Lecture 9

Poincare group:

The complete space time symmetry includes translation as well as homogeneous Lorentz transformation.

$$x^\mu \xrightarrow[(\Lambda,a)]{} x'^\mu = \Lambda^\mu{}_\nu x^\nu + a^\nu$$

These transfomations from a group since

(i) $(\Lambda_2, a_2)\,(\Lambda_1, a_1) = (\Lambda_2 \Lambda_1,\ \Lambda_2 a_1 + a_2)$

(ii) $(\Lambda_2, a)^{-1} = (\Lambda^{-1}, -\Lambda^{-1} a)$

etc.

The translation group: $(\mathbb{1}, a)$ is contained inside Poicare group, so is the Lorentz group: $(\Lambda, 0)$

The general element of Poicare group can be written as

$$(\Lambda, a) = (\mathbb{1}, a)\,(\Lambda, 0)$$

In quantum theory the associated Unitary operator $U[\Lambda, a]$, such that

$$U[\Lambda_2, a_2]\ U[\Lambda_1, a_1] = U[\Lambda_2\Lambda_1,\ \Lambda_2 a_1 + a_2] \rightarrow \text{defn of rep.}$$

for infinitesimal Lorentz tranformation and translation

$$U[\Lambda, a] = 1 - i\tfrac{1}{2}\,\omega^{\mu\nu} M_{\mu\nu} + i\, a^\mu P_\mu$$

$$M_{\mu\nu}^\dagger = M_{\mu\nu} \qquad P_\mu^\dagger = P_\mu$$

$$M_{\mu\nu} + M_{\nu\mu} = 0$$

We found that

(a) $[M_{\mu\nu}, M_{\rho\sigma}] = i\,(g_{\mu\sigma} M_{\nu\rho} + g_{\nu\rho} M_{\mu\sigma} - g_{\mu\rho} M_{\nu\sigma} - g_{\nu\sigma} M_{\mu\rho})$

(b) $[M_{\mu\nu}, P_\sigma] = i\,(g_{\nu\sigma} P_\mu - g_{\mu\sigma} P_\nu)$

(c) $[P_\mu, P_\nu] = 0$