$\Rightarrow$ The group is characterized by (Λ, a)

(1) $(\Lambda_2, a_2)(\Lambda_1, a_1) = (\Lambda_2\Lambda_1, \Lambda_2 a_1 + a_2)$

(2) $(\Lambda, a)^{-1} = (\Lambda^{-1}, -\Lambda^{-1}a)$

$(\mathbb{1}, a) \to$ Translation group $\equiv T[a]$ is contained within Poincare group

In quantum theory the associated unitary operator

$U[\Lambda, a]$ such that $U[\Lambda_2, a_2]\, U[\Lambda_1, a_1] = U[\Lambda_2\Lambda_1, \Lambda_2 a_1 + a_2]$

for infinitesimal transformation $U[\Lambda, a] = 1 - i/_2\, \omega^{\mu\nu} M_{\mu\nu} + i a^\mu P_\mu$

we found that

(a) $[M_{\mu\nu}, M_{\rho\sigma}] = i\left(g_{\mu\sigma} M_{\nu\rho} + g_{\nu\rho} M_{\mu\sigma} - g_{\mu\rho} M_{\nu\sigma} - g_{\nu\sigma} M_{\mu\rho}\right)$

(b) $[M_{\mu\nu}, P_\sigma] = i\left(g_{\nu\sigma} P_\mu - g_{\mu\sigma} P_\nu\right)$

(c) $[P_\mu, P_\nu] = 0$

using $J_i = \frac{1}{2}\epsilon_{ijk} M_{jk}$, $K_i = M_{0i}$, $P^\mu = (H, \vec{P})$

we showed

$[J_i, J_j] = i\epsilon_{ijk} J_k$

$[J_i, K_j] = i\epsilon_{ijk} K_k$

$[J_i, P_j] = i\epsilon_{ijk} P_k$

$[J_i, H] = 0$

$[K_i, K_j] = -i\epsilon_{ijk} K_k$

$[K_i, P_j] = i\delta_{ij} H$

$[K_i, H] = iP_i$